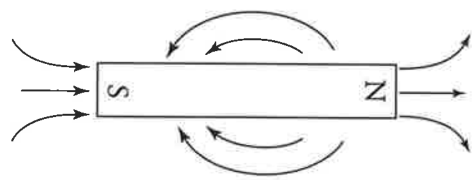
Chapter 7 Magnetism and Electromagnetism

INTRODUCTION

In a previous chapter, we learned that electric charges are the sources of electric fields and that other charges experience an electric force in those fields. The charges generating the field were assumed to be at rest, because if they weren't, then another force field would have been generated in addition to the electric field. Electric charges *that move* are the sources of **magnetic fields**, and other charges that move can experience a magnetic force in these fields.

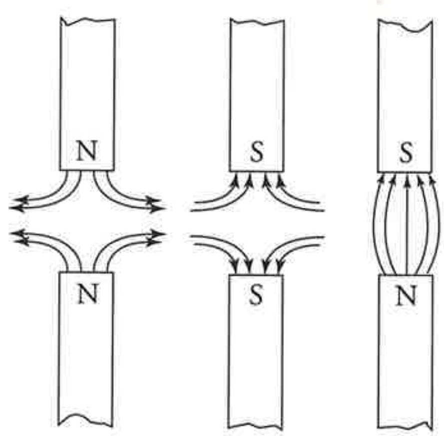
MAGNETIC FIELDS

Similar to our discussion about electric fields, the space surrounding a magnet is permeated by a magnetic field. The direction of the magnetic field is defined as pointing out of the north end of a magnet and into the south end of a magnet, as illustrated below.

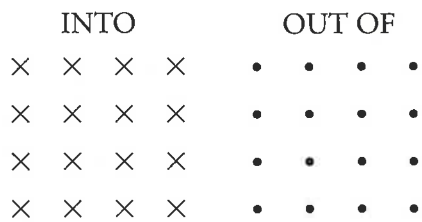


Magnetic fields have always been found to have both a north and a south pole, defined so that the field comes out of the north pole and enters back into the magnet at the south pole. If you look back at Chapter 5, electric fields could be created from a single positive or single negative charge. You will also see that a pair of charges, one positive and one negative, generated an electric dipole. Magnetic fields always have both a north and a south pole.

When two magnets get near each other, the magnetic fields interfere with each other and can be drawn as follows. Note that, for simplicity's sake, only the field lines closest to the poles are shown.

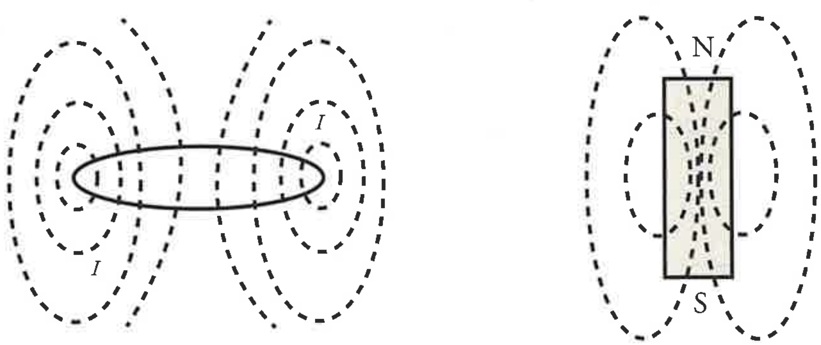


Notice there is a curve to the previous fields. We call the field uniform if the field lines are parallel and of equal strength. It is easy to recognize a uniform magnetic field to the right, left, top of the page, or bottom of the page. But you will also see a field going into or out of the page. A field into the page looks as if there were a north pole of a magnet above the page pointing down at the south pole of a magnet that is below the page. It is represented by an area with X's going into the page. A field coming out of the page looks as if there were a north pole of a magnet below the page pointing up at the south pole of a magnet that is above the page. It is represented by an area with dots (•) coming out of the page.



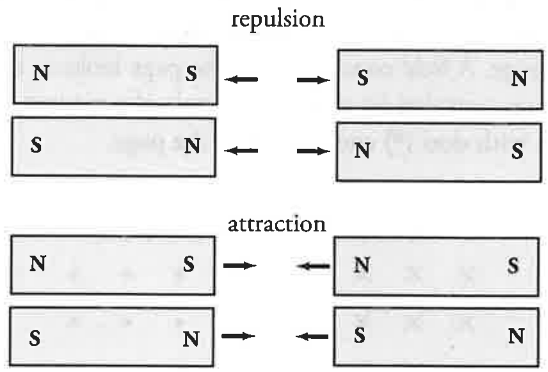
BAR MAGNETS

A permanent bar magnet creates a magnetic field that closely resembles the magnetic field produced by a circular loop of current-carrying wire:



By convention, the magnetic field lines emanate from the end of the magnet designated the north pole (N) and then curl around and reenter the magnet at the end designated the south pole (S). The magnetic field decreases in strength as distance from the magnet increases. The magnetic field created by a permanent bar magnet is due to the electrons, which have an intrinsic spin as they orbit the nuclei. They are literally charges in motion, which is the ultimate source of any and all magnetic fields. If a piece of iron is placed in an external magnetic field (for example, one created by a current-carrying solenoid), the individual magnetic dipole moments of the electrons will be forced to more or less line up. Because iron is ferromagnetic, these now-aligned magnetic dipole moments tend to retain this configuration, thus permanently magnetizing the bar and causing it to produce its own magnetic field. All materials have some **magnetic permeability**, μ, which determines how great a magnetic field an object will develop when placed in an external field.

As with electric charges, like magnetic poles repel each other, while opposite magnetic poles attract each other.



However, while you can have a positive electric charge all by itself, you can't have a single magnetic pole all by itself: remember, the existence of a lone magnetic pole has never been confirmed. That is, there are no magnetic monopoles; magnetic poles always exist in pairs. If you break a bar magnet into two pieces, it does not produce one piece with just an N and another with just an S; it produces two separate, complete magnets, each with an N-S pair.

THE MAGNETIC FORCE ON A MOVING CHARGE

If a particle with charge q moves with velocity **v** through a magnetic field **B**, it will experience a magnetic force, $\mathbf{F}_B$:

Equation Sheet

$$\mathbf{F}_B = q\mathbf{v} \times \mathbf{B}$$

with magnitude:

Equation Sheet

$$F_B = |q| vB \sin\theta$$

where θ is the angle between **v** and **B**. From this equation, we can see that if the charge is at rest, then $v = 0$ immediately gives us $F_B = 0$. This tells us that magnetic forces act only on moving charges. Also, if **v** is parallel (or antiparallel) to **B**, then $F_B = 0$ since, in either of these cases, $\sin\theta = 0$. So, only charges that cut across the magnetic field lines will experience a magnetic force. Furthermore, the magnetic force is maximized when **v** is perpendicular to **B**, since if $\theta = 90°$, then $\sin\theta$ is equal to 1, its maximum value.

The direction of $\mathbf{F}_B$ is always perpendicular to both **v** and **B** and depends on the sign of the charge q and the direction of $\mathbf{v} \times \mathbf{B}$ (which can be found by using the Right-Hand Rule).

If q is *positive*, use your *right* hand and the *Right*-Hand Rule.

If q is *negative*, you need your thumb to point in the direction of the product of q and v (which is the direction opposite of v), not just the direction of the velocity when you apply the Right-Hand Rule.

0 or 180
If the velocity **v** and the magnetic field **B** are parallel or antiparallel, which means oriented so the angle between the vectors is 180 degrees, the magnetic force $\mathbf{F}_B = 0$.

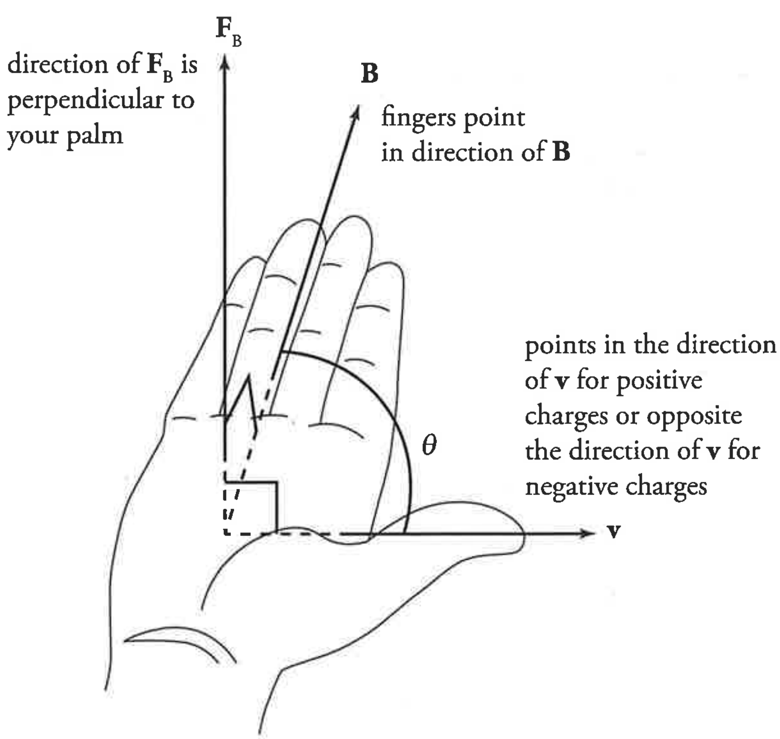
Whenever you use the Right-Hand Rule, keep your hand flat, and follow these steps:

1. Orient your hand so that your thumb points in the direction of the velocity **v**. If the charge is negative, turn your thumb by 180 degrees.
2. Without changing the direction of your thumb, rotate your hand to point your fingers in the direction of **B**.
3. The direction of $\mathbf{F}_B$ will then be perpendicular to your palm.

Think of your palm pushing with the force $\mathbf{F}_B$; the direction it pushes is the direction of $\mathbf{F}_B$.

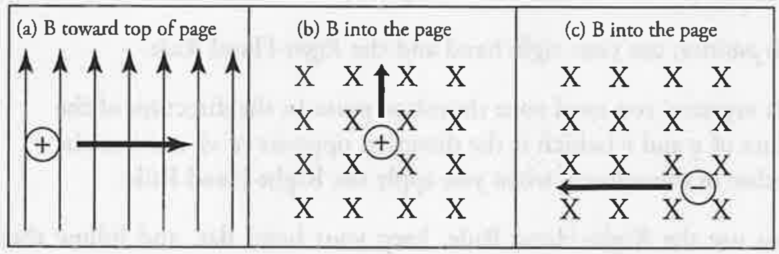
Right-Hand Rule:

For determining the direction of the magnetic force, $\mathbf{F}_B$, on a *positive* charge



Note that there are fundamental differences between the electric force and magnetic force on a charge. First, a magnetic force acts on a charge only if the charge is moving; the electric force acts on a charge whether it moves or not. Second, the direction of the magnetic force is always perpendicular to the magnetic field, while the electric force is always parallel (or antiparallel) to the electric field.

Example 1 For each of the following charged particles moving through a magnetic field, determine the direction of the force acting on the charge.



Solution.

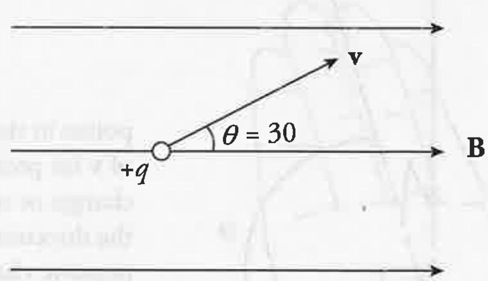
(a) If you point your fingers to the top of the page and thumb to the right of the page, your palm should point out of the page. The force is out of the page.

(b) If you point your fingers into the page and thumb to the top of the page, your palm should point to the left of the page. The force is to the left of the page.

(c) If you point your fingers into the page and thumb to the right of the page (remember, when the charge is negative, your thumb points in the opposite direction from the velocity), your palm should point to the top of the page. The force points up toward the top of the page.

The SI unit for the magnetic field is the **tesla** (abbreviated **T**), which is one newton per ampere-meter.

Example 2 A charge $+q = +6 \times 10^{-6}$ C moves with speed $v = 4 \times 10^5$ m/s through a magnetic field of strength $B = 0.4$ T, as shown in the figure below. What is the magnetic force experienced by q?

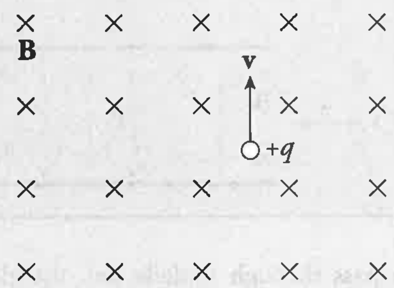


Solution. The magnitude of F_B is

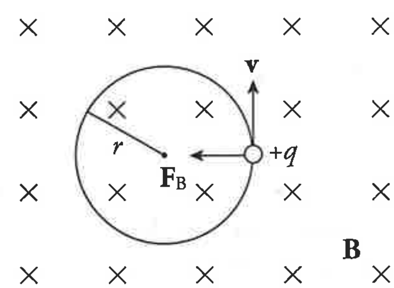
$$F_B = qvB \sin \theta = (6 \times 10^{-6}\text{ C})(4 \times 10^5\text{ m/s})(0.4\text{ T}) \sin 30° = 0.48\text{ N}$$

By the Right-Hand Rule, the direction is into the plane of the page, which is symbolized by ×.

Example 3 A particle of mass m and charge $+q$ is projected with velocity **v** (in the plane of the page) into a uniform magnetic field **B** that points into the page. How will the particle move?



Solution. Since **v** is perpendicular to **B**, the particle will feel a magnetic force of strength qvB, which will be directed perpendicular to **v** (and to **B**) as shown:

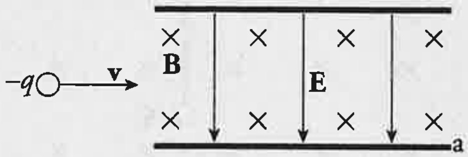


Since $\mathbf{F}_B$ is always perpendicular to **v**, the particle will undergo uniform circular motion; $\mathbf{F}_B$ will provide the centripetal force. Notice that, because $\mathbf{F}_B$ is always perpendicular to **v**, the magnitude of **v** will not change, just its direction. *Magnetic forces alone cannot change the speed of a charged particle; they can change only its direction of motion.* The radius of the particle's circular path is found from the equation $F_B = F_C$:

$$qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB}$$

Magnetic Fields
$\mathbf{F}_B$ is always perpendicular to both **v** and **B**. Magnetic forces cannot change the speed of an object, only its direction. The magnetic field does no work on any charges.

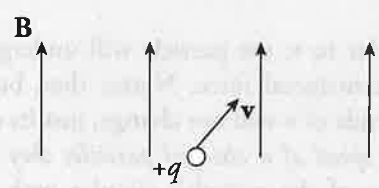
Example 4 A particle of charge $-q$ is shot into a region that contains an electric field, **E**, crossed with a perpendicular magnetic field, **B**. If $E = 2 \times 10^4$ N/C and $B = 0.5$ T, what must be the speed of the particle if it is to cross this region without being deflected?



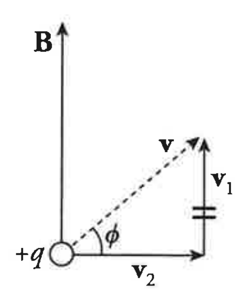
Solution. If the particle is to pass through undeflected, the electric force it feels has to be canceled by the magnetic force. In the diagram above, the electric force on the particle is directed upward (since the charge is negative and E is downward), and the magnetic force is directed downward by the Right-Hand Rule. So F_E and F_B point in opposite directions, and in order for their magnitudes to balance, qE must equal qvB, so v must equal E/B, which in this case gives

$$v = \frac{E}{B} = \frac{2 \times 10^4 \text{ N/C}}{0.5 \text{ T}} = 4 \times 10^4 \text{ m/s}$$

Example 5 A particle with charge $+q$, traveling with velocity **v**, enters a uniform magnetic field **B**, as shown below. Describe the particle's subsequent motion.



Solution. If the particle's velocity were parallel to **B**, then it would be unaffected by **B**. If **v** were perpendicular to **B**, then it would undergo uniform circular motion (as we saw in Example 2). In this case, **v** is neither purely parallel nor perpendicular to **B**. It has a component ($\mathbf{v}_1$) that's parallel to **B** and a component ($\mathbf{v}_2$) that's perpendicular to **B**.



Component $\mathbf{v}_1$ will not be changed by **B**, so the particle will continue upward in the direction of **B**. However, the presence of $\mathbf{v}_2$ will create circular motion. The superposition of these two types of motion will cause the particle's trajectory to be a *helix*; it will spin in circular motion while traveling upward with the speed $v_1 = v \sin \phi$:

THE MAGNETIC FORCE ON A CURRENT-CARRYING WIRE

Since magnetic fields affect moving charges, they should also affect current-carrying wires. After all, a wire that contains a current contains charges that move. Remember, a current is the flow of positive charges.

Let a wire of length ℓ be immersed in magnetic field **B**. If the wire carries a current I, then the force on the wire is

$$\mathbf{F}_B = I\boldsymbol{\ell} \times \mathbf{B}$$

Equation Sheet

with magnitude

$$F_B = BI\ell \sin \theta$$

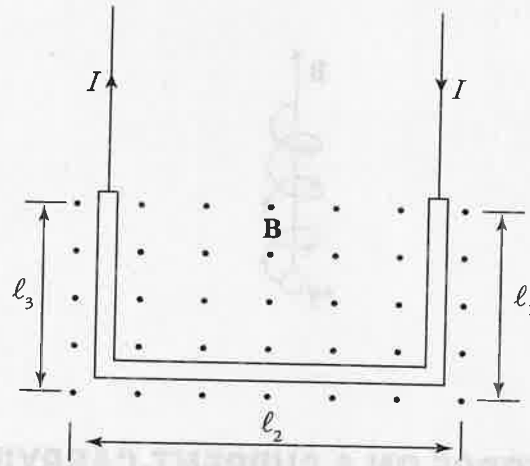
Equation Sheet

where θ is the angle between ℓ and **B**. Here, the direction of ℓ is the direction of the current, I. The direction of $\mathbf{F}_B$ can be found using the Right-Hand Rule and by letting your thumb point in the direction in which the current flows. Remember, conventional current is the flow of positive charges.

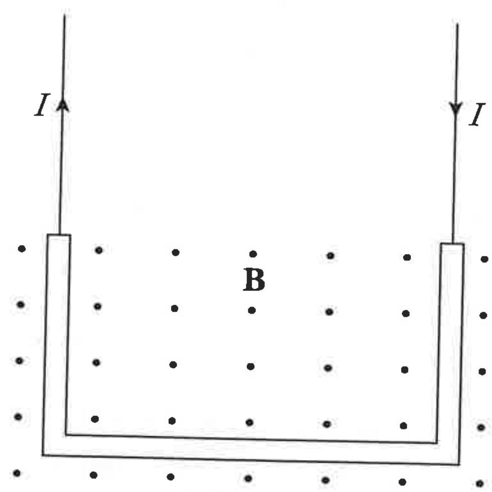
Another Way of Seeing It

Current I is charge over time (q/t) and the length of a wire I is a distance d. Hence, $I\ell = q(d/t)$. This gives us the same formula as before: $F_B = qvB\sin\theta = I\ell B\sin\theta$.

Example 6 A U-shaped wire of mass m is placed in a magnetic field **B** that points out of the plane of the page. How much current I must pass through the wire in order to cause the net force on the wire to be zero?



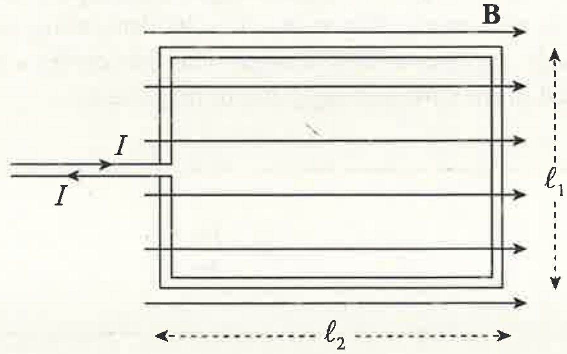
Solution. The total magnetic force on the wire is equal to the sum of the magnetic forces on each of the three sections of wire. The force on the first section (the right, vertical one), $\mathbf{F}_{B1}$, is directed to the left (applying the Right-Hand Rule), and the force on the third piece (the left, vertical one), $\mathbf{F}_{B3}$, is directed to the right. Since these pieces are the same length, these two oppositely directed forces have the same magnitude, $I\ell_1\text{B} = I\ell_3\text{B}$, and they cancel. So, the net magnetic force on the wire is the magnetic force on the middle piece. Since I points to the left and **B** is out of the page, the Right-Hand Rule tells us the force is upward.



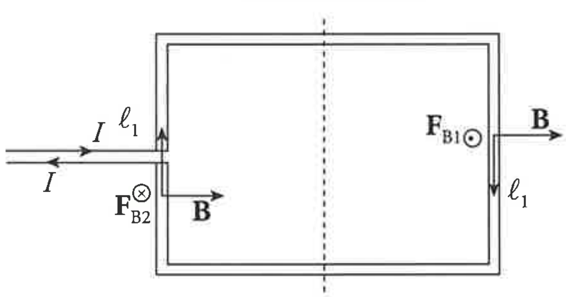
Since the magnetic force on the wire is $I\ell_2B$, directed upward, the amount of current must create an upward magnetic force that exactly balances the downward gravitational force on the wire. Because the total mass of the wire is m, the resultant force (magnetic + gravitational) will be zero if

$$I\ell_2\text{B} = mg \quad \Rightarrow \quad I = \frac{mg}{\ell_2\text{B}}$$

Example 7 A rectangular loop of wire that carries a current I is placed in a uniform magnetic field, **B**, as shown in the diagram below and is free to rotate. What torque does it experience?



Solution. Ignoring the tiny gap in the vertical left-hand wire, we have two wires of length ℓ_1 and two of length ℓ_2. There is no magnetic force on either of the sides of the loop of length ℓ_2 because the current in the top side is parallel to **B** and the current in the bottom side is antiparallel to **B**. The magnetic force on the right-hand side points out of the plane of the page, while the magnetic force on the left-hand side points into the plane of the page.



Each of these two forces exerts a torque that tends to turn the loop in such a way that the right-hand side rises out of the plane of the page while the left-hand side rotates into the page. Relative to the axis shown above (which cuts the loop in half), the torque of $\mathbf{F}_{B1}$ is

$$\tau_1 = rF_{B1} \sin \theta = (\tfrac{1}{2}\ell_2)(I\ell_1 B) \sin 90° = \tfrac{1}{2} I\ell_1\ell_2 B$$

and the torque of $\mathbf{F}_{B2}$ is

$$\tau_2 = rF_{B2} \sin \theta = (\tfrac{1}{2}\ell_2)(I\ell_1 B) \sin 90° = \tfrac{1}{2} I\ell_1\ell_2 B$$

Since both these torques rotate the loop in the same direction, the net torque on the loop is

$$\tau_1 + \tau_2 = I\ell_1\ell_2 B$$

MAGNETIC FIELDS CREATED BY CURRENT-CARRYING WIRES

In the previous section, we examined the force that a current-carrying wire experiences when it is subjected to an external magnetic field. The source of the magnetic field in the previous section was unspecified. As we said at the beginning of this chapter, the sources of magnetic fields are electric charges that move; they may spin, circulate, move through space, or flow down a wire. For example, consider a long, straight wire that carries a current *I*. The current generates a magnetic field in the surrounding space, of magnitude

Equation Sheet

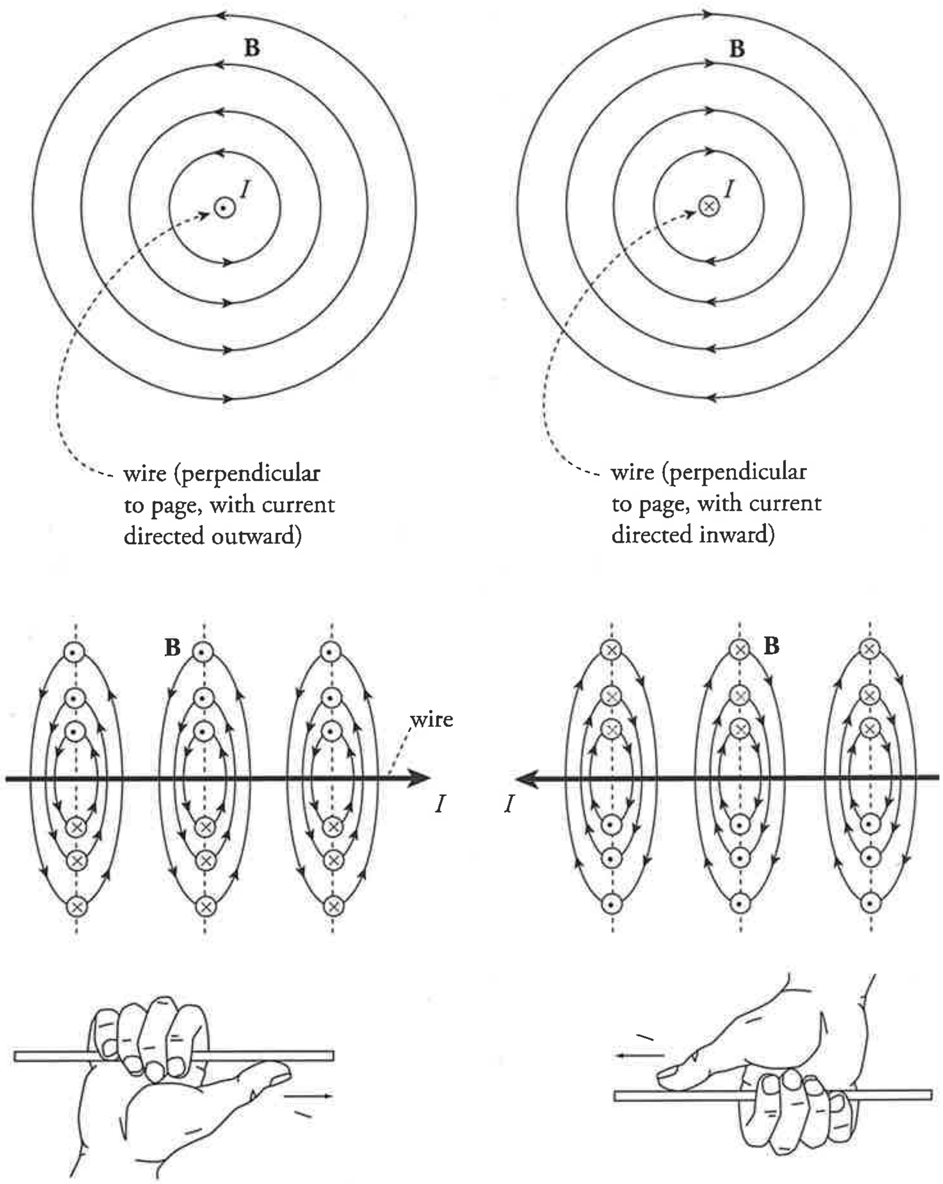
$$B = \frac{\mu_0}{2\pi}\frac{I}{r}$$

Magnetic Fields vs. Electric Fields
Unlike electric fields that start at a point charge and end at another point charge, magnetic fields are unending loops. Electric charges can be positive or negative and exist by themselves. There is no such thing as a monopole for magnets.

where r is the distance from the wire. The symbol μ_0 denotes a fundamental constant called the permeability of free space. Its value is

$$\mu_0 = 4\pi \times 10^{-7}\ \text{N/A}^2 = 4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}$$

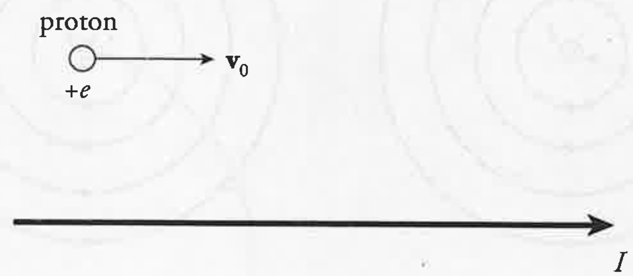
The magnetic field lines are actually circles whose centers are on the wire. The direction of these circles is determined by a variation of the Right-Hand Rule. Imagine grabbing the wire in your right hand with your thumb pointing in the direction of the current. Then, the direction in which your fingers curl around the wire gives the direction of the magnetic field lines.



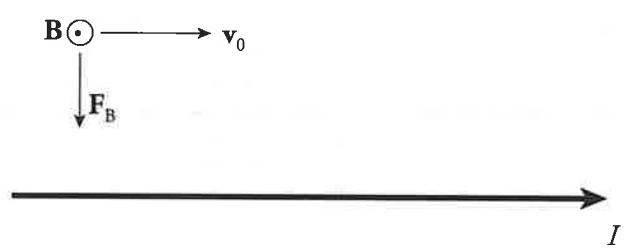
Right-Hand Rule for the Magnetic Field Created by a Current-Carrying Wire:

1. Put your thumb in the direction of the current or in the direction of a positive traveling charge.
2. Grab the wire/path.
3. As the fingers curl around your thumb, it represents the magnetic field going around the wire/path.

Example 8 The diagram below shows a proton moving with a speed of 2×10^5 m/s, initially parallel to, and 4 cm from, a long, straight wire. If the current in the wire is 20 A, what's the magnetic force on the proton?



Solution. Above the wire (where the proton is), the magnetic field lines generated by the current-carrying wire point out of the plane of the page, so $\mathbf{v}_0 \times \mathbf{B}$ points downward. Since the proton's charge is positive, the magnetic force $\mathbf{F}_B = q(\mathbf{v}_0 \times \mathbf{B})$ is also directed down, toward the wire.

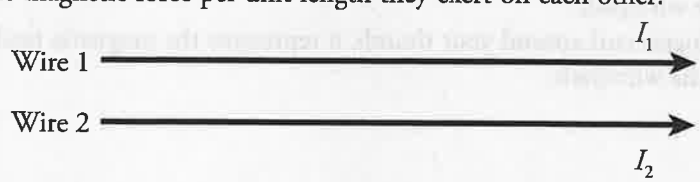


The strength of the magnetic force on the proton is

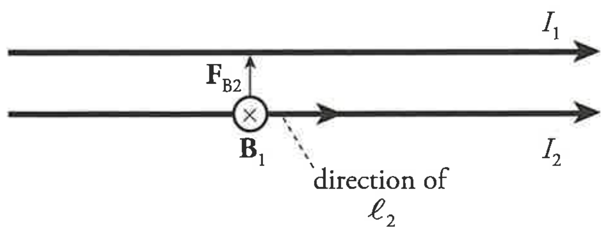
$$F_B = qv_0 B = ev_0 \frac{\mu_0}{2\pi}\frac{I}{r} = (1.6 \times 10^{-19}\ \text{C})(2 \times 10^5\ \text{m/s}) \frac{4\pi \times 10^{-7}\ \text{N/A}^2}{2\pi}\frac{20\ \text{A}}{0.04\ \text{m}}$$

$$= 3.2 \times 10^{-18}\ \text{N}$$

Example 9 The diagram below shows a pair of long, straight, parallel wires, separated by a small distance, r. If currents I_1 and I_2 are established in the wires, what is the magnetic force per unit length they exert on each other?



Solution. To find the force on Wire 2, consider the current in Wire 1 as the source of the magnetic field. Below Wire 1, the magnetic field lines generated by Wire 1 point into the plane of the page. Therefore, the force on Wire 2, as given by the equation $\mathbf{F}_{B2} = I_2(\ell_2 \times \mathbf{B}_1)$, points upward.



The magnitude of the magnetic force per unit length felt by Wire 2, due to the magnetic field generated by Wire 1, is found this way:

$$F_{B2} = I_2\ell_2 B_1 = I_2\ell_2 \frac{\mu_0}{2\pi}\frac{I_1}{r} \quad \Rightarrow \quad \frac{F_{B2}}{\ell_2} = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{r}$$

By Newton's Third Law, this is the same force that Wire 1 feels due to the magnetic field generated by Wire 2. The force is attractive because the currents point in the same direction; if one of the currents were reversed, then the force between the wires would be repulsive.

SOLENOIDS CREATE UNIFORM FIELDS

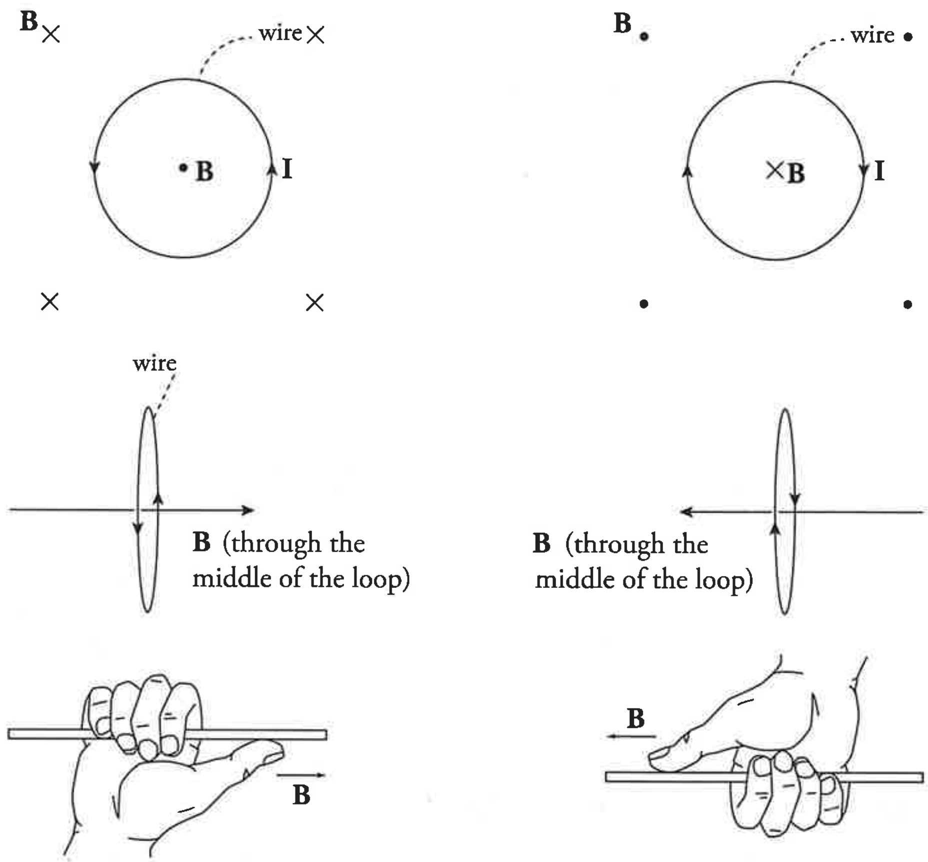
The parallel-plate capacitor that was discussed in the chapter on electric fields appears frequently on the AP Physics 2 Exam because it generates a constant electric field in the region between the plates. Knowing that a current-carrying wire creates a magnetic field, if you look back at the section on bar magnets, you see that the magnetic field produced by a bar magnet closely resembles that produced by a single closed loop of current-carrying wire.

A device called a solenoid is constructed by a series of coaxial loops of wires through which a continuous current flows. The magnetic fields from each of the individual loops of wire add together and within the region inside the solenoid, the magnetic field becomes uniform. If the solenoid is very long in comparison to its diameter and the coils are tightly packed together, the field inside a solenoid created in a laboratory is very nearly uniform in both direction and strength.

Unlike a bar magnet, a solenoid is an electromagnet. When there is a current flowing through the wires of the solenoid, the solenoid creates a magnetic field. When there is no current flowing through the wire, there is not a magnetic field. The bar magnet, on the other hand, always produces a magnetic field.

MAGNETIC FIELDS CREATED BY CURRENT LOOPS

When the magnetic field is created by a current that is flowing through a loop of wire or a solenoid, the above Right-Hand Rule still applies. However, there's another variation that can be used:



Right-Hand Rule for the Magnetic Field Created by a Current Loop or Solenoid:

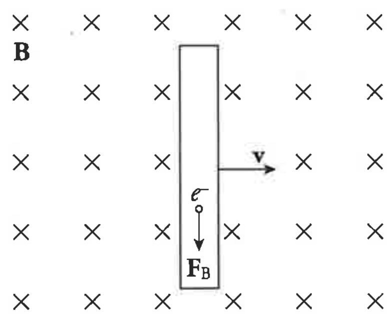
1. Curl your fingers in the direction of the current loop.
2. Make a thumbs up.
3. Your thumb is pointing in the direction of the magnetic field in the center of the loop/solenoid. Outside the loop, the magnetic field goes in the opposite direction.

You'll notice that it is very similar to the other Right-Hand Rule, except now your fingers are the current and your thumb is the magnetic field.

MOTIONAL EMF

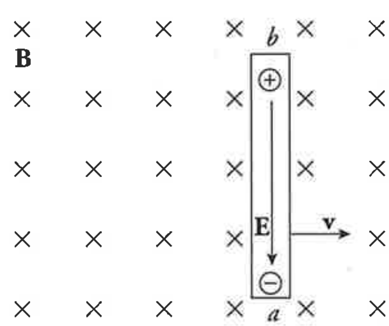
The simple act of moving a conducting rod in the presence of an external magnetic field *creates an electric field* within the rod. The figure below shows a conducting wire of length ℓ moving with constant velocity v in the plane of the page through a uniform magnetic field **B** that's perpendicular to the page. The presence of a moving conductor (recall there are many electrons present in the wire) inside a magnetic field results in the creation of an electric field that points in the same direction as the magnetic force on the electrons. (We know that the magnetic force is not directly responsible for the movement of the electrons in the rod because the magnetic force is *always* perpendicular to the direction of the motion of charges and therefore does no work. The magnetic field cannot be the source of the energy that causes the charges to move.) We can find the direction of the magnetic field using the Right-Hand Rule the instant the charges begin to move. The charges are negative and moving to the right, so $q\mathbf{v}$ points to the left (thumb of your right hand). **B** points into the page (fingers of your right hand). Your palm shows the direction of the magnetic force is downward. Therefore, the motion of the rod through the external magnetic field generates an electric field within the rod that causes electrons to move toward the bottom of the rod. This *induced electric field* points downward.

Vice-Versa
Earlier, we learned that moving charges generate magnetic fields. If we think about this backward, magnetic fields then can generate moving charges or current.



Right-Hand Rules
You will have to use a combination of your Right-Hand Rules in order to find the direction of current. Just pay attention to orientation.

As long as the rod continues to move at velocity **v**, the electric field will be maintained within the rod. Electric fields are generated by charge separations, and therefore there will be a surfeit of negative charges on the bottom of the rod and a deficit of electrons at the top of the rod.



A charge q in the wire feels two forces: an electric force, $\mathbf{F}_E = q\mathbf{E}$, and a magnetic force, $F_B = qvB\sin\theta = qvB$, because $\theta = 90°$.

If q is negative, $\mathbf{F}_E$ is upward and $\mathbf{F}_B$ is downward; if q is positive, $\mathbf{F}_E$ is downward and $\mathbf{F}_B$ is upward. So, in both cases, the forces act in opposite directions. Once the magnitude of $\mathbf{F}_E$ equals the magnitude of $\mathbf{F}_B$, the charges in the wire are in electromagnetic equilibrium. This occurs when $qE = qvB$; that is, when $E = vB$.

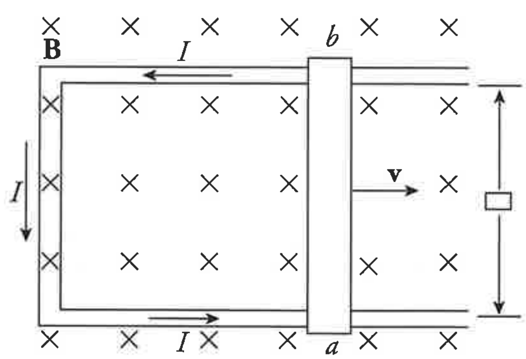
The presence of the electric field requires a potential difference between the ends of the rod. Since negative charge accumulates at the lower end (which we'll call point a) and positive charge accumulates at the upper end (point b), point b is at a higher electric potential. The potential difference V_{ba} is equal to $E\ell$ and, since $E = vB$, the potential difference can be written as $vB\ell$.

Now, imagine that the rod is sliding along a pair of conducting rails connected at the left by a stationary bar. The sliding rod now completes a rectangular circuit, and the potential difference V_{ba} causes current to flow.

Induced Current

An induced current can be created in three different ways:

1. changing the area of the loop of wire in a stationary magnetic field
2. changing the magnetic field strength through a stationary circuit
3. changing the angle between the magnetic field and the wire loop



Each of these three changes causes a change in the flux, as we will see in the next section. The motion of the sliding rod through the magnetic field creates an electromotive force, called **motional emf**:

Equation Sheet

$$\mathcal{E} = B\ell v$$

The existence of a current in the sliding rod causes the magnetic field to exert a force on it. Using the formula $F_B = BI\ell$, the fact that ℓ points upward (in the direction of the current) and **B** is into the page tells us that the direction of $\mathbf{F}_B$ on the rod is to the left. An external agent must provide this same amount of force to the right to maintain the rod's constant velocity and keep the current flowing. The power that the external agent must supply is $P = Fv = I\ell Bv$, and the electrical power delivered to the circuit is $P = IV_{ba} = I\mathcal{E} = IvB\ell$. Notice that these two expressions are identical. The energy provided by the external agent is transformed first into electrical energy and then thermal energy as the conductors making up the circuit dissipate heat.

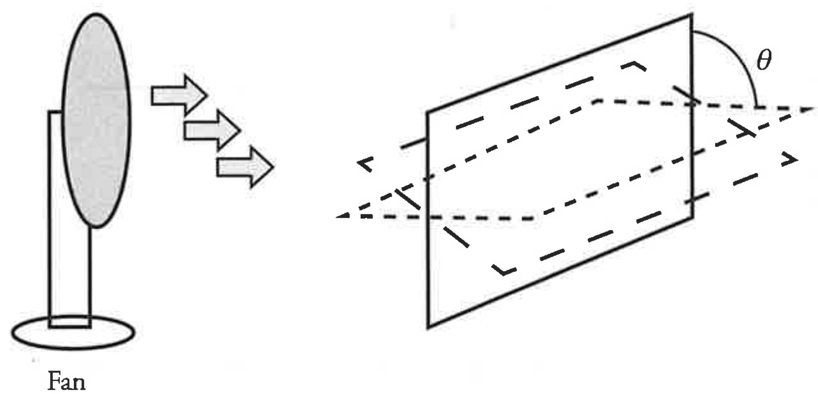
This relationship between current in a coil of wire and magnetic fields sets the basis for Faraday's Law.

FARADAY'S LAW OF ELECTROMAGNETIC INDUCTION

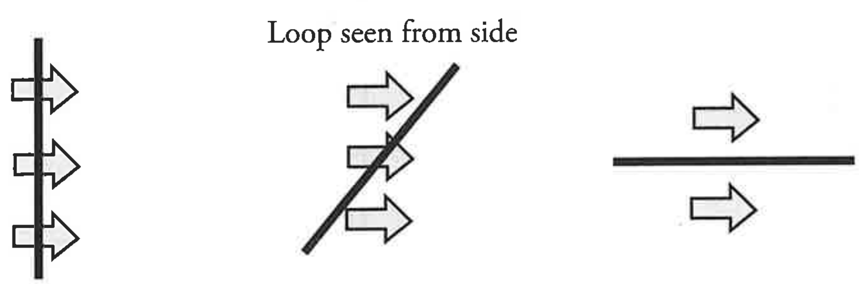
Electromotive force can be created by the motion of a conducting wire through a magnetic field, but there is another way to generate an emf from a magnetic field.

Faraday discovered that a current is induced when the magnetic flux passing through the coil or loop of wire changes. Magnetic flux helps us define what that amount of field passing into the loop means.

Imagine holding a loop of wire in front of a fan as shown:



The amount of air that flows through the loop depends on the area of the loop as well as its orientation (tilt angle θ).



The most effective airflow occurs when the loop is completely perpendicular, as in the situation to the left. The least effective occurs when the airflow and loop are in the situation to the right.

We can apply this idea to a magnetic field passing through a loop. The magnetic flux, Φ_B, through an area A is equal to the product of **A** and the magnetic field parallel to the area vector. The area vector points normal to the surface.

Equation Sheet

Faraday's Law

Course Description version:

$$\Phi_B = BA\cos\theta$$

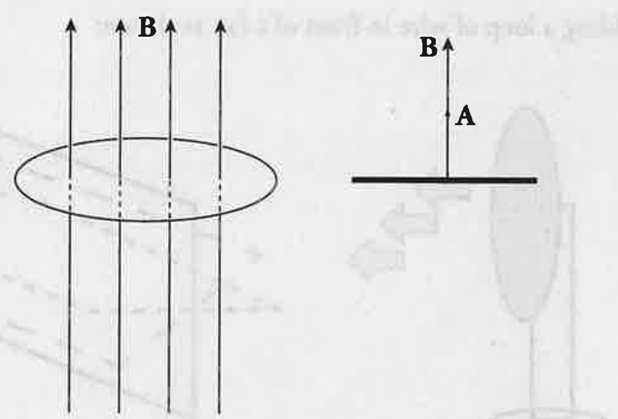
Equation Sheet version:

$$\Phi_B = \vec{B}\bullet\vec{A} = |\vec{B}|\cos\theta|\vec{A}|$$

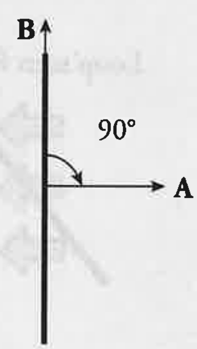
Magnetic flux measures the density of magnetic field lines that cross through an area. (Note that the direction of A is taken to be perpendicular to the plane of the loop.)

Magnetic Flux Units
The SI unit for magnetic flux is called the weber (Wb), which is equivalent to one Tesla meter-squared (T·m^2).

Example 10 The figure below shows two views of a circular loop of radius 3 cm placed within a uniform magnetic field, **B** (magnitude 0.2 T).



(a) What's the magnetic flux through the loop?
(b) What would be the magnetic flux through the loop if the loop were rotated 45°?
(c) What would be the magnetic flux through the loop if the loop were rotated 90°?



Solution.

(a) Since **B** is parallel to **A**, the magnetic flux is equal to BA:

$\Phi_B = BA = B \cdot \pi r^2 = (0.2\text{ T}) \cdot \pi(0.03\text{ m})^2 = 5.7 \times 10^{-4}\text{ T}\cdot\text{m}^2$

The SI unit for magnetic flux, the tesla meter^2, is called a **weber** (abbreviated **Wb**). So $\Phi_B = 5.7 \times 10^{-4}$ Wb.

(b) Since the angle between **B** and **A** is 45°, the magnetic flux through the loop is

$\Phi_B = BA \cos 45° = B \cdot \pi r^2 \cos 45° = (0.2\text{ T}) \cdot \pi(0.03\text{ m})^2 \cos 45° = 4.0 \times 10^{-4}\text{ Wb}$

(c) If the angle between **B** and **A** is 90°, the magnetic flux through the loop is zero, since cos 90° = 0.

The concept of magnetic flux is crucial because changes in magnetic flux induce emf. According to **Faraday's Law of Electromagnetic Induction**, the magnitude of the emf induced in a circuit is equal to the rate of change of the magnetic flux through the circuit. This can be written mathematically in the form

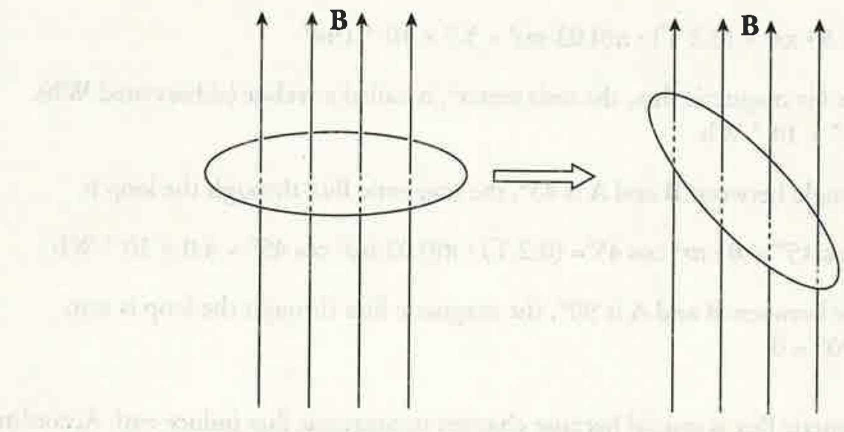
$$\left|\varepsilon_{avg}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right|$$

This induced emf can produce a current, which will then create its own magnetic field. The direction of the induced current is determined by the polarity of the induced emf and is given by **Lenz's Law**: the induced current will always flow in the direction that opposes the change in magnetic flux that produced it. If this were not so, then the magnetic flux created by the induced current would magnify the change that produced it, and energy would not be conserved. Lenz's Law can be included mathematically with Faraday's Law by the introduction of a minus sign; this leads to a single equation that expresses both results:

$$|\varepsilon| = \left|\frac{\Delta\Phi_B}{\Delta t}\right|$$

Equation Sheet

Example 11 The circular loop of Example 10 rotates at a constant angular speed through 45° in 0.5 s.



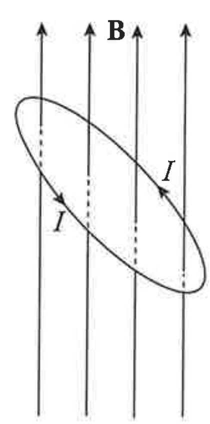
(a) What's the induced emf in the loop?
(b) In which direction will current be induced to flow?

Solution.

(a) As we found in Example 10, the magnetic flux through the loop changes when the loop rotates. Using the values we determined earlier, Faraday's Law gives

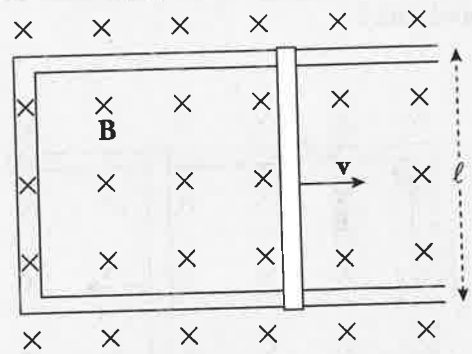
$$\mathcal{E}_{avg} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{(4.0\times10^{-4}\text{ Wb})-(5.7\times10^{-4}\text{ Wb})}{0.5\text{ s}} = 3.4\times10^{-4}\text{ V}$$

(b) The original magnetic flux was 5.7×10^{-4} Wb upward, and it was decreased to 4.0×10^{-4} Wb. So the change in magnetic flux is -1.7×10^{-4} Wb upward, or, equivalently, $\Delta\Phi_B = 1.7 \times 10^{-4}$ Wb downward. To oppose this change, we would need to create some magnetic flux upward. The current would be induced in the counterclockwise direction (looking down on the loop), because the Right-Hand Rule tells us that then the current would produce a magnetic field that would point up.



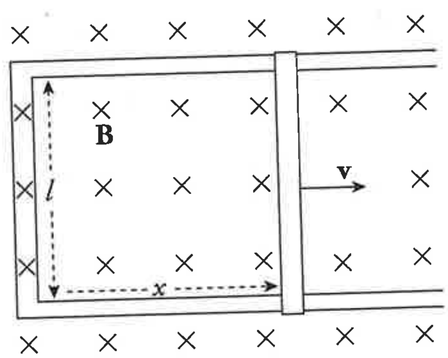
The current will flow only while the loop rotates, because emf is induced only when magnetic flux is changing. If the loop rotates 45° and then stops, the current will disappear.

Example 12 Again consider the conducting rod that's moving with constant velocity **v** along a pair of parallel conducting rails (separated by a distance ℓ), within a uniform magnetic field directed into the page, **B**:



Find the induced emf and the direction of the induced current in the rectangular circuit.

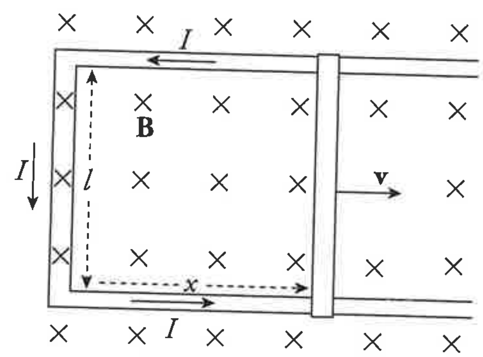
Solution. The area of the rectangular loop is ℓx, where x is the distance from the left-hand bar to the moving rod:



Because the area is changing, the magnetic flux through the loop is changing, which means that an emf will be induced in the loop. To calculate the induced emf, we first write $\Phi_B = BA = B\ell x$; then since $\Delta x/\Delta t = v$, we get

$$\mathcal{E}_{avg} = -\frac{\Delta\Phi_B}{\Delta t} = -\frac{\Delta(B\ell x)}{\Delta t} = -B\ell\frac{\Delta x}{\Delta t} = -B\ell v$$

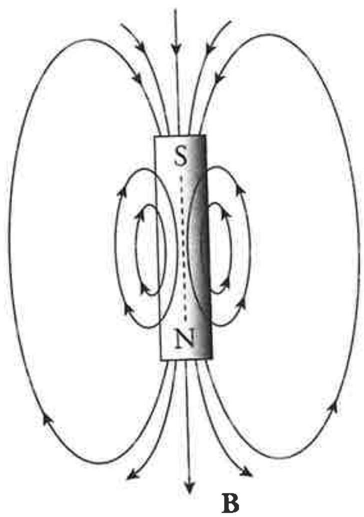
We can figure out the direction of the induced current from Lenz's Law. As the rod slides to the right, the magnetic flux into the page increases. How do we oppose an increasing into-the-page flux? By producing out-of-the-page flux. In order for the induced current to generate a magnetic field that points out of the plane of the page, the current must be directed counterclockwise (according to the Right-Hand Rule).



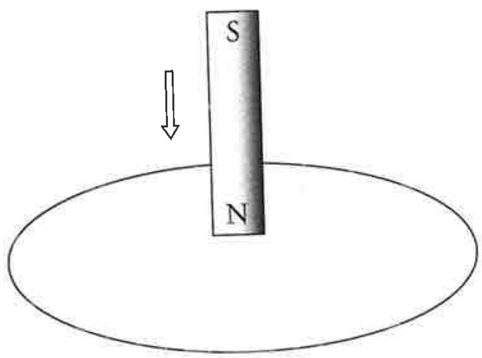
Note that the magnitude of the induced emf and the direction of the current agree with the results we derived earlier, in the section on motional emf.

This example also shows how a violation of Lenz's Law would lead directly to a violation of the Law of Conservation of Energy. The current in the sliding rod is directed upward, as given by Lenz's Law, so the conduction electrons are drifting downward. The force on these drifting electrons—and thus, the rod itself—is directed to the left, opposing the force that's pulling the rod to the right. If the current were directed downward, in violation of Lenz's Law, then the magnetic force on the rod would be to the right, causing the rod to accelerate to the right with ever-increasing speed and kinetic energy, without the input of an equal amount of energy from an external agent.

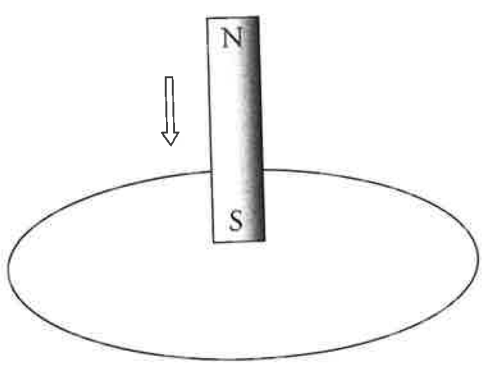
Example 13 A permanent magnet creates a magnetic field in the surrounding space. The end of the magnet at which the field lines emerge is designated the **north pole** (N), and the other end is the **south pole** (S):



(a) The figure below shows a bar magnet moving down, through a circular loop of wire. What will be the direction of the induced current in the wire?

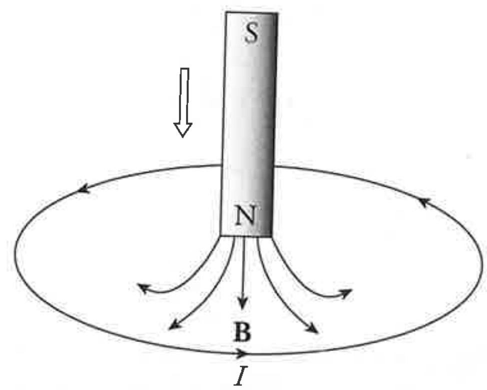


(b) What will be the direction of the induced current in the wire if the magnet is moved as shown in the following diagram?

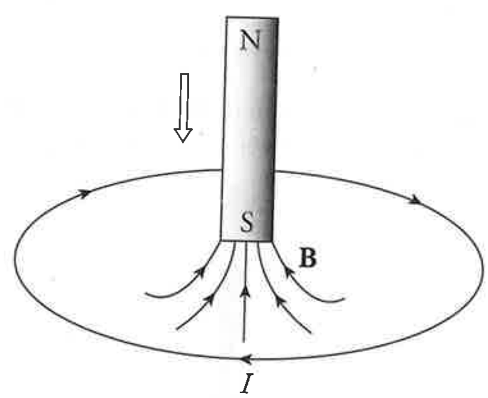


Solution.

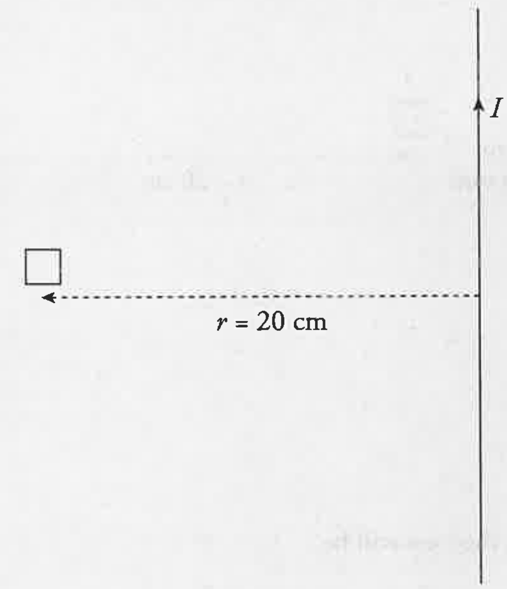
(a) The magnetic flux down, through the loop, increases as the magnet is moved. By Lenz's Law, the induced emf will generate a current that opposes this change. How do we oppose a change of *more flux downward*? By creating flux *upward*. So, according to the Right-Hand Rule, the induced current must flow counterclockwise (because this current will generate an upward-pointing magnetic field):



(b) In this case, the magnetic flux through the loop is upward and, as the south pole moves closer to the loop, the magnetic field strength increases so the magnetic flux through the loop increases upward. How do we oppose a change of *more flux upward*? By creating flux *downward*. Therefore, in accordance with the Right-Hand Rule, the induced current will flow clockwise (because this current will generate a downward-pointing magnetic field):



Example 14 A square loop of wire 2 cm on each side contains 5 tight turns and has a total resistance of 0.0002 Ω. It is placed 20 cm from a long, straight, current-carrying wire. If the current in the straight wire is increased at a steady rate from 20 A to 50 A in 2 s, determine the magnitude and direction of the current induced in the square loop. (Because the square loop is at such a great distance from the straight wire, assume that the magnetic field through the loop is uniform and equal to the magnetic field at its center.)



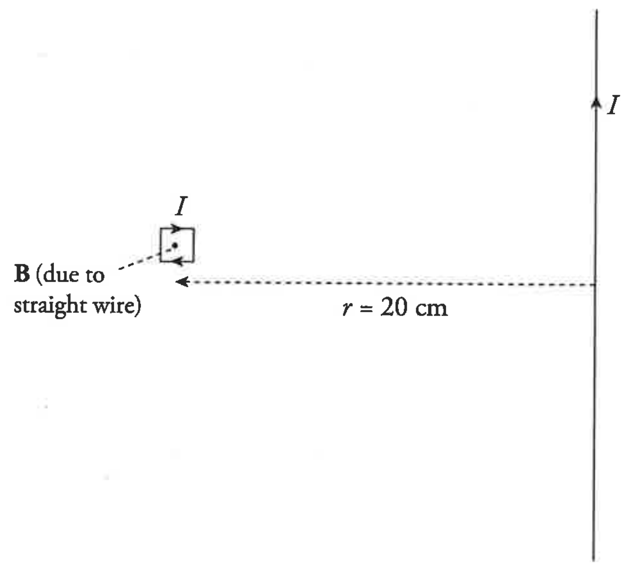
Solution. At the position of the square loop, the magnetic field due to the straight wire is directed out of the plane of the page, and its strength is given by the equation $B = (\mu_0/2\pi)(I/r)$. As the current in the straight wire increases, the magnetic flux through the turns of the square loop changes, inducing an emf and current. There are $N = 5$ turns; each loop contributes the same flux, so the total flux becomes the number of loops N times the flux of each individual loop, Φ_B. Faraday's Law becomes $\mathcal{E}_{avg} = -N(\Delta\Phi_B/\Delta t)$, and

$$\mathcal{E}_{avg} = -N\frac{\Delta\Phi_B}{\Delta t} = -N\frac{\Delta(BA)}{\Delta t} = -NA\frac{\Delta B}{\Delta t} = -NA\frac{\mu_0}{2\pi r}\frac{\Delta I}{\Delta t}$$

Substituting the given numerical values, we get

$$\begin{aligned}\mathcal{E}_{avg} &= -NA\frac{\mu_0}{2\pi r}\frac{\Delta I}{\Delta t}\\ &= -(5)(0.02\text{ m})^2\frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi(0.20\text{ m})}\frac{(50\text{ A}-20\text{ A})}{2\text{ s}}\\ &= -3\times10^{-8}\text{ V}\end{aligned}$$

The magnetic flux through the loop is out of the page and increases as the current in the straight wire increases. To oppose an increasing out-of-the-page flux, the direction of the induced current should be clockwise, thereby generating an into-the-page magnetic field (and flux).



The value of the current in the loop will be

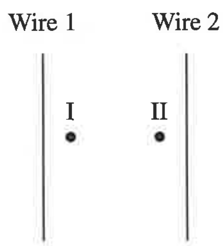
$$I = \frac{\mathcal{E}}{R} = \frac{3\times10^{-8}\ \text{V}}{0.0002\ \Omega} = 1.5\times10^{-4}\ \text{A}$$

Chapter 7 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review



Two long parallel wires are lain as shown above. If the current through wire 1 is larger than the current through wire 2, which of the following scenarios and points could there be a magnetic field of strength 0 T?

(A) Point I when the currents flow in the same direction

(B) Point II when the currents flow in the same direction

(C) Point I when the currents flow in opposite directions

(D) Point II when the currents flow in opposite directions

2 Mark for Review

An experiment is performed with a long current-carrying wire in a region free from any other magnetic fields. The field strength is recorded at various distances from the wire. Which of the following statements about a graph of B versus I is true?

(A) The slope of the graph is directly proportional to the square of the distance the magnetic field strength was measured from the wire.

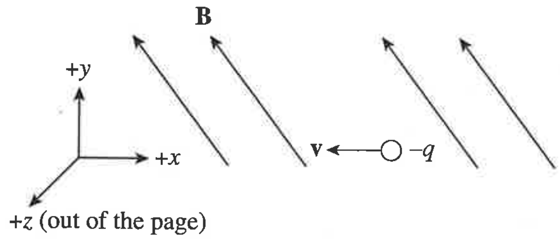
(B) The slope of the graph is directly proportional to the distance the magnetic field strength was measured from the wire.

(C) The slope of the graph is inversely proportional to the square of the distance the magnetic field strength was measured from the wire.

(D) The slope of the graph is inversely proportional to the distance the magnetic field strength was measured from the wire.

3 Mark for Review

A particle of charge $-q$ is launched to the left at speed **v** through a uniform magnetic field **B** which points up and to the left, as shown below.



Another particle is then launch through a uniform magnetic field and experiences a magnetic force of the same magnitude and direction as the first particle. Which of the following could NOT be the conditions for the second particle?

(A) **B** is unchanged, but q is now positive and **v** is directed to the right.

(B) The charge is still negative, but **v** and **B** are rotated 90° into the plane of the paper so that **v** is directed along the $-z$-axis.

(C) The charge is still negative, but **v** and **B** are rotated 90° counterclockwise so that **v** is directed along the $-y$-axis.

(D) **v** is unchanged, but q is now positive and **B** is rotated 180° to point into the fourth quadrant.

4 Mark for Review

Which of the following situations would result in the largest measurement of magnetic field?

(A) Measuring at a distance r from a wire carrying a current of I

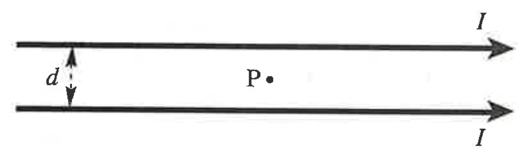
(B) Measuring at a distance $2r$ from a wire carrying a current of $I/2$

(C) Measuring at a distance $r/2$ from a wire carrying a current of $2I$

(D) All three measurements would be identical.

5 Mark for Review

In the figure below, what is the magnetic field at the Point P, which is midway between the two wires?



(A) $2\mu_0 I/(\pi d)$, into the plane of the page

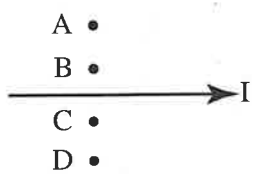
(B) $\mu_0 I/(2\pi d)$, out of the plane of the page

(C) $\mu_0 I/(2\pi d)$, into the plane of the page

(D) Zero

6 Mark for Review

Here is a section of a wire with a current moving to the right. Where is the magnetic field strongest and pointing INTO the page?



Ⓐ A

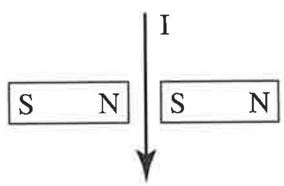
Ⓑ B

Ⓒ C

Ⓓ D

7 Mark for Review

What is the direction of force acting on the current-carrying wire as shown below?



Ⓐ To the bottom of the page

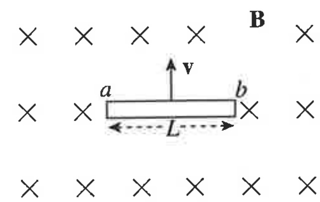
Ⓑ Into the page

Ⓒ Out of the page

Ⓓ To the right of the page

8 Mark for Review

A metal rod of length L is pulled upward with constant velocity **v** through a uniform magnetic field **B** that points into the plane of the page.



What is the potential difference between points a and b?

Ⓐ 0

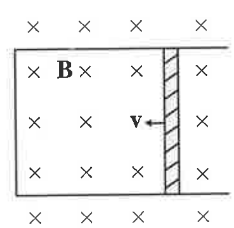
Ⓑ $\frac{1}{2}vBL$, with point b at the higher potential

Ⓒ vBL, with point a at the higher potential

Ⓓ vBL, with point b at the higher potential

9 Mark for Review

A conducting rod of length 0.2 m and resistance 10 ohms between its endpoints slides without friction along a U-shaped conductor in a uniform magnetic field B of magnitude 0.5 T perpendicular to the plane of the conductor, as shown in the diagram below.



If the rod is moving with velocity **v** = 3 m/s to the left, what is the magnitude and direction of the current induced in the rod?

Ⓐ 0.03A down the rod

Ⓑ 0.03A up the rod

Ⓒ 0.3A down the rod

Ⓓ 0.3A up the rod

10 Mark for Review

As shown in the figures below, a bar magnet is moved at a constant speed through a loop of wire. Figure A shows the bar magnet when it is at a position below the loop of wire, and Figure B shows the loop of wire after the bar magnet has passed completely through the loop.

Figure A

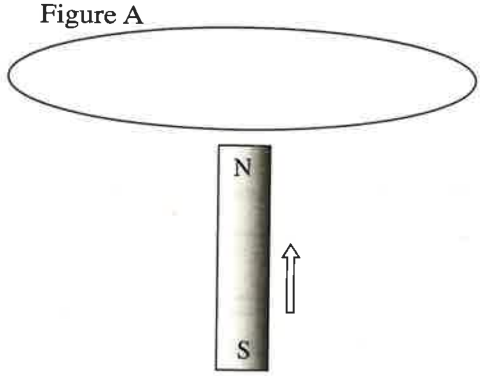
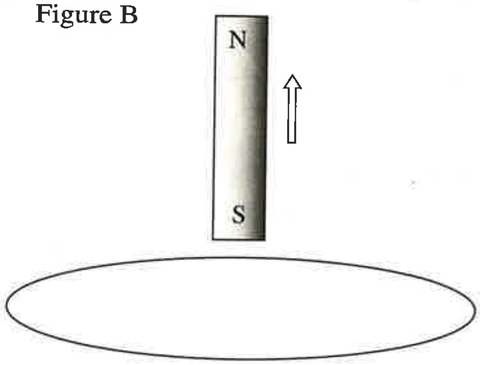


Figure B



Which of the following best describes the direction or directions of the current induced in the loop when the loop is looked at from above? Note that when looking at the loop from above, the bar magnet will be moving toward the viewer.

(A) Always clockwise

(B) Always counterclockwise

(C) First clockwise, then counterclockwise

(D) First counterclockwise, then clockwise

11 Mark for Review

Which of the following statements about induced emf is true?

(A) An emf will be induced when moving a loop of wire perpendicularly through the magnetic field.

(B) Rotating a wire in the presence of a magnetic field always induces an emf.

(C) Shrinking the size of a loop of wire in constant magnetic field will induce an emf in the wire.

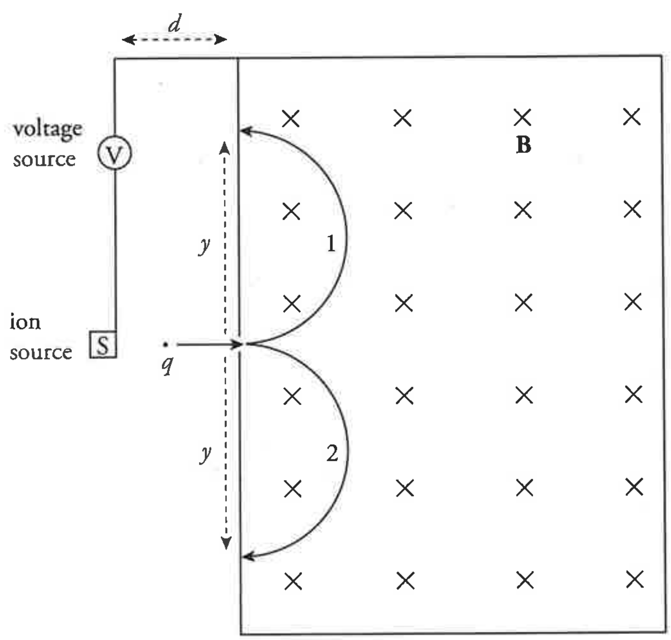
(D) A time varying magnetic field is required to induce an emf.

Section II: Free Response

1 Mark for Review

Figure 1 shows a simple mass spectrograph. It consists of a source of ions (charged atoms) that are accelerated (essentially from rest) by the voltage V and enter a region containing a uniform magnetic field $\mathbf{B}$. The polarity of V may be reversed so that both positively charged ions (cations) and negatively charged ions (anions) can be accelerated. Once the ions enter the magnetic field, they follow a semicircular path and strike the front wall of the spectrograph, on which photographic plates are constructed to record the impact. Assume that the ions have mass m.

Figure 1



A. What is the acceleration of an ion of charge q just before it enters the magnetic field?

B. Find the speed with which an ion of charge q enters the magnetic field.

C.

i. Which semicircular path, 1 or 2, would a cation follow?

ii. Which semicircular path, 1 or 2, would an anion follow?

D. Determine the mass of a cation entering the apparatus in terms of y, q, $\mathbf{B}$, and V.

E. Once a cation of charge q enters the magnetic field, how long does it take to strike the photographic plate?

F. What is the work done by the magnetic force in the spectrograph on a cation of charge q?

2 Mark for Review

A region of uniform magnetic field directed into the page (in the $-z$ direction) is generated. A positively charged particle traveling in the $+x$ direction at a speed v enters the region of the magnetic field, as shown in Figure 1.

Figure 1

A. What direction is the magnetic force on the particle at the instant when it enters the magnetic field region?

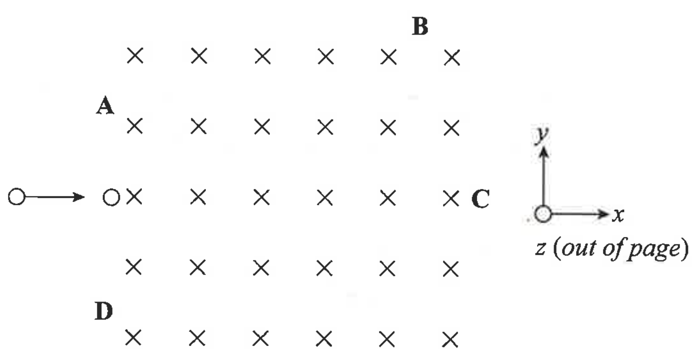
B. Some time passes while the particle continues to move in the region of the magnetic field under the influence of the magnetic force.

i. How will the new speed v_{new} compare to the initial speed v?

ii. How will the new velocity $\overrightarrow{v_{new}}$ compare to the initial velocity $\vec{v}$? Describe both the magnitude of the velocity and its components when comparing it to the initial velocity.

C. More time passes and the particle leaves the region of the magnetic field. For each position A, B, C, and D in Figure 2, state whether that position could be the position where the particle leaves the region of the magnetic field or not. For each position that you state is a possible exit position, draw the path on the diagram below that the particle would take from the dot where it enters the magnetic field to the letter where it exits.

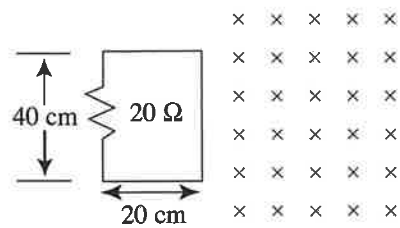
Figure 2



3 Mark for Review

A rectangular wire is pulled through a uniform magnetic field of 2 T going into the page, as shown. The resistor has a resistance of 20 Ω.

Figure 1



A. What is the voltage across the resistor as the wire is pulled horizontally at a velocity of 1 m/s and it just enters the field?

B. What is the current through the circuit in the above case and in what direction does it flow?

C. The region containing the magnetic field is 1 meter long in the direction the loop is being pulled. If the loop is pulled at the constant velocity of 1 m/s, describe the current in the loop from a time before the loop encounters the left edge of the region of the magnetic field until the time that the left edge of the loop completely passes out of the magnetic field region. Indicate the values and the direction of the current.

D. The loop of wire is rotated 90° clockwise so that the 20 cm side is vertical and the 40 cm side is horizontal at the top. Explain, without relying solely on equations, whether the loop would have to be pulled faster than 1 m/s, at exactly 1 m/s, or slower than 1 m/s for the current to have the same magnitude as in part B.

Chapter 7 Summary

- Charges moving through a magnetic field experience a force whose magnitude is given by $F_B = qvB\sin\theta$ and whose direction is given by the Right-Hand Rule.
- Because the force is always perpendicular to the direction of velocity, the charge may experience uniform circular motion. It would then follow all the appropriate circular motion relationships and orbit in a radius given by $r = \dfrac{mv}{qB}$.
- Because wires have charges moving through them, if a wire is carrying a current, it will experience a force if placed in a magnetic field. This is expressed by $F_B = BI\ell\sin\theta$.
- A current-carrying wire will produce a magnetic field whose strength is given by $B = \dfrac{\mu_o}{2\pi}\dfrac{I}{r}$, where $\mu_0 = 4\pi \times 10^{-7}$ T·m/A, I is the current through the wire, and r is the radial distance from the wire.
- An electromotive force is produced as a conducting wire's position changes or as the magnetic field changes. This idea is summarized by $\varepsilon = B\ell v$.
- Lenz's Law states that the electromagnetic force produced by Faraday's Law will act to produce a current that will create a magnetic field with direction that opposes the change in the magnetic flux.
- Faraday's Law of Induction says that if a wire is formed in a loop, an electromotive force is produced if the magnetic flux through the loop changes with time. This idea can be summarized by $|\varepsilon| = \left|\dfrac{\Delta\Phi_B}{\Delta t}\right|$.