Chapter 11
Fluids

"This world was once a fluid haze of light,

Till toward the centre set the starry tides,

And eddied into suns, that wheeling cast

The planets: then the monster, then the man."

—Lord Alfred Tennyson

Fluids Moving Around

Starting with the 2024–2025 school year, the College Board moved the topic of Fluids from AP Physics 2 to AP Physics 1.

INTRODUCTION

In this chapter, we'll discuss some of the fundamental concepts dealing with substances that can flow, which are known as **fluids.** The term *fluid* refers to both liquids and gases. While there are distinctions between liquids and gases, this chapter focuses on the similarities of all fluids.

DENSITY

Although the concept of *mass* is central to your study of mechanics (because of the all-important equation $\mathbf{F}_{net} = m\mathbf{a}$), it is the substance's *density* that turns out to be more useful in fluid mechanics.

By definition, the density of a substance is its mass per unit volume, and it's typically denoted by the letter ρ (the Greek letter *rho*):

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

Note that this equation immediately implies that the mass, m, of an object is equal to the product of the density, ρ, of that object, and V, the volume of that object, whenever the density of the object is constant.

For example, if 10^{-3} m^3 of oil has a mass of 0.8 kg, then the density of this oil is:

$$\rho = \frac{m}{V} = \frac{0.8\ \text{kg}}{10^{-3}\ \text{m}^3} = 800\ \frac{\text{kg}}{\text{m}^3}$$

While it is not required knowledge for an AP Physics 1 Exam, it is useful to know that the density of water (at "standard temperature and pressure," or STP) is very close to 1000 kg/m^3. The density of fluids and gases changes at varying temperatures and pressures, hence the inclusion of the phrase "at standard temperature and pressure." STP is defined at a temperate of 0°C and a pressure of 1×10^5 Pa, which is standard atmospheric pressure at the surface of the Earth listed on the equation sheet. With this density of water in mind, you can see that the oil calculated in the example above has a lower density than water, and therefore the oil would float on top of the water, as we will see in the next section.

PRESSURE

If we place an object in a fluid, the fluid exerts a contact force on the object. How that force is distributed over any small area of the object's surface defines the **pressure**:

$$\text{pressure} = \frac{\text{force}_\perp}{\text{area}}$$
$$P = \frac{F_\perp}{A}$$

The subscript ⊥ (which means *perpendicular*) is meant to emphasize that the pressure is defined to be the magnitude of the force that acts perpendicular to the surface divided by the area. Because force is measured in newtons (N) and area is expressed in square meters (m^2), the SI unit for pressure is the newton per square meter. This unit is given its own name: the **pascal**, abbreviated Pa:

$$\text{SI unit of pressure: } 1 \text{ pascal} = 1 \text{ Pa} = 1 \frac{\text{N}}{\text{m}^2}$$

One pascal is a very tiny amount of pressure; for example, a nickel on a table exerts about 140 Pa of pressure just due to its weight alone. For this reason, you'll often see pressures expressed in kPa (kilopascals, where 1 kPa = 10^3 Pa) or even in MPa (megapascals, where 1 MPa = 10^6 Pa). Another common unit for pressure is the atmosphere (atm). At sea level, atmospheric pressure, P_{atm}, is about 100,000 Pa; this is 1 **atmosphere.**

Example 1 A vertical column made of cement has a base area of 0.5 m^2. If its height is 2 m and the density of cement is 3000 kg/m^3, how much pressure does this column exert on the ground?

Solution. The force the column exerts on the ground is equal to its weight, *mg*, so we'll find the pressure it exerts by dividing this by the base area, *A*. The mass of the column is equal to ρV, which we calculate as follows:

$$m = \rho V = \rho A h = (3 \times 10^3 \ \frac{\text{kg}}{\text{m}^3})(0.5 \text{ m}^2)(2 \text{ m}) = 3 \times 10^3 \text{ kg}$$

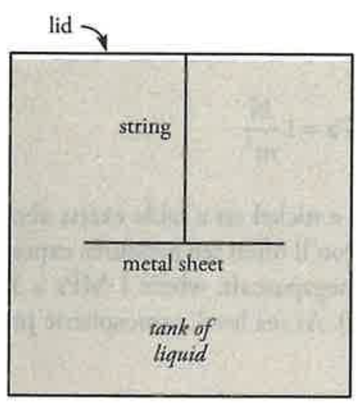
Therefore,

$$P = \frac{F}{A} = \frac{mg}{A} = \frac{(3 \times 10^3 \text{ kg})(9.8 \frac{\text{N}}{\text{kg}})}{0.5 \text{ m}^2} = 5.88 \times 10^4 \text{ Pa}$$

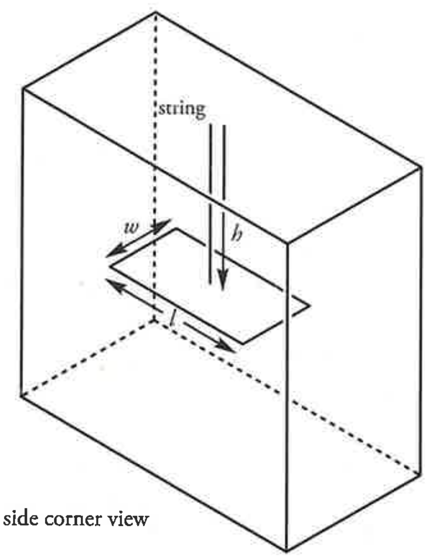
The units given for g, the acceleration due to gravity at the Earth's surface, are m/s^2 when calculated from the time-rate of change in velocity. These units are equivalent to N/kg when the acceleration is instead calculated using $F = ma$. In studying fluids, N/kg are typically more useful units.

HYDROSTATIC PRESSURE

Imagine that we have a tank with a lid on top, filled with some liquid. Suspended from this lid is a string attached to a thin sheet of metal that hangs horizontally. The figures below show two views of this tank:



front view



side corner view

The weight of the liquid produces a force that pushes down on the metal sheet. If the sheet has length l and width w, and is at depth h below the surface of the liquid, then the weight of the liquid on top of the sheet is:

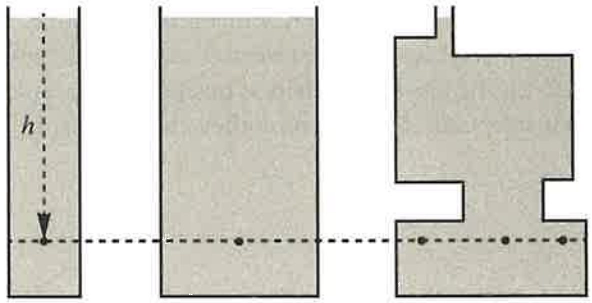
$$F_g = mg = \rho Vg = \rho(lwh)g$$

where ρ is the liquid's density. If we divide this weight by the area of the sheet ($A = lw$), we get the pressure due to the liquid:

$$P_{\text{liquid}} = \frac{\text{force}}{\text{area}} = \frac{F_{g\text{liquid}}}{A} = \frac{\rho(lwh)g}{lw} = \rho gh$$

Since the liquid is at rest, this is known as **hydrostatic pressure.**

Note that the hydrostatic pressure due to the liquid, $P_{\text{liquid}} = \rho gh$, depends only on the density of the liquid and the depth below the surface; in fact, it's proportional to both of these quantities. One important consequence of this is that the shape of the container doesn't matter. For example, if all the containers in the figure below are filled with the same liquid, then the pressure is the same at every point along the horizontal dashed line (and within a container), simply because every point on this line is at the same depth, *h*, below the surface of the liquid.



If the liquid in the tank were open to the atmosphere, then the total (or absolute) pressure at depth *h* would be equal to the pressure pushing down on the surface—the atmospheric pressure, P_{atm}—plus the pressure due to the liquid alone. In general, the pressure on the surface of the liquid is some pressure P_0 and the total pressure is:

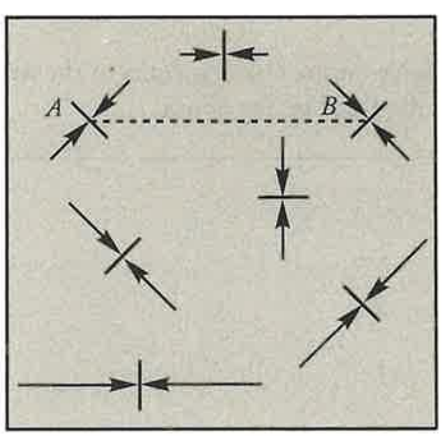
total (absolute) pressure: $P_{\text{total}} = P_0 + P_{\text{liquid}} = P_0 + \rho gh$

When the pressure at the top of the liquid is atmospheric pressure (as in the scenario above), then our equation for total pressure reduces to:

$$P_{\text{total}} = P_{\text{atm}} + \rho gh$$

Because pressure is the *magnitude* of the force per area, pressure is a scalar. It has no direction. The direction of the force due to the pressure on any small surface is perpendicular to that surface. For example, in the figure below, the pressure at Point *A* is the same as the pressure at Point *B* because they're at the same depth.

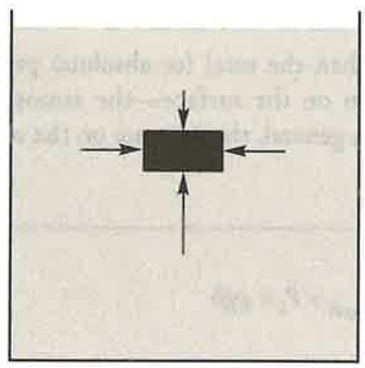
It's All About Depth
The pressure exerted on an object by a fluid is proportional only to the density of the fluid and the depth of the object but is independent of the object's mass. An elephant and a feather at the same depth in the ocean will feel the same pressure from the water.



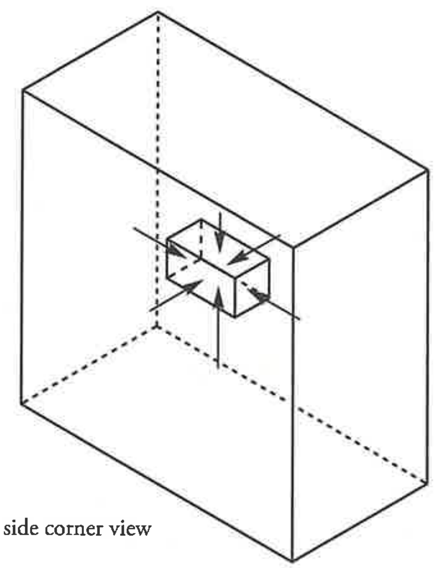
But, as you can see, the direction of the force due to the pressure varies depending on the orientation of the surface—and even which side of the surface—the force is pushing on.

BUOYANCY

Let's place a block in our tank of fluid. Because the pressure on each side of the block depends on its average depth, we see that there's more pressure on the bottom of the block than there is on the top. Because the block is rectangular and the top and bottom have the same area, there's a greater force pushing up on the block than there is pushing down on it. The forces due to the pressure on the other four sides cancel out (because they are at the same depth), so the net force on the block is upward.



front view



side corner view

This net upward force is called the **buoyant force** (or just **buoyancy** for short), denoted $\mathbf{F}_{buoy}$. We calculate the magnitude of the buoyant force using **Archimedes' Principle**; in words, Archimedes' Principle says:

> The strength of the buoyant force on the object is equal to the weight of the fluid displaced by the object.

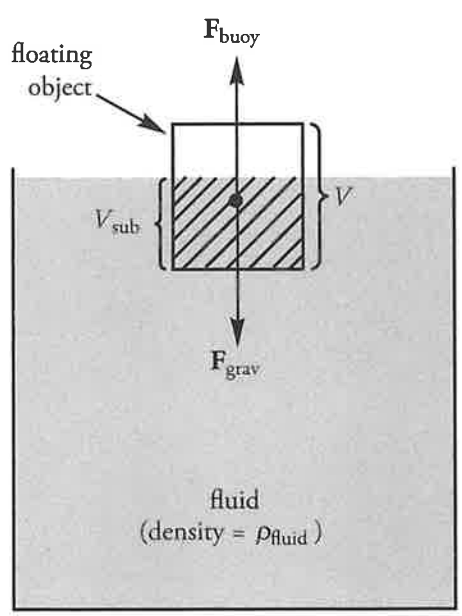
When an object is partially or completely submerged in a fluid, the volume of the object submerged, which we call V_{sub}, is the volume of the fluid displaced. By multiplying this volume by the density of the fluid, we get the mass of the fluid displaced; then, multiplying this mass by g gives us the weight of the fluid displaced. So, here's Archimedes' Principle as a mathematical equation:

$$\text{Buoyant force: } F_{buoy} = \rho_{fluid} V_{sub}\, g$$

When an object floats, its submerged volume is just enough to make the buoyant force it feels balance its weight. So, if an object's density is ρ_{object} and its (total) volume is V, its weight will be $mg = \rho_{object} Vg$. The buoyant force it feels is $\rho_{fluid} V_{sub}\, g$. Setting these equal to each other, we find that:

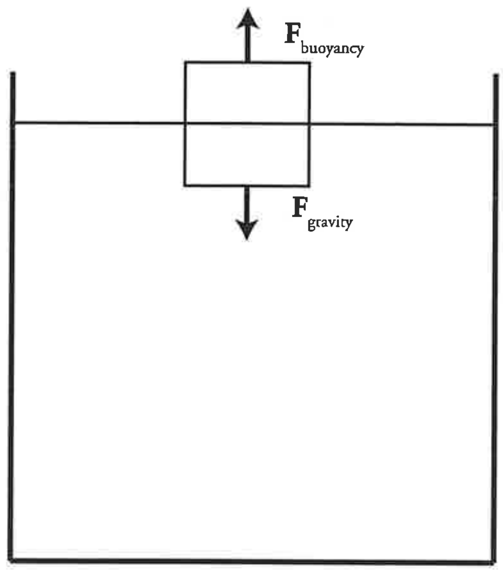
$$\frac{V_{sub}}{V} = \frac{\rho_{object}}{\rho_{fluid}}$$

So, if $\rho_{object} < \rho_{fluid}$, then the object will float, and the fraction of its volume submerged is the same as the ratio of its density to the fluid's density. For example, if the object's density is 2/3 the density of the fluid, then the object will float, and 2/3 of the object will be submerged.



If an object is denser than the fluid, it will sink. In this case, even if the entire object is submerged (in an attempt to maximize V_{sub} and the buoyant force), its weight is still greater than the buoyant force, and down it goes. And if an object just happens to have the same density as the fluid, it will be happy hovering (in static equilibrium) anywhere underneath the fluid.

Let's summarize and go through the steps of an object floating and then an object as it sinks and hits the bottom of a tank. In the diagram below, we have an object that is floating in a liquid.



$$\rho_{object} < \rho_{fluid}$$

$$\mathbf{F}_{net} = 0$$

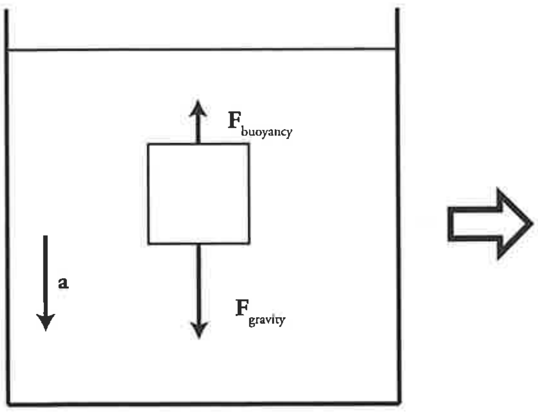
$$\mathbf{F}_{buoyancy} = \mathbf{F}_{gravity}$$

$$\rho_{fluid} V_{fluid} g = m_{object} g$$

$$\rho_{fluid} V_{sub} g = \rho_{object} V_{object} g$$

$$\frac{V_{sub}}{V_{object}} = \frac{\rho_{object}}{\rho_{fluid}}$$

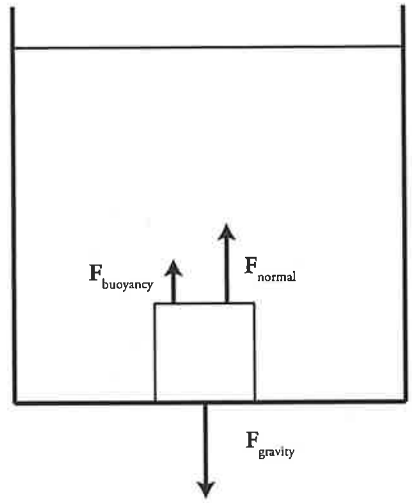
On the facing page is a diagram of an object with a density greater than the fluid density as it sinks to the bottom of the tank.



$\rho_{\text{object}} > \rho_{\text{fluid}}$

$\mathbf{F}_{\text{net}} = m\mathbf{a}$

$\mathbf{F}_{\text{net}} = F_{\text{gravity}} - F_{\text{buoyancy}}$



$\rho_{\text{object}} > \rho_{\text{fluid}}$

$\mathbf{F}_{\text{net}} = 0$

$F_{\text{gravity}} = F_{\text{buoyancy}} + F_{\text{normal}}$

When we solve for F_{normal}, we get:

Apparent Weight
$F_{\text{normal}} = F_{\text{gravity}} - F_{\text{buoyancy}}$

Because the buoyant force is always directed upward, and the force of gravity is defined to point downward, these forces are always directed in opposite directions. The result of the buoyant force opposing gravity is that any object in a fluid appears to weigh less than when it is not in the fluid, and as the fluid density increases, the apparent weight decreases. The apparent weight of an object is commonly referred to as the normal force of that object.

Example 2 An object with a mass of 150 kg and a volume of 0.75 m^3 is floating in ethyl alcohol, whose density is 800 kg/m^3. What fraction of the object's volume is above the surface of the fluid?

Solution. The density of the object is:

$$\rho_{\text{object}} = \frac{m}{V} = \frac{150 \text{ kg}}{0.75 \text{ m}^3} = 200 \frac{\text{kg}}{\text{m}^3}$$

The ratio of the object's density to the fluid's density is:

$$\frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} = \frac{200 \text{ kg/m}^3}{800 \text{ kg/m}^3} = \frac{1}{4}$$

This means that 1/4 of the object's volume is *below* the surface of the fluid; therefore, the fraction *above* the surface is 1 – (1/4) = 3/4. Make sure you know exactly what the question is asking. Typically, you use this equation to find the volume submerged, but this question (Example 2) asks for the volume *not* submerged.

Example 3 A brick, of density 2000 kg/m³ and volume 1.5×10^{-3} m³, is dropped into a swimming pool full of water, with a density of 1000 kg/m³.

(a) Explain briefly why the brick will sink.

(b) When the brick is lying on the bottom of the pool, what is the apparent weight of the brick?

Solution.

(a) The brick has a greater density than the surrounding fluid (water), so it will sink.

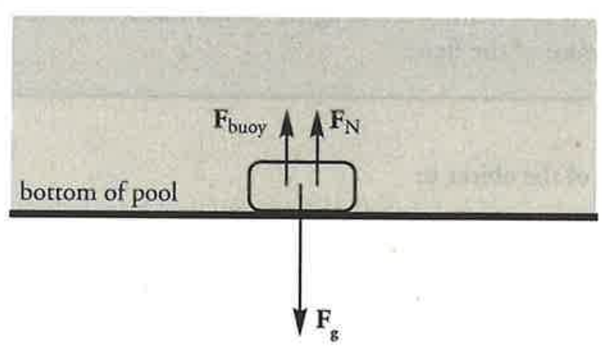
(b) When the brick is lying on the bottom surface of the pool, it is totally submerged, so $V_{sub} = V$; this means the buoyant force on the brick is:

$$\begin{aligned} F_{buoy} &= \rho_{fluid} V_{sub} g = \rho_{water} V g \\ &= (1000 \text{ kg/m}^3)(1.5 \times 10^{-3} \text{ m}^3)(10 \text{ N/kg}) \\ &= 15 \text{ N} \end{aligned}$$

The weight of the brick is:

$$\begin{aligned} F_g &= mg = \rho_{brick} V g \\ &= (2000 \text{ kg/m}^3)(1.5 \times 10^{-3} \text{ m}^3)(10 \text{ N/kg}) \\ &= 30 \text{ N} \end{aligned}$$

When the brick is lying on the bottom of the pool, the net force it feels is zero.



Therefore, we must have $F_{buoy} + F_N = F_g$, so $F_N = F_g - F_{buoy}$ = 30 N – 15 N = 15 N. So the apparent weight of the brick, that is, the normal force it feels, is 15 N.

Example 4 A helium balloon has a volume of 0.03 m^3. Ignoring the weight of the plastic of the balloon, calculate the net force on the balloon if it's surrounded by air. (Note: The density of helium is 0.2 kg/m^3, and the density of air is 1.2 kg/m^3.)

Solution. The balloon will feel a buoyant force upward and the force of gravity—the balloon's weight—downward. Because the balloon is completely surrounded by air, $V_{sub} = V$, and the buoyant force is:

$$\begin{aligned} F_{buoy} = \rho_{fluid} V_{sub} g = \rho_{air} V g \\ = (1.2 \text{ kg/m}^3)(0.03 \text{ m}^3)(10 \text{ N/kg}) \\ = 0.36 \text{ N} \end{aligned}$$

The weight of the balloon is:

$$\begin{aligned} F_g = mg = \rho_{helium} V g = (0.2 \text{ kg/m}^3)(0.03 \text{ m}^3)(10 \text{ N/kg}) \\ = 0.06 \text{ N} \end{aligned}$$

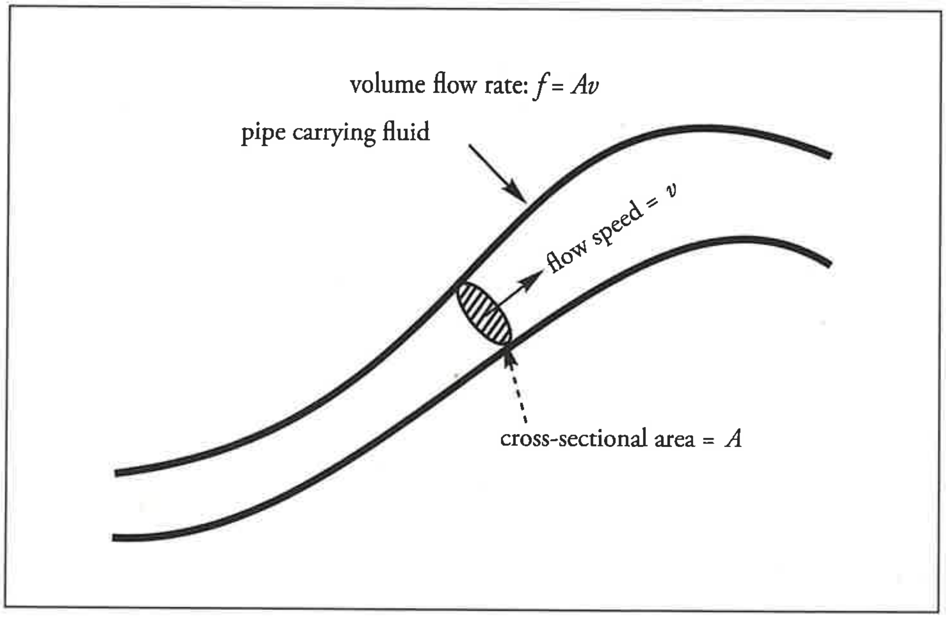
Because $F_{buoy} > w$, the net force on the balloon is upward and has magnitude:

$$F_{net} = F_{buoy} - F_g = 0.36 \text{ N} - 0.06 \text{ N} = 0.3 \text{ N}$$

This is why a helium balloon floats away if you let go of its string.

THE CONTINUITY EQUATION: FLOW RATE AND CONSERVATION OF MASS

Consider a pipe through which fluid is flowing. The **volume flow rate,** f, is the volume of fluid that passes a particular point per unit time. In SI units, flow rate is expressed in m^3/s. To find the volume flow rate, all we need to do is multiply the size of the pipe at a particular location by the average speed of the flow at that point. In this context, the size of the pipe means the area of a cross-section of the pipe that is perpendicular to the direction of the flow.



Be careful not to confuse *volume flow rate* with *flow speed*; volume flow rate tells us how *much* volume of fluid flows per unit time; flow speed tells us how *fast* it's moving. The volume flow rate is measured in cubic meters per second, while the flow speed is measured in meters per second.

An incompressible fluid has a density that does not change. For an incompressible fluid, if we imagine some fluid moving past a particular point in the pipe during some small amount of time, then we can conclude the same amount of mass must flow past another point in the pipe in the same amount of time because mass is conserved. The amount of mass m flowing for distance Δl past any point with area A during a time interval Δt is given by $\Delta m/\Delta t = \rho\Delta V/\Delta t = \rho A(\Delta l/\Delta t) = \rho A\Delta v$. The final two terms in this equation are simply the volume flow rate we described above. Because the density of the fluid is constant, we get the **Continuity Equation:**

$$A_1v_1 = A_2v_2$$

Because the product Av is a constant, the flow speed will increase where the pipe narrows (decreases in area) and decrease where the pipe widens (increases in area). In fact, we can say that the flow speed is inversely proportional to the cross-sectional area—or to the square of the radius (in the case of a circular cross-section)—of the pipe.

Example 5 A circular pipe of non-uniform diameter carries water. At one point in the pipe, the radius is 2 cm and the flow speed is 6 m/s.

(a) What is the volume flow rate?

(b) Without performing a new flow rate calculation, explain what the volume flow rate and flow speed will be if, at a later point, the radius of the pipe decreases to 1 cm.

Solution.

(a) At any point, the volume flow rate, f, is equal to the cross-sectional area of the pipe multiplied by the flow speed:

$$f = Av = \pi r^2 v = \pi (2 \times 10^{-2} \text{ m})^2 (6 \text{ m/s}) \approx 75 \times 10^{-4} \text{ m}^3/\text{s} = 7.5 \times 10^{-3} \text{ m}^3/\text{s}$$

(b) The volume flow rate will be unchanged. Then, from the Continuity Equation, we know that v, the flow speed, is inversely proportional to A, the cross-sectional area of the pipe. If the pipe's radius decreases by a factor of 2 (from 2 cm to 1 cm), then A decreases by a factor of 4, because A is proportional to r^2.

If A decreases by a factor of 4, then v will increase by a factor of 4. Therefore, the flow speed at a point where the pipe's radius is 1 cm will be $4 \cdot (6 \text{ m/s}) = 24 \text{ m/s}$.

BERNOULLI'S EQUATION: CONSERVATION OF ENERGY IN FLUIDS

The most important equation in fluid mechanics is **Bernoulli's Equation**, which is the statement of Conservation of Energy for ideal fluid flow. First, let's describe the conditions that make fluid flow *ideal.*

- *The fluid is incompressible.*

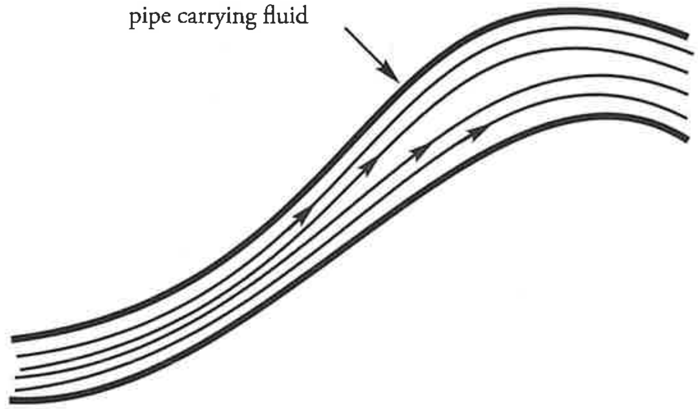
  This works very well for liquids and also applies to gases if the pressure changes are small.

- *The fluid's viscosity is negligible.*

  Viscosity is the force of cohesion between molecules in a fluid; think of viscosity as internal friction for fluids. For example, maple syrup is sticky and has a greater viscosity than water: there's more resistance to a flow of maple syrup than to a flow of water. While Bernoulli's Equation would give good results when applied to a flow of water, it would not give good results if it were applied to a flow of maple syrup.

- *The flow is streamline.*

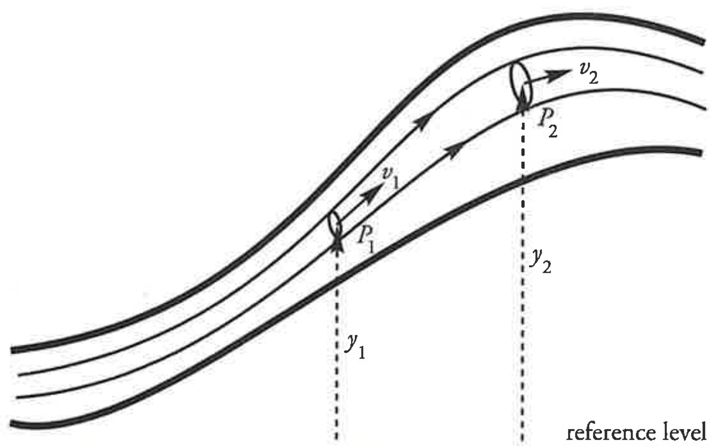
In a tube carrying a flowing fluid, a **streamline** is just what it sounds like: it's a *line* in the *stream*. If we were to inject a drop of dye into a clear glass pipe carrying, say, water, we'd see a streak of dye in the pipe, indicating a streamline.



When the flow is streamline, the fluid moves smoothly through the tube. (The opposite of streamline flow is **turbulent** flow, which is characterized by rapidly swirling whirlpools; such chaotic flow is unpredictable.)

If the three conditions described above hold, and the flow rate, f, is steady (meaning it doesn't change with time), Bernoulli's Equation can be applied to any pair of points along a streamline within the flow. Let ρ be the density of the fluid that's flowing. Label the points we want to compare as Point 1 and Point 2. Choose a horizontal reference level, and let y_1 and y_2 be the heights of these points above this level. If the pressures at Points 1 and 2 are P_1 and P_2, and if the flow speeds at these points are v_1 and v_2, then Bernoulli's Equation says:

$$P_1 + \rho g y_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$



Conservation of Energy

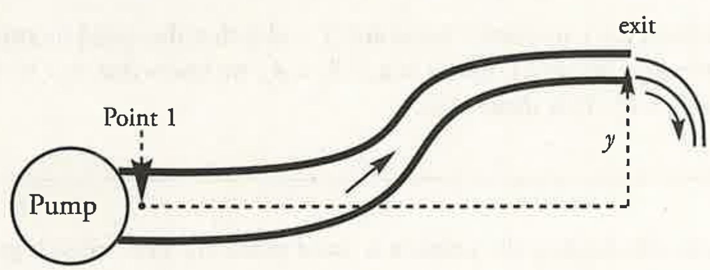
This equation looks very similar to a previous equation dealing with Conservation of Energy with total mechanical energy. In fact, Bernoulli's Equation deals with the Conservation of Energy for fluids.

An alternative, but equivalent, way of stating Bernoulli's Equation is to say that the quantity

$$P + \rho g y + \tfrac{1}{2}\rho v^2$$

is constant along a streamline. We mentioned earlier that Bernoulli's Equation is a statement of Conservation of Energy. You should notice the similarity between $\rho g y$ and mgh (gravitational potential energy) as well as between $\frac{1}{2}\rho v^2$ and $\frac{1}{2}mv^2$ (kinetic energy). The term P, indicating the pressure due to external fluids, is similar to the work done by an external force in a statement of Conservation of Energy.

Example 6 In the figure below, a pump forces water at a constant flow rate through a pipe whose cross-sectional area, A, gradually decreases: at the exit point, A has decreased to 1/3 its value at the beginning of the pipe. If y = 60 cm and the flow speed of the water just after it leaves the pump (Point 1 in the figure) is 1 m/s, what is the gauge pressure at Point 1?



Solution. We'll apply Bernoulli's Equation to Point 1 and the exit point, Point 2. We'll choose the level of Point 1 as the horizontal reference level; this makes $y_1 = 0$. Now, because the cross-sectional area of the pipe decreases by a factor of 3 between Points 1 and 2, the Continuity Equation tells us that the flow speed must increase by a factor of 3; that is, $v_2 = 3v_1$. Since the pressure at Point 2 is P_{atm}, Bernoulli's Equation becomes:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_{atm} + \rho g y_2 + \tfrac{1}{2}\rho v_2^2$$

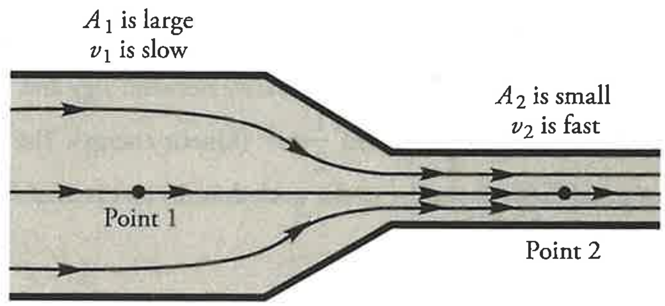
Now, P_1 is the total pressure at Point 1. Recall that gauge pressure is $P_{tot} - P_{atm}$, so $P_{gauge} = P_1 - P_{atm}$.

Therefore,

$$\begin{aligned} P_1 - P_{atm} &= \rho g y_2 + \tfrac{1}{2}\rho v_2^2 - \tfrac{1}{2}\rho v_1^2 \\ &= \rho g y_2 + \tfrac{1}{2}\rho (3v_1)^2 - \tfrac{1}{2}\rho v_1^2 \\ &= \rho(g y_2 + 4v_1^2) \\ &= (1000\ \text{kg/m}^3)[(10\ \text{m/s}^2)(0.6\ \text{m}) + 4(1\ \text{m/s})^2] \\ &= 10^4\ \text{Pa} \end{aligned}$$

The Bernoulli Effect

Consider the two points labeled in the pipe shown below:



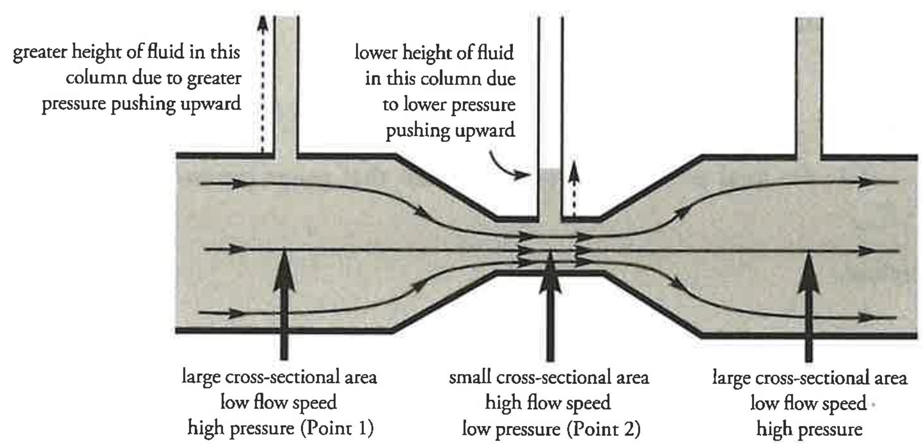
Since the height of the fluid flow is constant in this case, the terms in Bernoulli's Equation that involve height will cancel, leaving us with:

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

We already know from the Continuity Equation ($f = Av$) that the speed increases as the cross-sectional area of the pipe decreases; that is, since $A_2 < A_1$, we know that $v_2 > v_1$, so the equation above tells us that $P_2 < P_1$. This shows that:

> At comparable heights, the pressure is lower where the flow speed is greater.

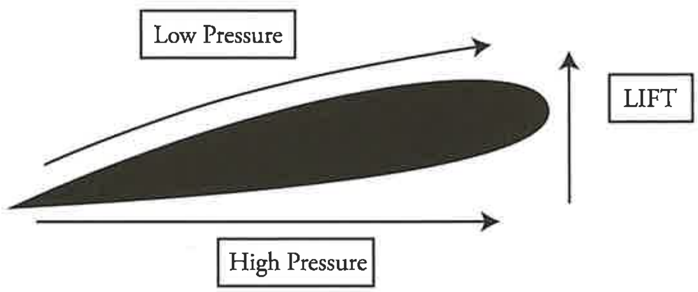
This is known as the **Bernoulli** (or **Venturi**) **Effect**, and is illustrated in the figure below.



Study Break
You're almost done with all of the content chapters! Way to go! Wrap up Chapter 11 and then give yourself a break—go for a walk, take a dance break, get a snack—before you continue on and tackle a practice test. You deserve it!

The height of the liquid column above Point 2 is less than the height of the liquid column above Point 1 because the pressure at Point 2 is lower than the pressure at Point 1 due to the fact that the flow speed at Point 2 is greater than the flow speed at Point 1.

The Bernoulli Effect also accounts for many everyday phenomena. It's what allows airplanes to fly, curve balls to curve, and tennis balls hit with top spin to drop quickly. You may have seen skydivers or motorcycle riders wearing a jacket that seems to puff out as they move rapidly through the air. The essentially stagnant air trapped inside the jacket is at a much higher pressure than the air whizzing by outside, and as a result, the jacket expands outward. The drastic drop in air pressure that accompanies the high winds in a hurricane or tornado is yet another example. In fact, if high winds streak across the roof of a house whose windows are closed, the outside air pressure can be reduced so much that the air pressure inside the house (where the air speed is essentially zero) can be great enough to blow the roof off.



Air flow: The upper side of the wing is longer, so air takes more time to reach the edge. If it takes the same time (but longer distance) [$\uparrow d = \uparrow v\ t$], then the top air is moving faster than the bottom air. The air on the bottom has greater pressure and pushes up on the wing, giving airplanes lift force.

CHAPTER 11 KEY TERMS

fluids
pressure
pascal
atmosphere
hydrostatic pressure
buoyant force (buoyancy)
Archimedes' Principle
volume flow rate
Continuity Equation
Bernoulli's Equation
viscosity
streamline
turbulent
Bernoulli (or Venturi) Effect

Chapter 11 Review Questions

Answers and explanations can be found in Chapter 12.

Section I: Multiple Choice

1 Mark for Review

A large tank is filled with water to a depth of 6 m. If Point X is 1 m from the bottom and Point Y is 2 m from the bottom, how does P_X, the hydrostatic pressure due to the water at Point X, compare to P_Y, the hydrostatic pressure due to the water at Point Y?

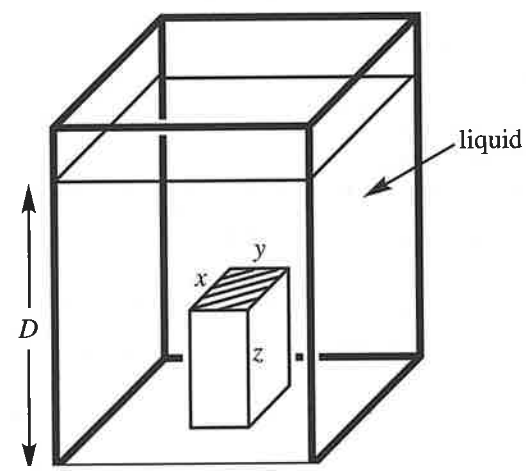
(A) $P_X = 2P_Y$

(B) $2P_X = P_Y$

(C) $5P_X = 4P_Y$

(D) $4P_X = 5P_Y$

2 Mark for Review



In the figure above, a box of dimensions x, y, and z rests on the bottom of a tank filled to depth D with a liquid of density ρ. If the tank is open to the atmosphere, what is the force on the (shaded) top of the box?

(A) $xy(P_{atm} + \rho gD)$

(B) $xyz[P_{atm} - \rho g(z - D)]$

(C) $xy[P_{atm} + \rho g(D - z)]$

(D) $xyz[P_{atm} + \rho gD]$

Questions 3 through 4 refer to the following.

An experiment is performed in which a cube is suspended from a spring scale. The cube is lowered into a beaker of water.

3 Mark for Review

If the independent variable is the distance the cube is lowered and the dependent variable is the reading on the scale, what will a graph of the data show?

(A) The graph will show a positive correlation and be linear.

(B) The graph will show a negative correlation and be linear.

(C) The graph will show a positive correlation, but will be nonlinear.

(D) The graph will show a negative correlation, but will be nonlinear.

4 Mark for Review

If the cube is replaced with a sphere and the same experiment is performed, which correctly describes the graph of the variables for the new experiment?

(A) The graph will be the same for both experiments because both experiments have the same independent and dependent variables.

(B) The graph will be different for both experiments because both experiments have the same independent and dependent variables.

(C) The graph will be different for both experiments because they have different independent variables but the same dependent variable.

(D) The graph will be different for both experiments because they have different dependent variables but the same independent variable.

5 Mark for Review

A block of Styrofoam, with a density of ρ_S and volume V, is pushed completely beneath the surface of a liquid whose density is ρ_L and released from rest. Given that $\rho_L > \rho_S$, which of the following expressions gives the magnitude of the block's initial upward acceleration?

(A) $(\rho_L - \rho_S)g$

(B) $\left(\frac{\rho_L}{\rho_S} - 1\right)g$

(C) $\left(\frac{\rho_L}{\rho_S} + 1\right)g$

(D) $\left[\left(\frac{\rho_L}{\rho_S}\right)^2 - 1\right]g$

6 Mark for Review

An object with a density of 2000 kg/m^3 weighs 100 N less when it's weighed while completely submerged in water than when it's weighed in air. What is the actual weight of this object?

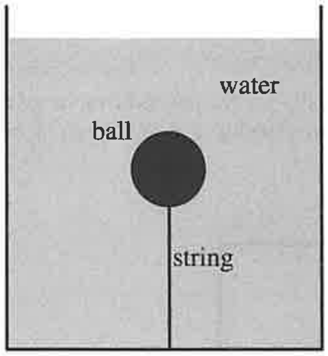
(A) 200 N

(B) 300 N

(C) 400 N

(D) 600 N

7 Mark for Review



A ball is tied to a string and placed in a container of water as shown. The water is slowly drained from the container until the water level is below the position of the ball shown, but the draining stops while there is still water in the container. The tension in the string is measured while this occurs. Which describes why the tension measurements decrease?

Ⓐ The tension decreases because there is less pressure on the ball when the water level drops because of the change in the water density.

Ⓑ The tension decreases because there is less pressure on the ball when the water level drops because the water is moving at a faster speed.

Ⓒ The tension decreases because the force of gravity on the ball decreases.

Ⓓ The tension decreases because the buoyant force on the ball decreases.

8 Mark for Review

A pump is used to send water through a hose, the diameter of which is 10 times that of the nozzle through which the water exits. If the nozzle is 1 m higher than the pump, and the water flows through the hose at 0.4 m/s, what is the difference in pressure between the pump and the atmosphere?

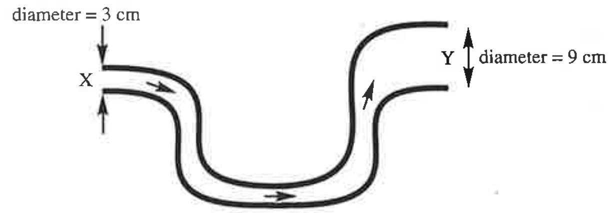
Ⓐ 108 kPa

Ⓑ 260 kPa

Ⓒ 400 kPa

Ⓓ 810 kPa

9 Mark for Review



In the pipe shown above, which carries water, the flow speed at Point X is 6 m/s. What is the flow speed at Point Y ?

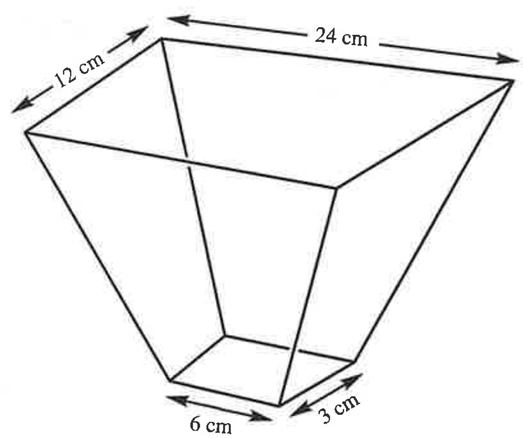
Ⓐ $\frac{2}{3}$ m/s

Ⓑ 2 m/s

Ⓒ 18 m/s

Ⓓ 54 m/s

10 Mark for Review



The figure above shows a portion of a conduit for water, one with rectangular cross-sections. If the flow speed at the top is v, what is the flow speed at the bottom?

Ⓐ $4v$

Ⓑ $8v$

Ⓒ $12v$

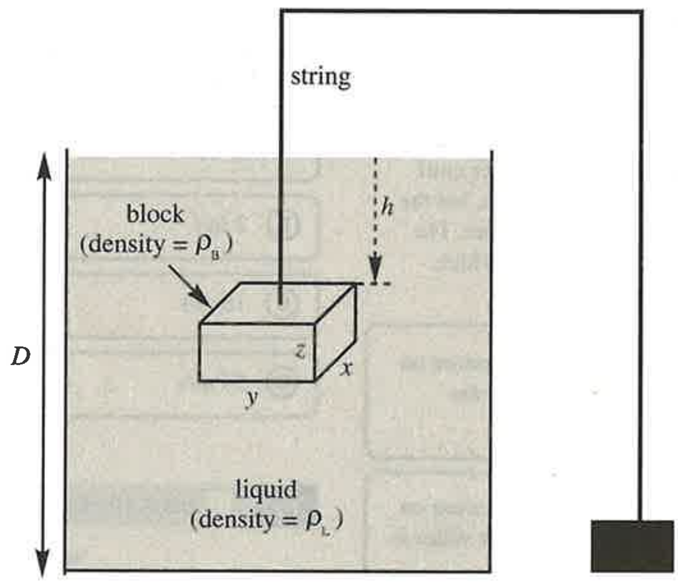
Ⓓ $16v$

Section II: Free Response

1 Mark for Review

Figure 1 shows a tank open to the atmosphere and filled to depth D with a liquid of density ρ_L. Suspended from a string is a block of density ρ_B (which is greater than ρ_L), whose dimensions are x, y, and z (meters). The top of the block is at a depth of h meters below the surface of the liquid.

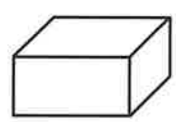
Figure 1



In each of the following, write your answer in simplest form in terms of ρ_L, ρ_B, x, y, z, h, D, and g.

A. Find the force due to the pressure on the top surface of the block and on the bottom surface. Sketch these forces on Figure 2:

Figure 2



B. What are the average forces due to the pressure on the other four sides of the block? Sketch these forces on the diagram above.

C. What is the total force on the block due to the pressure?

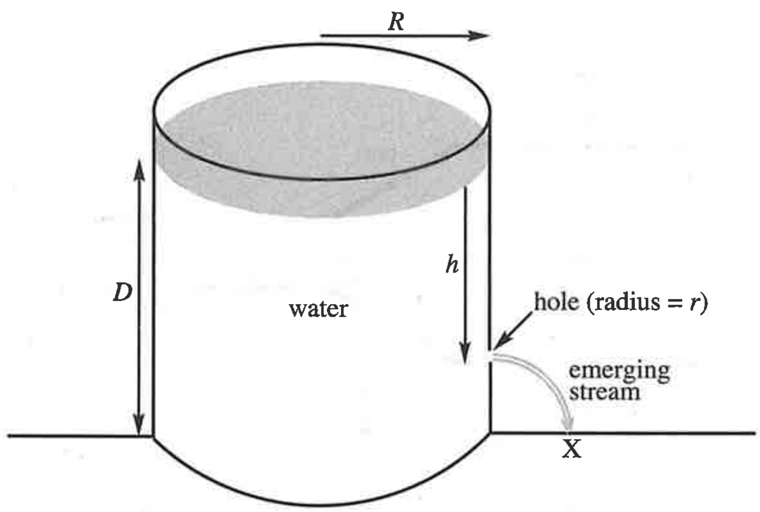
D. Find an expression for the buoyant force on the block. How does your answer here compare to your answer to part C?

E. What is the tension in the string?

2 Mark for Review

Figure 1 shows a large, cylindrical tank of water, open to the atmosphere, filled with water to depth D. The radius of the tank is R. At a depth h below the surface, a small circular hole of radius r is punctured in the side of the tank, and the point where the emerging stream strikes the level ground is labeled X.

Figure 1



In parts A through C, assume that the speed with which the water level in the tank drops is negligible.

A. At what speed does the water emerge from the hole?

B. How far is point X from the edge of the tank?

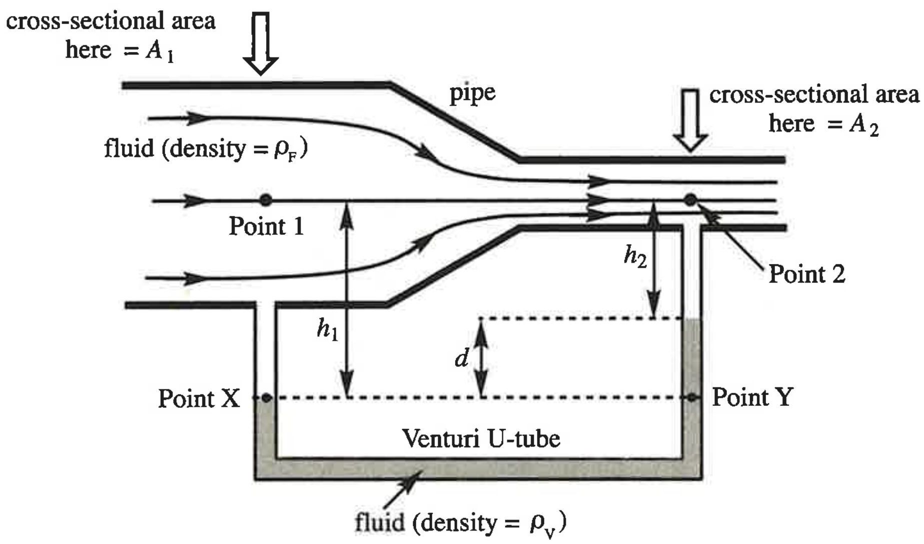
C. Assume that a second small hole is punctured in the side of the tank, a distance of $h/2$ directly above the hole shown in the figure. If the stream of water emerging from this second hole also lands at point X, find h in terms of D.

D. For this part, do *not* assume that the speed with which the water level in the tank drops is negligible, and derive an expression for the speed of efflux from the hole punctured at depth h below the surface of the water. Write your answer in terms of r, R, h, and g.

3 Mark for Review

Figure 1 shows a pipe fitted with a Venturi U-tube. Fluid of density ρ_F flows at a constant flow rate and with negligible viscosity through the pipe, which constricts from a cross-sectional area A_1 at Point 1 to a smaller cross-sectional area A_2 at Point 2. The upper portion of both sides of the Venturi U-tube contains the same fluid that's flowing through the pipe, while the lower portion is filled with a fluid of density ρ_V (which is greater than ρ_F). At Point 1 in the pipe, the pressure is P_1 and the flow speed is v_1 ; at Point 2 in the pipe, the pressure is P_2 and the flow speed is v_2. All the fluid within the Venturi U-tube is stationary.

Figure 1



A. What is P_X, the hydrostatic pressure at Point X? Write your answer in terms of P_1, ρ_F, h_1, and g.

B. What is P_Y, the hydrostatic pressure at Point Y? Write your answer in terms of P_2, ρ_F, ρ_V , h_2, d, and g.

C. Write down the result of Bernoulli's Equation applied to Points 1 and 2 in the pipe, and solve for $P_1 - P_2$.

D. Since $P_X = P_Y$, set the expressions you derived in parts A and B equal to each other, and use this equation to find $P_1 - P_2$.

E. Derive an expression for the flow speed, v_2, and the flow rate, f, in terms of A_1, A_2, d, ρ_F, ρ_V, and g. Show that v_2 and f are proportional to $\sqrt{d}$.

Chapter 11 Summary

- Density is given by $\rho = \frac{m}{V}$. Pressure is given by $P = \frac{F}{A}$. Hydrostatic pressure can be found using $P = P_0 + \rho gh$, where ρgh is the pressure at a given depth below the surface of the fluid and P_0 is the pressure right above the surface of the fluid.
- The buoyant force is an upward force any object immersed in a fluid experiences due to the displaced fluid. The buoyant force is given by $F_{buoy} = \rho Vg$, where ρ is the density of the fluid and V is the volume of the fluid displaced.
- The Continuity Equation is a statement of Conservation of Energy. It says that the flow rate through a pipe (cross-sectional area times flow speed) is constant so that $A_1v_1 = A_2v_2$. This expresses the idea that a larger cross-sectional area of pipe will experience fluids traveling at a lower flow speed.
- Bernoulli's Equation is a statement of Conservation of Energy.

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{constant}$$