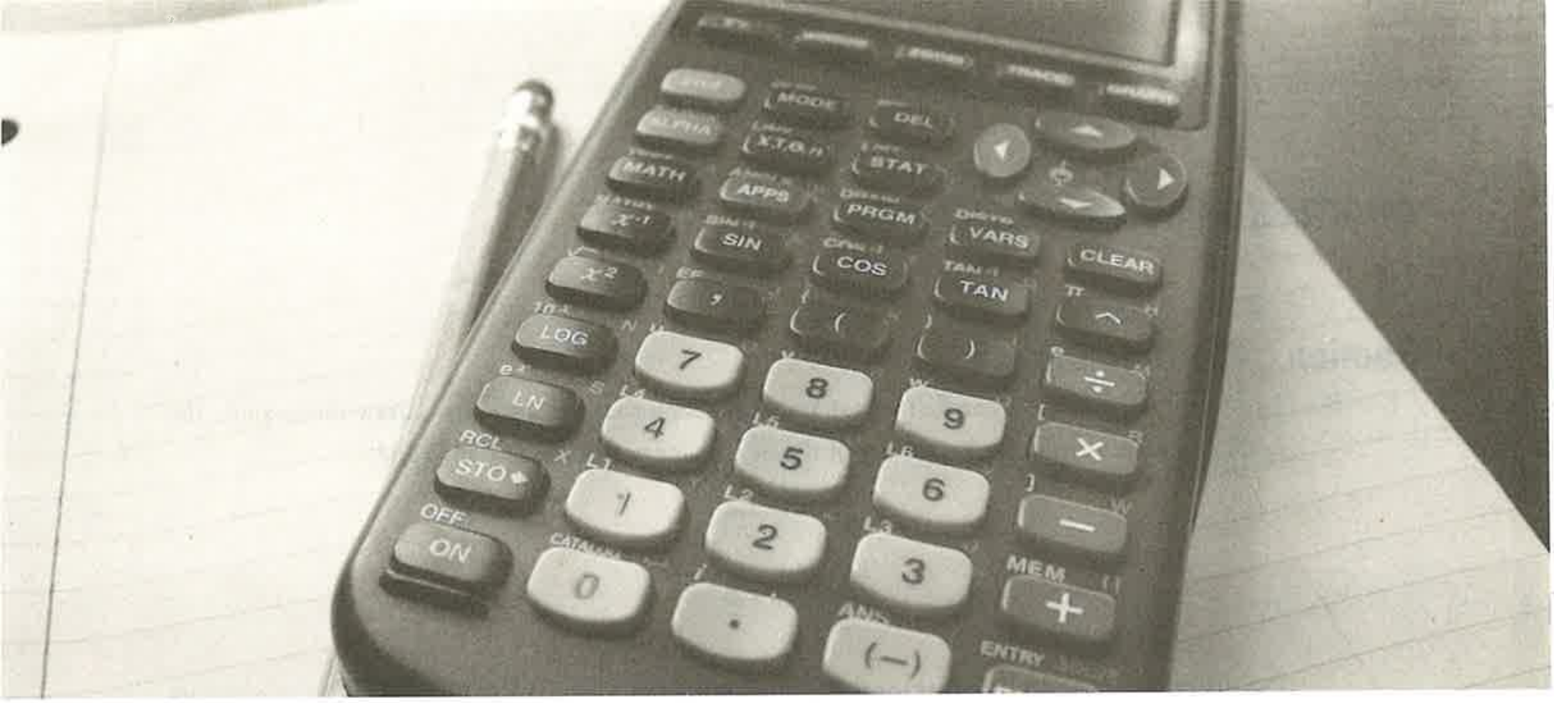
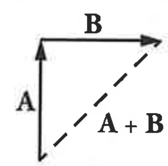


Chapter 12
Answers and Explanations to the Chapter Review Questions

CHAPTER 3 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B** To add the vectors, draw the first vector and then, from the end of the first, draw the second. The resultant vector is from the beginning of the first vector to the end of the second:



The direction of the resultant vector is therefore northeast, eliminating (C) and (D). Since vectors **A** and **B** are perpendicular to each other and equal in magnitude, the magnitude of the resultant vector can be found using the Pythagorean Theorem:

$$a^2 + b^2 = c^2 \Rightarrow m^2 + m^2 = c^2 \Rightarrow 2m^2 = c^2 \Rightarrow m\sqrt{2} = c$$

This makes (B) correct.

2. **B** To add the three vectors, add their components separately:

$$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = (0 - 10 + 5)\hat{\mathbf{i}} + (-20 + 0 + 10)\hat{\mathbf{j}} = -5\hat{\mathbf{i}} - 10\hat{\mathbf{j}}$$

3. **C** The magnitude of a vector is given by:

$$A = \sqrt{(Ax)^2 + (Ay)^2}$$

If both components of the vector are doubled, the new magnitude, A', will be:

$$A' = \sqrt{(2Ax)^2 + (2Ay)^2} = \sqrt{4(Ax)^2 + 4(Ay)^2} = \sqrt{4[(Ax)^2 + (Ay)^2]} = 2\sqrt{(Ax)^2 + (Ay)^2} = 2A$$

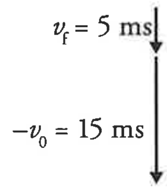
so the magnitude of the vector will also be doubled.

The angle/direction of a vector is given by:

$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right)$$

As can be observed in this equation, doubling the magnitude of both components of the vector will have no effect on the direction of the vector.

4. **D** Subtracting the vector $\boldsymbol{v}_0$ is equivalent to adding the negative of $\boldsymbol{v}_0$ to $\boldsymbol{v}_f$. Since $\boldsymbol{v}_f$ and $-\boldsymbol{v}_0$ both point south, adding the two vectors results in a vector that is the sum of their two magnitudes and also points south:



This corresponds to (D).

5. **B** The components of the vector must satisfy the equation:

$$A = 10 = \sqrt{(Ax)^2 + (Ay)^2}$$

The only answer choice with components that satisfy this equation is (B): $10 = \sqrt{6^2 + 8^2}$.

6. **B** The vector sum of **A** + **B** can be found by adding the individual components:

$$\mathbf{A} + \mathbf{B} = (1 + 4)\hat{\imath} + (-2 - 5)\hat{\jmath} = 5\hat{\imath} - 7\hat{\jmath}$$

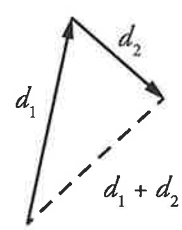
The angle that this vector makes with the x-axis is then found with:

$$\theta = \tan^{-1}\frac{(\mathbf{A} + \mathbf{B})_y}{(\mathbf{A} + \mathbf{B})_x} = \tan^{-1}\frac{7}{5}$$

This matches (B).

7. **A** If the object travels along the vectors $\boldsymbol{d}_1$ and then $\boldsymbol{d}_2$, then the total distance traveled will be the sum of the two vectors $\boldsymbol{d}_1 + \boldsymbol{d}_2$:

$$\boldsymbol{d}_1 + \boldsymbol{d}_2 = (4 + 2)\hat{\imath} + (5 + (-3))\hat{\jmath}$$
$$= 6\hat{\imath} + 2\hat{\jmath}$$

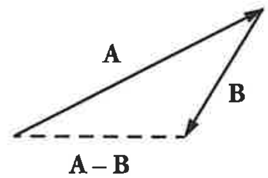


The total distance of the object from the starting position is therefore the magnitude $\boldsymbol{d}_1 + \boldsymbol{d}_2$:

$$|d_1 + d_2| = \sqrt{((d_1 + d_2)_x)^2 + ((d_1 + d_2)_y)^2} = \sqrt{6^2 + 2^2} = \sqrt{40} \approx 6.3 \text{ m}$$

That's (A).

8. **A** Subtracting vector **B** from **A** is equivalent to adding the negative of the second vector to the first:



This results in a vector pointing the direction of (A).

9. **C** The x-component of vector **A** can be used to calculate the magnitude of **A**:

$$A_x = A\cos\theta$$
$$42 = A\cos 50°$$
$$A = \frac{42}{\cos 50°}$$

The y-component can then be calculated:

$$A_y = A\sin\theta = \left(\frac{42}{\cos 50°}\right)\sin 50° = 42\tan 50° \approx 50.1$$

which corresponds to (C).

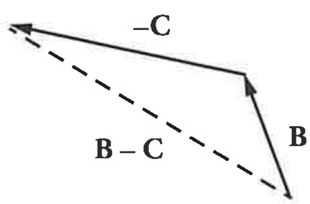
10. **B** If the two vectors were pointed in the same direction, their magnitudes would add, and since magnitudes are positive, the result could not be zero, eliminating (A). If the vectors point in opposite directions, then the magnitude of the resultant vector would be obtained by subtracting the individual magnitudes of the two vectors. The only way for the difference to equal zero is if these two vectors have the same magnitude, eliminating (C) and (D). Choice (B) is therefore correct.

Section II: Free Response

1. A. The magnitude of A is:

$$|A| = \sqrt{(A_x)^2 + (A_y)^2} = \sqrt{3^2 + 6^2} = \sqrt{9+36} = \sqrt{45} \approx 6.7$$

B. Vector **C** is subtracted from **B** by adding –**C** to **B**:



The components of **B** – **C** are found by subtracting the individual components:

$$\mathbf{B} - \mathbf{C} = (-1 - 5)\hat{\imath} + (4 - (-2))\hat{\jmath}$$
$$= -6\hat{\imath} + 6\hat{\jmath}$$

C. The components of 2**B** are found by multiplying each component of **B** by 2:

$$2\mathbf{B} = (2 \times -1)\hat{\imath} + (2 \times 4)\hat{\jmath} = -2\hat{\imath} + 8\hat{\jmath}$$

The components of **A** + 2**B** are found by adding the individual components:

$$\mathbf{A} + 2\mathbf{B} = (3 + (-2))\hat{\imath} + (6 + 8)\hat{\jmath} = \hat{\imath} + 14\hat{\jmath}$$

D. The components of **A** – **B** – **C** are found by working with the individual components:

$$\mathbf{A} - \mathbf{B} - \mathbf{C} = (3 - (-1) - 5)\hat{\imath} + (6 - 4 - (-2))\hat{\jmath} = -1\hat{\imath} + 4\hat{\jmath}$$

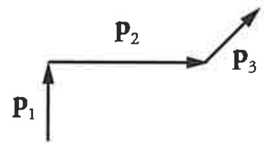
Let **D** = **A** – **B** – **C**. The magnitude of **D** is found using:

$$D = \sqrt{(D_x)^2 + (D_y)^2} = \sqrt{(-1)^2 + 4^2} = \sqrt{17} \approx 4.1$$

and the direction of **D** relative to the horizontal is found using:

$$\theta = \tan^{-1}\frac{4}{-1} \approx -76°$$

As the resultant vector has components $-1\hat{\imath} + 4\hat{\jmath}$, it is located in Quadrant II. To obtain an angle in Quadrant II, add 180° to get the correct answer of $\theta \approx 104°$ *or* 76° north of west.

2. A. 

B. The distance the ant has traveled, d, is the magnitude of the sum of the individual vectors in the ant's path: $\mathbf{d} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3$. To do this addition, the components of $\mathbf{p}_3$ must be calculated. If due east is the x-direction and due north is the y-direction, then northeast is 45° about the horizontal. Thus,

$$p_{3,x} = p_3 \cos\theta = (14 \text{ cm}) \cos 45° \approx 10 \text{ cm}$$
$$p_{3,y} = p_3 \sin\theta = (14 \text{ cm}) \sin 45° \approx 10 \text{ cm}$$

Adding the individual components,

$$\mathbf{d} = \mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = (0 + 30 \text{ cm} + 10 \text{ cm})\hat{\mathbf{i}} + (20 \text{ cm} + 0 + 10 \text{ cm})\hat{\mathbf{j}}$$
$$= 40 \text{ cm}\,\hat{\mathbf{i}} + 30 \text{ cm}\,\hat{\mathbf{j}}$$

The magnitude of this vector is:

$$d = \sqrt{(40 \text{ cm})^2 + (30 \text{ cm})^2} = \sqrt{2500 \text{ cm}^2} = 50 \text{ cm}$$

C. The ant's position is $\mathbf{d} = 40 \text{ cm}\,\hat{\mathbf{i}} + 30 \text{ cm}\,\hat{\mathbf{j}}$. If the ant walks due north, the x-component of its position will not change, but the y-component will increase. Since its final position vector will have a magnitude of 80 cm, the y-component of the final position vector, $\mathbf{f}$, can be calculated:

$$\mathbf{f} = \sqrt{(f_x)^2 + (f_y)^2} \Rightarrow 80 \text{ cm} = \sqrt{(40 \text{ cm})^2 + (f_y)^2} \Rightarrow (80 \text{ cm})^2 - (40 \text{ cm})^2 = (f_y)^2 \Rightarrow f_y \approx 69.3 \text{ cm}$$

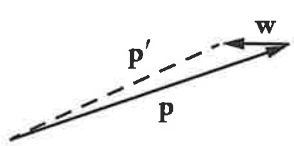
Since the ant was already 30 cm north of his original position, he can walk another 69.3 cm – 30 cm = 39.3 cm due north.

3. A. The components of $\mathbf{p}$ = 140 m directed 10° above the horizontal are given by:

$$p_x = p\cos\theta = (140 \text{ m}) \cos 10° = 138 \text{ m}$$
$$p_y = p\sin\theta = (140 \text{ m}) \sin 10° = 24.3 \text{ m}$$

Thus, $\mathbf{p} = 138 \text{ m}\,\hat{\mathbf{i}} + 24.3 \text{ m}\,\hat{\mathbf{j}}$.

B. The vector addition $\mathbf{p}' = \mathbf{p} + \mathbf{w}$ is accomplished by starting to draw the tail of the second vector at the end of the first:



C. Adding the components of $\mathbf{p} + \mathbf{w}$ yields $p' = (138 \text{ m} - 30 \text{ m})\hat{\mathbf{i}} + 24.3 \text{ m}\,\hat{\mathbf{j}} = 108 \text{ m}\,\hat{\mathbf{i}} + 24.3 \text{ m}\,\hat{\mathbf{j}}$. The new angle with the horizontal is therefore $\theta = \tan^{-1}\frac{24.3}{108} = 12.7°$.

4. A. Since **v** depends on **c** and **r**, it makes sense to use **v** as the resultant vector. The diagram shows that **r** and **c** are connected tip-to-tail, indicating that these vectors are being added together, and thus, **v** = **r** + **c**.

B. If the angle between **v** and **c** is 90°, a right triangle is formed, so the Pythagorean Theorem can be used to solve for the magnitude of **v** as a function of the other two vectors:

$$v^2 + c^2 = r^2 \Rightarrow v^2 = r^2 - c^2 \Rightarrow v = \sqrt{r^2 - c^2}$$

C. Let due east be the $\hat{\mathbf{i}}$ direction and due north be the $\hat{\mathbf{j}}$ direction. Then the vectors **c** and **v** are defined as: $\mathbf{c} = 5\,\text{m/s}\,\hat{\mathbf{i}}$ and $\mathbf{v} = -10\,\text{m/s}\,\hat{\mathbf{j}}$. Solving the vector equation from part A for **r** yields:

$$\mathbf{r} = \mathbf{v} - \mathbf{c} = (0 - 5\,\text{m/s})\hat{\mathbf{i}} + (-10\,\text{m/s} - 0)\hat{\mathbf{j}} = -5\,\text{m/s}\,\hat{\mathbf{i}} - 10\,\text{m/s}\,\hat{\mathbf{j}}$$

Using these components, the magnitude and direction of **r** are given by:

$$|r| = \sqrt{(r_x)^2 + (r_y)^2} = \sqrt{(-5)^2 + (-10)^2} = \sqrt{125} = 5\sqrt{5} \approx 11.2\text{ m/s}$$

$$\theta = \tan^{-1}\frac{-10}{-5} = -116.6° = 26.6° \text{ west of south or } 63.4° \text{ south of west}$$

CHAPTER 4 REVIEW QUESTIONS

Section I: Multiple Choice

1. **C** Distance is the length of the path traveled, which is the circumference of the circular path, so the displacement is not zero, eliminating (A). The average speed, which is total distance traveled divided by elapsed time, cannot be zero since the distance traveled is zero, eliminating (B). Average velocity is the displacement divided by time. The object starts and ends at the same position if it travels once around the path, so its displacement is zero. Therefore, its average velocity is also zero, so (C) is correct. Instantaneous acceleration is related to the change in velocity. As the object travels in a circle, the direction of its velocity is constantly changing, so the acceleration is not zero, eliminating (D).

2. **D** Section 1 represents a constant positive speed. Section 2 shows an object slowing down, moving in the positive direction. Section 3 represents an object speeding up in the negative direction. Section 4 demonstrates a constant negative speed, and section 5 represents the correct answer: slowing down moving in the negative direction. Though the slope is positive, this corresponds to acceleration, indicating that the direction of acceleration is opposite to the direction of velocity and thus is slowing down. However, the section remains in the negative quadrant, and the velocity becomes slower but is still negative.

3. **D** In parabolic motion, a projectile experiences only the constant direction due to gravity, but the velocity does not point in the same direction, so eliminate (A). Zero acceleration means no change in speed (or direction); therefore (B) is incorrect, and (D) is correct. An object whose speed remains constant but whose velocity vector is changing direction is accelerating, so eliminate (C).

4. **B** The baseball is still under the influence of Earth's gravity. Its acceleration throughout the *entire* flight is constant, equal to g downward.

5. **A** Use Big Five #3 with $v_0 = 0$:

$$x = x_0 + v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta x}{a}} = \sqrt{\frac{2(200\text{ m})}{5\text{ m/s}^2}} = 9\text{ s}$$

6. **C** Use Big Five #5 with $v_0 = 0$ (calling *down* the positive direction):

$$v^2 = v_0^2 + 2a(x - x_0) = 2a(x - x_0) \quad \Rightarrow \quad (x - x_0) = \frac{v^2}{2a} = \frac{v^2}{2g} = \frac{(30\text{ m/s})^2}{2(10\text{ m/s}^2)} = 45\text{ m}$$

7. **C** Apply Big Five #3 to the vertical motion, calling *down* the positive direction:

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = \frac{1}{2}a_y t^2 = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2\Delta y}{g}} = \sqrt{\frac{2(80\text{ m})}{10\text{ m/s}^2}} = 4\text{ s}$$

Note that the stone's initial horizontal speed (v_{0x} = 10 m/s) is irrelevant.

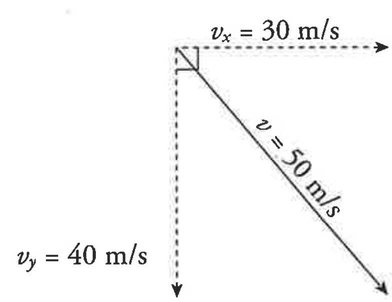
8. **B** First, determine the time required for the ball to reach the top of its parabolic trajectory (which is the time required for the vertical velocity to drop to zero).

$$v_y \overset{\text{set}}{=} 0 \quad \Rightarrow \quad v_{0y} - gt = 0 \quad \Rightarrow \quad t = \frac{v_{0y}}{g}$$

The total flight time is equal to twice this value:

$$t_t = 2t = 2\frac{v_{0y}}{g} = 2\frac{v_0 \sin\theta_0}{g} = \frac{2(10\text{ m/s})\sin 30°}{10\text{ m/s}^2} = 1\text{ s}$$

9. **C** After 4 seconds, the stone's vertical speed has changed by $\Delta v_y = a_y t$ = (10 m/s²)(4 s) = 40 m/s. Since v_{0y} = 0, the value of v_y at t = 4 is 40 m/s. The horizontal speed does not change. Therefore, when the rock hits the water, its velocity has a horizontal component of 30 m/s and a vertical component of 40 m/s.



By the Pythagorean Theorem, the magnitude of the total velocity, v, is 50 m/s.

10. **D** Since the acceleration of the projectile is always downward (because of its gravitational acceleration), the vertical speed decreases as the projectile rises and increases as the projectile falls. Choices (A), (B), and (C) are all false.

11. **B** Use Big Five #2:

$$v = v_0 + at = 5\text{ m/s} + (-10\text{ m/s}^2)(3\text{ s}) = -25\text{ m/s}$$

Because you called up the positive direction, the negative sign for the velocity indicates that the stone is traveling downward.

12. **C** The variables involved in this question are the initial velocity (given in both cases), the acceleration (constant), final velocity (0 in both cases), and displacement (the skidding distance). As the missing variable is time, use Big Five #5 to solve for the displacement:

$$v^2 = v^2 + 2a(x - x_0)$$

$$x - x_0 = \frac{v^2 - v_0^2}{2a}$$

As the initial position, x_0, and the final velocity, v^2, are equal to 0, this equation simplifies to:

$$x = \frac{-v_0^2}{2a}$$

When the initial velocity is doubled, the final position quadruples. (Note that the acceleration in this problem is negative, as the brakes cause the car to decelerate.)

Section II: Free Response

1. A. At time $t = 1$ s, the car's velocity starts to decrease as the acceleration (which is the slope of the given velocity-versus-time graph) changes from positive to negative.

B. The average velocity between $t = 0$ and $t = 1$ s is $\frac{1}{2}(v_{t=0} + v_{t=1}) = \frac{1}{2}(0 + 20 \text{ m/s}) = 10$ m/s, and the average velocity between $t = 1$ and $t = 5$ is $\frac{1}{2}(v_{t=1} + v_{t=5}) = \frac{1}{2}(20 \text{ m/s} + 0) = 10$ m/s. The two average velocities are the same.

C. The displacement is equal to the area bounded by the graph and the t-axis, taking areas above the t-axis as positive and those below as negative. In this case, the displacement from $t = 0$ to $t = 5$ s is equal to the area of the triangular region whose base is the segment along the t-axis from $t = 0$ to $t = 5$ s:

$$\Delta x\ (t = 0 \text{ to } t = 5 \text{ s}) = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2}(5 \text{ s})(20 \text{ m/s}) = 50 \text{ m}$$

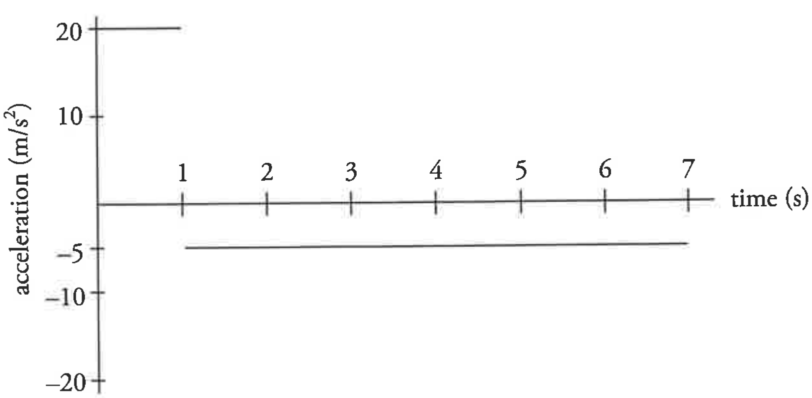
The displacement from $t = 5$ s to $t = 7$ s is equal to the negative of the area of the triangular region whose base is the segment along the t-axis from $t = 5$ s to $t = 7$ s:

$$\Delta x\ (t = 5 \text{ s to } t = 7 \text{ s}) = -\frac{1}{2} \times \text{base} \times \text{height} = -\frac{1}{2}(2 \text{ s})(10 \text{ m/s}) = -10 \text{ m}$$

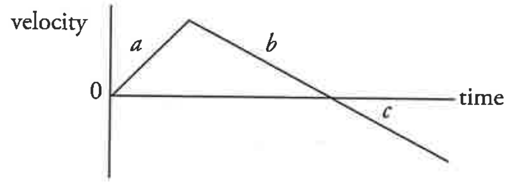
Therefore, the displacement from $t = 0$ to $t = 7$ s is:

$$\Delta x\ (t = 0 \text{ to } t = 5 \text{ s}) + \Delta s\ (t = 5 \text{ s to } t = 7 \text{ s}) = 50 \text{ m} + (-10 \text{ m}) = 40 \text{ m}$$

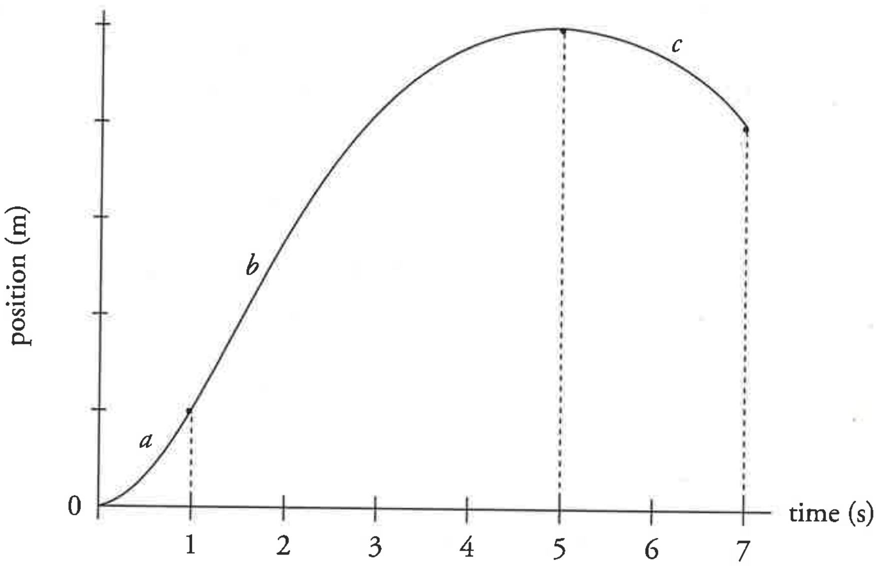
D. The acceleration is the slope of the velocity-versus-time graph. The segment of the graph from $t = 0$ to $t = 1$ s has a slope of $a = \Delta v/\Delta t = (20 \text{ m/s} - 0)/(1 \text{ s} - 0) = 20 \text{ m/s}^2$, and the segment of the graph from $t = 1$ s to $t = 7$ s has a slope of $a = \Delta v/\Delta t = (-10 \text{ m/s} - 20 \text{ s})/(7 \text{ s} - 1 \text{ s}) = -5 \text{ m/s}^2$. Therefore, the acceleration-versus-time graph is:



E.



Section a shows that the object is speeding up in the positive direction. Section b shows that the object is slowing down, yet still moving in the positive direction. At five seconds, the object has stopped for an instant. Section c shows that the object is moving in the negative direction and speeding up. The corresponding position-versus-time graph for each section would look like this:



2. A. The maximum height of the projectile occurs at the time at which its vertical velocity drops to zero:

$$v_y \overset{\text{set}}{=} 0 \quad \Rightarrow \quad v_{0y} - gt = 0 \quad \Rightarrow \quad t = \frac{v_{0y}}{g}$$

The vertical displacement of the projectile at this time is computed as follows:

$$\Delta y = v_{0y}t - \frac{1}{2}gt^2 \quad \Rightarrow \quad H = v_{0y}\frac{v_{0y}}{g} - \frac{1}{2}g\left(\frac{v_{0y}}{g}\right)^2 = \frac{v_{0y}^2}{2g} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

B. The total flight time is equal to twice the time computed in part A:

$$t_t = 2t = 2\frac{v_{0y}}{g}$$

The horizontal displacement at this time gives the projectile's range:

$$\Delta x = v_{0x}t \quad \Rightarrow \quad R = v_{0x}t_t = \frac{v_{0x} \cdot 2v_{0y}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g} \text{ or } \frac{v_0^2 \sin 2\theta_0}{g}$$

C. For any given value of v_0, the range,

$$\Delta x = v_{0x}t \quad \Rightarrow \quad R = v_{0x}t_t = \frac{v_{0x} \cdot 2v_{0y}}{g} = \frac{2v_0^2 \sin\theta_0 \cos\theta_0}{g} \text{ or } \frac{v_0^2 \sin 2\theta_0}{g}$$

will be maximized when $\sin 2\theta_0$ is maximized. This occurs when $2\theta_0 = 90°$, that is, when $\theta_0 = 45°$.

D. Set the general expression for the projectile's vertical displacement equal to h and solve for the two values of t (assuming that $g = +10 \text{ m/s}^2$):

$$v_{0y}t - \frac{1}{2}gt^2 \overset{\text{set}}{=} h \quad \Rightarrow \quad \frac{1}{2}gt^2 - v_{0y}t + h = 0$$

Applying the quadratic formula, find that

$$t = \frac{v_{0y} \pm \sqrt{(-v_{0y})^2 - 4(\frac{1}{2}g)(h)}}{2(\frac{1}{2}g)} = \frac{v_{0y} \pm \sqrt{v_{0y}^2 - 2gh}}{g}$$

Therefore, the two times at which the projectile crosses the horizontal line at height h are

$$t_1 = \frac{v_{0y} - \sqrt{v_{0y}^2 - 2gh}}{g} \quad \text{and} \quad t_2 = \frac{v_{0y} + \sqrt{v_{0y}^2 - 2gh}}{g}$$

so the amount of time that elapses between these events is:

$$\Delta t = t_2 - t_1 = \frac{2\sqrt{v_{0y}^2 - 2gh}}{g}$$

3. A. The cannonball will certainly reach the wall (which is only 220 m away) since the ball's range is:

$$R = \frac{v_0^2 \sin 2\theta_0}{g} = \frac{(50 \text{ m/s})^2 \sin 2(40°)}{9.8 \text{ m/s}^2} = 251 \text{ m}$$

You simply need to make sure that the cannonball's height is less than 30 m at the point where its horizontal displacement is 220 m (so that the ball actually hits the wall rather than flying over it). To do this, find the time at which $x = 220$ m by first writing:

$$x = v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}} = \frac{x}{v_0 \cos\theta_0} \qquad (1)$$

Thus, the cannonball's vertical position can be written in terms of its horizontal position as follows:

$$y = v_{0y}t - \tfrac{1}{2}gt^2 = v_0 \sin\theta_0 \frac{x}{v_0 \cos\theta_0} - \tfrac{1}{2}g\left(\frac{x}{v_0 \cos\theta_0}\right)^2$$
$$= x\tan\theta_0 - \frac{gx^2}{2v_0^2 \cos^2\theta_0} \qquad (2)$$

Substituting the known values for x, θ_0, g, and v_0, you get:

$$y(\text{at } x = 220 \text{ m}) = (220 \text{ m})\tan 40° - \frac{(9.8 \text{ m/s}^2)(220 \text{ m})^2}{2(50 \text{ m/s})^2 \cos^2 40°}$$
$$= 23 \text{ m}$$

This is indeed less than 30 m, as desired.

B. From Equation (1) derived in part A,

$$t = \frac{x}{v_0 \cos\theta_0} = \frac{220 \text{ m}}{(50 \text{ m/s})\cos 40°} = 5.7 \text{ s}$$

C. The height at which the cannonball strikes the wall was determined in part A to be 23 m.

4\. A. For parabolic trajectories, the total flight time can be determined by doubling the amount of time it takes for the projectile to reach its apex. However, this trajectory is NOT parabolic. As the initial position is 25 m, the final position is 0 m, the initial velocity is $v_0 = v_0 \sin\theta = 40 \sin 30° = 40\left(\frac{1}{2}\right) = 20$ m/s, and the acceleration is –10 m/s^2; the missing variable is the final velocity, so the flight time of the cannonball can be computed using Big Five #3:

$$x_y = x_0 + v_0 t + \frac{1}{2}at^2$$

$$0 = 25 \text{ m} + 20 \text{ m/s} \cdot t + \frac{1}{2}(-10 \text{ m/s}^2) \cdot t^2$$

$$0 = 25 \text{ m} + 20 \text{ m/s} \cdot t - 5 \text{ m/s}^2 \cdot t^2$$

$$0 = -5 \text{ m/s}^2 \cdot t^2 + 20 \text{ m/s} \cdot t + 25 \text{ m}$$

Applying the quadratic formula, find that,

$$t = \frac{-20 \pm \sqrt{20^2 - 4(25)(-5)}}{2(-5)} = \frac{-20 \pm \sqrt{400 + 500}}{-10} = \frac{-20 \pm \sqrt{900}}{-10} = \frac{-20 \pm 30}{-10}$$

$$t = -1 \text{ s or } t = 5 \text{ s}$$

As time cannot be a negative value, the flight time of the cannonball is 5 seconds.

B. As the horizontal speed of a projectile is constant, the range is given by:

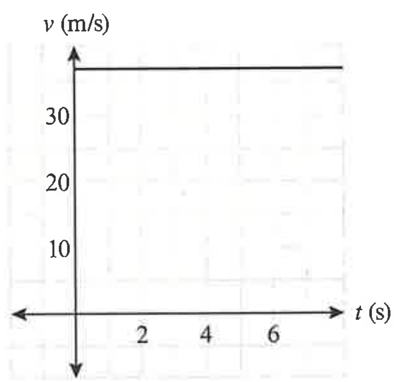
$$\Delta x = v_{0x} t = 40 \text{ m/s} \cdot \cos 30° \cdot 5 \text{ s} = 173 \text{ m}$$

C. The cannonball is fired with an initial horizontal velocity of:

$$v_{0y} = 40 \text{ m/s} \cdot \cos 30° = 34.6 \text{ m/s}$$

As there is no horizontal acceleration, the horizontal speed of the projectile remains constant throughout the entire flight, leading to a flat line with a y-value of 34.6 m/s over the flight time of 5 s. The horizontal speed-versus-time graph can then be plotted.

Horizontal Speed Versus Time



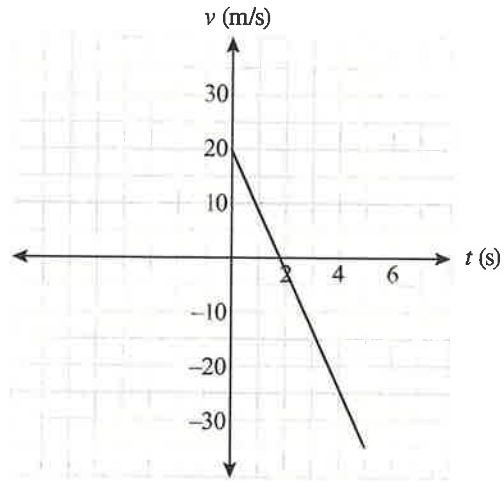
The initial vertical velocity of the cannonball is:

$$v_{0y} = 40 \text{ m/s} \cdot \sin 30° = 20 \text{ m/s}$$

In the vertical direction, there is a constant acceleration due to gravity (a = 10 m/s^2). As the slope of the velocity-versus-time graph is equal to the acceleration, the resulting velocity-versus-time graph is a linear line with a negative slope since acceleration due to gravity points downward. As the magnitude of the acceleration of gravity is 10 m/s^2, the vertical velocity of the cannonball thus decreases by 10 m/s every second. With an initial velocity of 20 m/s, this means that after 2 s, the vertical velocity of the projectile is 0 m/s. After a total of 5 s, the vertical velocity of the projectile is –30 m/s.

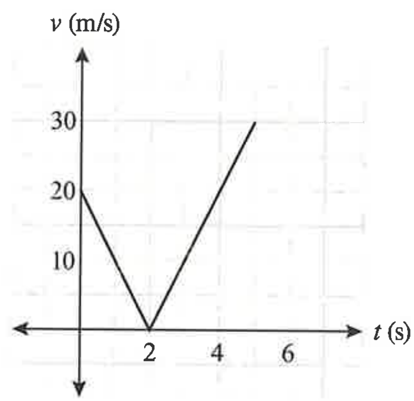
This can be visualized in the vertical velocity versus time graph below:

Vertical Velocity Versus Time



As the problem asks for the graph of the vertical speed versus time, the absolute value of the graph must be drawn to get the correct plot.

Vertical Speed Versus Time

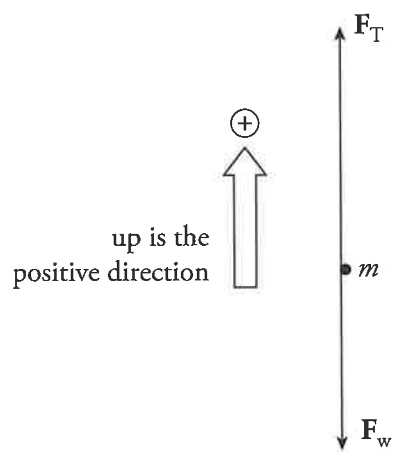


CHAPTER 5 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B** Because the person is not accelerating, the net force he feels must be zero. Therefore, the magnitude of the upward normal force from the floor must balance that of the downward gravitational force. Although these two forces have equal magnitudes, they do not form an action-reaction pair because they both act on the same object (namely, the person). The forces in an action-reaction pair always act on different objects. The correct action-reaction pair in this situation is the Earth's pull on the person (the weight) and the person's pull on the Earth.

2. **D** First, draw a free-body diagram:



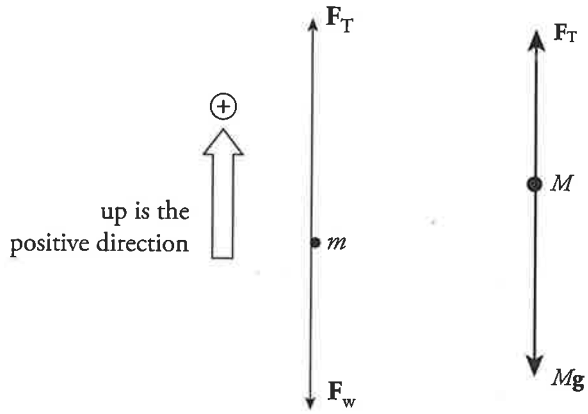
The person exerts a downward force on the scale, and the scale pushes up on the person with an equal (but opposite) force, $\mathbf{F}_N$. Thus, the scale reading is F_N, the magnitude of the normal force. Since $F_N - F_w = ma$, you get $F_N = F_w + ma = (800\text{ N}) + [800\text{ N}/(10\text{ m/s}^2)](5\text{ m/s}^2) = 1200\text{ N}$.

3. **A** The net force that the object feels on the inclined plane is $mg\sin\theta$, the component of the gravitational force that is parallel to the ramp. Since $\sin\theta = (5\text{ m})/(20\text{ m}) = \frac{1}{4}$, you get $F_{net} = (2\text{ kg})(10\text{ N/kg})(\frac{1}{4}) = 5\text{ N}$.

4. **C** The net force on the block is $F - F_f = F - \mu_k F_N = F - \mu_k F_w = (18\text{ N}) - (0.4)(20\text{ N}) = 10\text{ N}$. Since $F_{net} = ma = (F_w/g)a$, you get $10\text{ N} = [(20\text{ N})/(10\text{ m/s}^2)]a$, which gives $a = 5\text{ m/s}^2$.

5. **A** The force pulling the block down the ramp is $mg\sin\theta$, and the maximum force of static friction is $\mu_s F_N = \mu_s mg\cos\theta$. If $mg\sin\theta$ is greater than $\mu_s mg\cos\theta$, then there is a net force down the ramp, and the block will accelerate down. So, the question becomes, "Is $\sin\theta$ greater than $\mu_s\cos\theta$?" Since $\theta = 30°$ and $\mu_s = 0.5$, the answer is "yes."

6. **D** One way to attack this question is to notice that if the two masses happen to be equal, that is, if $M = m$, then the blocks won't accelerate (because their weights balance). The only expression given that becomes zero when $M = m$ is the one given in (D). Draw a free-body diagram:



Newton's Second Law gives the following two equations:

$F_T - mg = ma$ (1)

$F_T - Mg = M(-a)$ (2)

Subtracting these equations yields $Mg - mg = ma + Ma = (M + m)a$, so

$$a = \frac{Mg - mg}{M + m} = \frac{M - m}{M + m}g$$

7. **D** If $F_{net} = 0$, then $a = 0$. No acceleration means constant speed (possibly, but not necessarily zero) with no change in direction. Therefore, (B) and (C) are false, and (A) is not necessarily true.

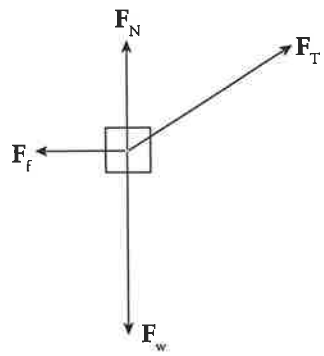
8. **C** The horizontal motion across the frictionless tables is unaffected by (vertical) gravitational acceleration. It would take as much force to accelerate the block across the table on Earth as it would on the Moon. (If friction *were* taken into account, then the smaller weight of the block on the Moon would imply a smaller normal force by the table and hence a smaller frictional force. Less force would be needed on the Moon in this case.)

9. **D** The maximum force which static friction can exert on the crate is $\mu_s F_N = \mu_s F_w = \mu_s mg =$ (0.4)(100 kg)(10 N/kg) = 400 N. Since the force applied to the crate is only 344 N, static friction is able to apply that same magnitude of force on the crate, keeping it stationary. Choice (B) is incorrect because the static friction force is *not* the reaction force to **F**; both **F** and $\mathbf{F}_{f\,(static)}$ act on the same object (the crate) and therefore cannot form an action-reaction pair.

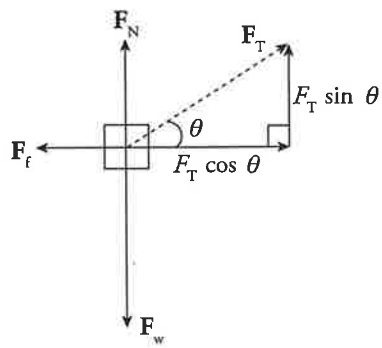
10. **A** With Crate #2 on top of Crate #1, the force pushing downward on the floor is greater, so the normal force exerted by the floor on Crate #1 is greater, which increases the friction force. Choices (B), (C), and (D) are all false.

Section II: Free Response

1. A. The forces acting on the crate are $\mathbf{F}_T$ (the tension in the rope), $\mathbf{F}_w$ (the weight of the block), $\mathbf{F}_N$ (the normal force exerted by the floor), and $\mathbf{F}_f$ (the force of kinetic friction):



B. First, break $\mathbf{F}_T$ into its horizontal and vertical components:



Since the net vertical force on the crate is zero, you get $F_N + F_T \sin\theta = F_w$, so $F_N = F_w - F_T \sin\theta = mg - F_T \sin\theta$.

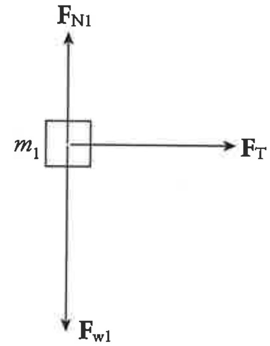
C. From part B, notice that the net horizontal force acting on the crate is:

$$F_T \cos\theta - F_f = F_T \cos\theta - \mu F_N = F_T \cos\theta - \mu(mg - F_T \sin\theta)$$

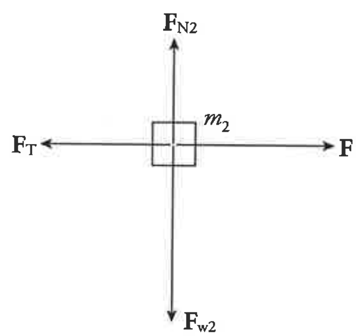
so the crate's horizontal acceleration across the floor is:

$$a = \frac{F_{net}}{m} = \frac{F_T \cos\theta - \mu(mg - F_T \sin\theta)}{m}$$

2. A. The forces acting on Block #1 are $\mathbf{F}_T$ (the tension in the string connecting it to Block #2), $\mathbf{F}_{w1}$ (the weight of the block), and $\mathbf{F}_{N1}$ (the normal force exerted by the tabletop):



B. The forces acting on Block #2 are $\mathbf{F}$ (the pulling force), $\mathbf{F}_T$ (the tension in the string connecting it to Block #1), $\mathbf{F}_{w2}$ (the weight of the block), and $\mathbf{F}_{N2}$ (the normal force exerted by the tabletop):



C. Newton's Second Law applied to Block #2 yields $F - F_T = m_2a$ and applied to Block #1 yields $F_T = m_1a$. Adding these equations, you get $F = (m_1 + m_2)a$, so:

$$a = \frac{F}{m_1 + m_2}$$

D. Substituting the result of part C into the equation $F_T = m_1a$, you get:

$$F_T = m_1a = \frac{m_1}{m_1 + m_2}F$$

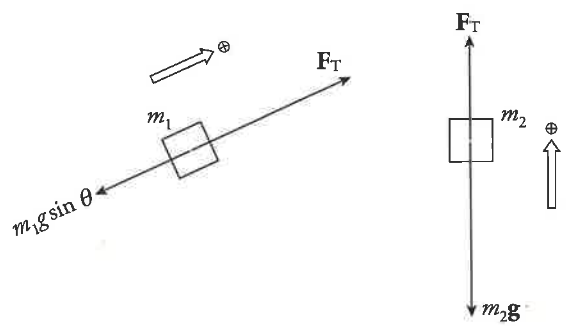
E. i. Since the force $\mathbf{F}$ must accelerate all three masses—m_1, m, and m_2—the common acceleration of all parts of the system is:

$$a = \frac{F}{m_1 + m + m_2}$$

ii. Let $\mathbf{F}_{T1}$ denote the tension force in the connecting string acting on Block #1, and let $\mathbf{F}_{T2}$ denote the tension force in the connecting string acting on Block #2. Then, Newton's Second Law applied to Block #1 yields $F_{T1} = m_1a$ and applied to Block #2 yields $F - F_{T2} = m_2a$. Therefore, using the value for a computed above, you get:

$$\begin{aligned} F_{T2} - F_{T1} &= (F - m_2a) - m_1a \\ &= F - (m_1 + m_2)a \\ &= F - (m_1 + m_2)\frac{F}{m_1 + m + m_2} \\ &= F\left(1 - \frac{m_1 + m_2}{m_1 + m + m_2}\right) \\ &= F\frac{m}{m_1 + m + m_2} \end{aligned}$$

3. A. First, draw free-body diagrams for the two boxes:



Applying Newton's Second Law to the boxes yields the following two equations:

$$F_T - m_1 g \sin\theta = m_1 a \qquad (1)$$

$$F_T - m_2 g = m_2(-a) \qquad (2)$$

Subtract the equations and solve for a:

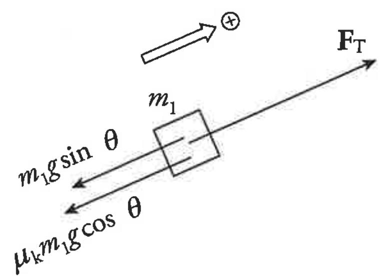
$$m_2 g - m_1 g \sin\theta = (m_1 + m_2)a$$

$$a = \frac{m_2 - m_1 \sin\theta}{m_1 + m_2}g$$

i. For a to be positive, you must have $m_2 - m_1 \sin\theta > 0$, which implies that $\sin\theta < m_2/m_1$, or, equivalently, $\theta < \sin^{-1}(m_2/m_1)$.

ii. For a to be zero, you must have $m_2 - m_1 \sin\theta = 0$, which implies that $\sin\theta = m_2/m_1$, or, equivalently, $\theta = \sin^{-1}(m_2/m_1)$.

B. Including the force of kinetic friction, the force diagram for m_1 is:



Since $F_f = \mu_k F_N = \mu_k m_1 g \cos\theta$, applying Newton's Second Law to the boxes yields these two equations:

$$F_T - m_1 g \sin\theta - \mu_k mg \cos\theta = m_1 a \qquad (1)$$

$$m_2 g - F_T = m_2 a \qquad (2)$$

Add the equations and solve for a:

$$m_2 g - m_1 g \sin\theta - \mu_k mg \cos\theta = (m_1 + m_2)a$$

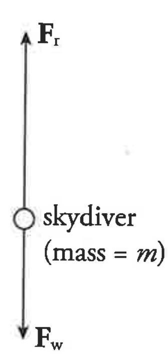
$$a = \left(\frac{m_2 - m_1(\sin\theta + \mu_k \cos\theta)}{m_1 + m_2}\right)g$$

In order for a to be equal to zero (so that the box of mass m_1 slides up the ramp with constant velocity),

$$m_2 - m_1(\sin\theta + \mu_k \cos\theta) = 0$$

$$\sin\theta + \mu_k \cos\theta = \frac{m_2}{m_1}$$

4. A. The forces acting on the skydiver are $\mathbf{F}_r$, the force of air resistance (upward), and $\mathbf{F}_w$, the weight of the skydiver (downward):



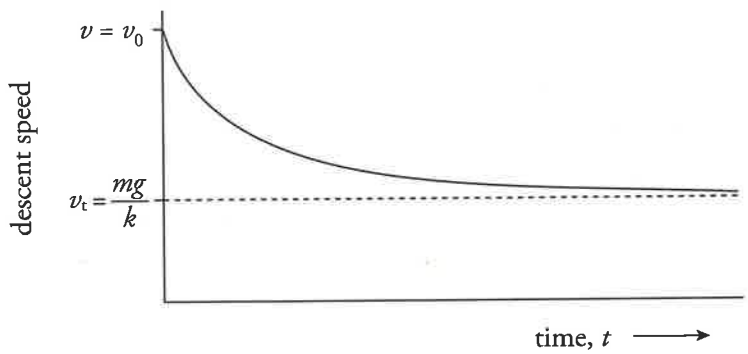
B. Since $F_{net} = F_w - F_r = mg - kv$, the skydiver's acceleration is:

$$a = \frac{F_{net}}{m} = \frac{mg - kv}{m}$$

C. Terminal speed occurs when the skydiver's acceleration becomes zero, since then the descent velocity becomes constant. Setting the expression derived in part B equal to 0, find the speed $v = v_t$ at which this occurs:

$$v = v_t \text{ when } a = 0 \quad \Rightarrow \quad \frac{mg - kv_t}{m} = 0 \quad \Rightarrow \quad v_t = \frac{mg}{k}$$

D. The skydiver's descent speed is initially v_0 and the acceleration is (close to) g. However, once the parachute opens, the force of air resistance provides a large (speed-dependent) upward acceleration, causing her descent velocity to decrease. The slope of the velocity-versus-time graph (the acceleration) is not constant but instead decreases to zero as her descent speed decreases from v_0 to v_t. Therefore, the graph is not linear.



CHAPTER 6 REVIEW QUESTIONS

Section I: Multiple Choice

1. **A** Velocity is a vector quantity which consists of a magnitude and direction. As the object moves in a circular path, the direction of its velocity is continuously changing, so its velocity is always changing, which means (A) is correct, whereas (B) is incorrect. Because the velocity is changing, the acceleration is not zero, eliminating (D). In circular motion, the acceleration vector points toward the center of the curve. As the objects move in a circle, the direction towards the center of the curve changes, so the acceleration is not constant, eliminating (C).

2. **B** When the bucket is at the lowest point in its vertical circle, it feels a tension force $\mathbf{F}_T$ upward and the gravitational force $\mathbf{F}_w$ downward. The net force toward the center of the circle, which is the centripetal force, is $F_T - F_w$. Thus,

$$F_T - F_w = m\frac{v^2}{r} \quad \Rightarrow \quad v = \sqrt{\frac{r(F_T - mg)}{m}} = \sqrt{\frac{(0.60 \text{ m})[50 \text{ N} - (3 \text{ kg})(10 \text{ N/kg})]}{3 \text{ kg}}} = 2 \text{ m/s}$$

3. **C** When the bucket reaches the topmost point in its vertical circle, the forces acting on the bucket are its weight, $\mathbf{F}_w$, and the downward tension force, $\mathbf{F}_T$. The net force, $\mathbf{F}_w + \mathbf{F}_T$, provides the centripetal force. In order for the rope to avoid becoming slack, $\mathbf{F}_T$ must not vanish. Therefore, the cut-off speed for ensuring that the bucket makes it around the circle is the speed at which $\mathbf{F}_T$ just becomes zero; any greater speed would imply that the bucket would make it around. Thus,

$$F_w + F_T = m\frac{v^2}{r} \quad \Rightarrow \quad F_w + 0 = m\frac{v^2_{\text{cut-off}}}{r} \quad \Rightarrow \quad v_{\text{cut-off}} = \sqrt{\frac{rF_w}{m}} = \sqrt{gr}$$
$$= \sqrt{(10 \text{ m/s}^2)(0.60 \text{ m})}$$
$$= 2.4 \text{ m/s}$$

4. **D** Centripetal acceleration is given by the equation $a_c = v^2/r$. Since the object covers a distance of $2\pi r$ in 1 revolution, its speed is $2\pi r$. Therefore,

$$a_c = \frac{v^2}{r} = \frac{(2\pi r)^2}{r} = 4\pi^2 r$$

5. **A** Gravitational force obeys an inverse-square law: $F_{grav} \propto 1/r^2$. Therefore, if r increases by a factor of 2, then F_{grav} decreases by a factor of $2^2 = 4$.

6. **D** Mass is an intrinsic property of an object and does not change with location. This eliminates (B). If an object's height above the surface of the Earth is equal to $2R_E$, then its distance from the center of the Earth is $3R_E$. Thus, the object's distance from the Earth's center increases by a factor of 3, so its weight decreases by a factor of $3^2 = 9$.

7. **C** The gravitational force that the Moon exerts on the planet is equal in magnitude to the gravitational force that the planet exerts on the Moon (Newton's Third Law).

8. **D** The gravitational acceleration at the surface of a planet of mass M and radius R is given by the equation $g = GM/R^2$. Therefore, for the dwarf planet Pluto,

$$g_{\text{Pluto}} = G\frac{M_{\text{Pluto}}}{R_{\text{Pluto}}^2} = G\frac{\frac{1}{500}M_{\text{Earth}}}{\left(\frac{1}{15}R_{\text{Earth}}\right)^2} = \frac{15^2}{500}\cdot G\frac{M_{\text{Earth}}}{R_{\text{Earth}}^2} = \frac{225}{500}(10g\text{ m/s}^2) = \frac{225}{50}\text{ m/s}^2$$

9. **D** The gravitational pull by Jupiter provides the centripetal force on its moon:

$$G\frac{Mm}{R^2} = \frac{mv^2}{R}$$

$$G\frac{M}{R} = v^2$$

$$G\frac{M}{R} = \left(\frac{2\pi R}{T}\right)^2$$

$$G\frac{M}{R} = \frac{4\pi^2 R^2}{T^2}$$

$$M = \frac{4\pi^2 R^3}{(GT^2)}$$

10. **D** Let the object's distance from Body A be x; then its distance from Body B is $R - x$. In order for the object to feel no net gravitational force, the gravitational pull by A must balance the gravitational pull by B. Therefore, if you let M denote the mass of the object, then

$$G\frac{m_A M}{x^2} = G\frac{m_B M}{(R-x)^2}$$

$$\frac{m}{x^2} = \frac{4m}{(R-x)^2}$$

$$\frac{(R-x)^2}{x^2} = \frac{4m}{m}$$

$$\left(\frac{R-x}{x}\right)^2 = 4$$

$$\left(\frac{R}{x}-1\right)^2 = 4$$

$$\frac{R}{x}-1 = 2$$

$$x = \frac{R}{3}$$

11. **B** Because the planet is spinning clockwise and the velocity is tangent to the circle, the velocity must point down. The acceleration and force point toward the center of the circle.

12. **B** If there were no forces or balanced forces, in and out, the satellite would have a net force of zero. If the net force were zero, the satellite would continue in a straight line and not orbit the planet, so (A) and (C) are incorrect. There is no outward force of the object, eliminating (D). (In addition, if the only force were outward, the satellite would not stay in orbit.) The force of gravity is the only force that acts on the object that keeps the object in orbit.

Section II: Free Response

1. A. Given a mass of 1 kg, a weight (F_g) of 5 N, and a diameter of 8×10^6 m (this gives you $r = 4 \times 10^6$ m), you can fill in this equation:

$$F_G = \frac{Gm_1m_2}{r^2} \Rightarrow m_1 = \frac{F_G r^2}{Gm_2}$$

This becomes

$$m_1 = \frac{(5\text{ N})(4\times10^6\text{ m})^2}{\left(6.67\times10^{-11}\dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1\text{ kg})} = 1.2 \times 10^{24}\text{ kg}$$

B. $$g = \frac{Gm_1}{r^2} \Rightarrow g = \frac{\left(6.67\times10^{-11}\dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)1.2\times10^{24}\text{ kg}}{(4\times10^6\text{ m})^2} = 5\text{ m/s}^2$$

Note that you could have also observed that, because a 1 kg mass (which normally weighs 10 N on the surface of the Earth) only weighed 5 N, gravity on this planet must be half the Earth's gravity.

If you want to look at it in terms of g, g is 10 m/s² on Earth, so you can simply convert:

$$5\text{ m/s}^2\left(\frac{1g}{10\text{ m/s}^2}\right) = 0.5g$$

C. Density is given by mass per unit volume ($\rho = \frac{m}{V}$). In addition, use the equation for the volume of a sphere as $V = \frac{4}{3}\pi r^3$ to get:

$$\rho = \frac{m}{\frac{4}{3}\pi r^3} \Rightarrow \rho = \frac{3m}{4\pi r^3} \Rightarrow \rho = \frac{3(1.2\times10^{24}\text{ kg})}{4(3.14)(4\times10^6\text{ m})^3} \text{ or}$$

$$\rho = 4480\frac{\text{kg}}{\text{m}^3}$$

2. A. The centripetal acceleration is given by the equation $a_c = \frac{v^2}{R}$. You also know that for objects traveling in circles (Earth's orbit can be considered a circle), $v = \frac{2\pi R}{T}$. Substituting this v into the previous equation for centripetal acceleration, you get $a_c = \frac{4\pi^2 R}{T^2}$. This becomes:

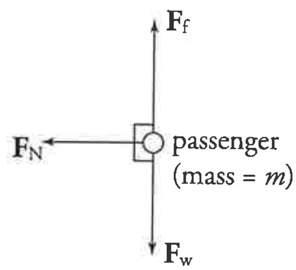
$$a_c = \frac{4\pi^2 1.5\times10^{11}\text{ m}}{(3.15\times10^7\text{ s})^2} = 6.0\times10^{-3}\text{ m/s}^2$$

B. The gravitational force is the force that keeps the Earth traveling in a circle around the Sun. More specifically, $ma_c = (6\times10^{24}\text{ kg})(6.0\times10^{-3}\text{ m/s}^2) = 3.6\times10^{22}$ N.

C. Use the Universal Law of Gravitation: $F = GmM/r^2$. You know F from the previous question, G is constant, m is the mass of the Earth (given in the question), and r is the radius from Earth to the Sun (also given in the question). So you can rearrange this equation to solve for M (mass of the Sun) as $M = F \cdot r^2/(G \cdot m)$ and then just plug in the appropriate values.

$$F = GmM/r^2 \rightarrow M = Fr^2/(Gm) =$$
$$(3.6 \times 10^{22}\text{ N})(1.5 \times 10^{11}\text{ m})^2/\{[6.67 \times 10^{-11}\text{ Nm}^2/(\text{kg}^2)][6.0 \times 10^{24}\text{ kg}]\} = 2.0 \times 10^{30}$$

3. A. The forces acting on a person standing against the cylinder wall are gravity ($\mathbf{F}_w$, downward), the normal force from the wall ($\mathbf{F}_N$, directed toward the center of the cylinder), and the force of static friction ($\mathbf{F}_f$, directed upward):



B. In order to keep a passenger from sliding down the wall, the maximum force of static friction must be at least as great as the passenger's weight: $F_{f\,(max)} \geq mg$. Since $F_{f\,(max)} = \mu_s F_N$, this condition becomes $\mu_s F_N \geq mg$.

Now, consider the circular motion of the passenger. Neither $\mathbf{F}_f$ nor $\mathbf{F}_w$ has a component toward the center of the path, so the centripetal force is provided entirely by the normal force:

$$F_N = \frac{mv^2}{r}$$

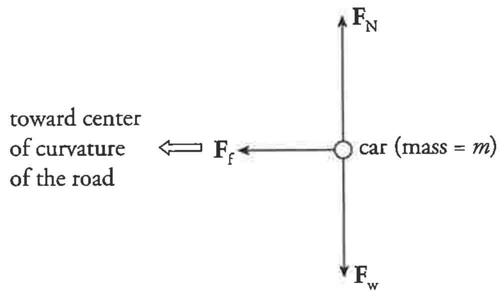
Substituting this expression for F_N into the previous equation, you get

$$\mu_s \frac{mv^2}{r} \geq mg$$
$$\mu_s \geq \frac{gr}{v^2}$$

Therefore, the coefficient of static friction between the passenger and the wall of the cylinder must satisfy this condition in order to keep the passenger from sliding down.

C. Since the mass m canceled out in deriving the expression for μ_s, the conditions are independent of mass. Thus, the inequality $\mu_s \geq gr/v^2$ holds for both the adult passenger of mass m and the child of mass $m/2$.

4. A. The forces acting on the car are gravity ($\mathbf{F}_w$, downward), the normal force from the road ($\mathbf{F}_N$, upward), and the force of static friction ($\mathbf{F}_f$, directed toward the center of curvature of the road):



B. The force of static friction (assume static friction because you *don't* want the car to slide) provides the necessary centripetal force:

$$F_f = \frac{mv^2}{r}$$

Therefore, to find the maximum speed at which static friction can continue to provide the necessary force, write:

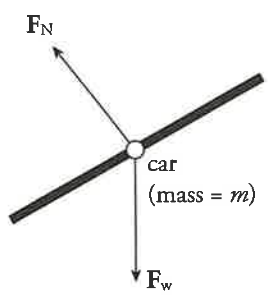
$$F_{f\,(max)} = \frac{mv_{max}^2}{r}$$

$$\mu_s F_N = \frac{mv_{max}^2}{r}$$

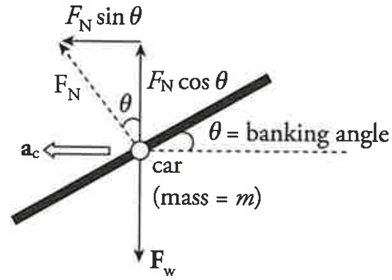
$$\mu_s mg = \frac{mv_{max}^2}{r}$$

$$v_{max} = \sqrt{\mu_s gr}$$

C. Ignoring friction, the forces acting on the car are gravity ($\mathbf{F}_w$, downward) and the normal force from the road ($\mathbf{F}_N$, which is now tilted toward the center of curvature of the road):



D. Because of the banking of the turn, the normal force is tilted toward the center of curvature of the road. The component of $\mathbf{F}_N$ toward the center can provide the centripetal force, making reliance on friction unnecessary.



However, there's no vertical acceleration, so there is no net vertical force. Therefore, $F_N \cos\theta = F_w = mg$, so $F_N = mg/\cos\theta$. The component of F_N toward the center of curvature of the turn, $F_N \sin\theta$, provides the centripetal force:

$$F_N \sin\theta = \frac{mv^2}{r}$$

$$\frac{mg}{\cos\theta}\sin\theta = \frac{mv^2}{r}$$

$$g\tan\theta = \frac{v^2}{r}$$

$$\theta = \tan^{-1}\frac{v^2}{gr}$$

CHAPTER 7 REVIEW QUESTIONS

Section I: Multiple Choice

1. **A** Since the force **F** is perpendicular to the displacement, the work it does is zero.

2. **B** By the Work–Energy Theorem,

$$W = \Delta K = \frac{1}{2}m(v^2 - v_0^2) = \frac{1}{2}(4 \text{ kg})[(6 \text{ m/s})^2 - (3 \text{ m/s})^2] = 54 \text{ J}$$

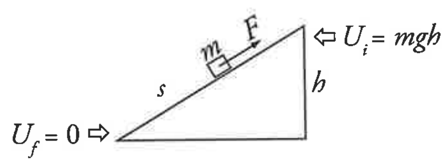
3. **B** Since the box (mass *m*) falls through a vertical distance of *h*, its gravitational potential energy decreases by *mgh*. The length of the ramp is irrelevant here. If friction were involved, then the length of the plane would matter.

4. **A** The gravitational force points downward while the book's displacement is upward. Therefore, the work done by gravity is $-mgh = -(2 \text{ kg})(10 \text{ N/kg})(1.5 \text{ m}) = -30$ J.

5. **B** The gravitational pull by the Earth provides the centripetal force on the satellite, so $\frac{GMm}{R^2} = R^2 = \frac{mc^2}{R}$. This gives $\frac{1}{2}mv^2 = \frac{GMm}{2R}$, so the kinetic energy *K* of the satellite is inversely proportional to *R*. Therefore, if *R* increases by a factor of 2, then *K* decreases by a factor of 2.

6. **B** Since the centripetal force always points along a radius toward the center of the circle and the velocity of the object is always tangent to the circle (and thus perpendicular to the radius), the work done by the centripetal force is zero. Alternatively, since the object's speed remains constant, the Work–Energy Theorem tells you that no work is being performed.

7. **C** Since a nonconservative force (namely, friction) is acting during the motion, use the modified Conservation of Mechanical Energy equation:



$$K_i + U_i + W_{\text{friction}} = K_i + U_f$$

$$0 + mgh - Fs = K_f + 0$$

$$mgh - Fs = K_f$$

8. **D** Apply Conservation of Mechanical Energy (including the negative work done by $\mathbf{F}_r$, the force of air resistance):

$$K_i + U_i + W_r = K_f + U_f$$
$$0 + mgh - F_r h = \frac{1}{2}mv^2 + 0$$
$$v = \sqrt{\frac{2h(mg - F_r)}{m}}$$
$$= \sqrt{\frac{2(40\text{ m})[(4\text{ kg})(10\text{ N/kg}) - 20\text{ N}]}{4\text{ kg}}}$$
$$= 20\text{ m/s}$$

9. **D** Because the rock has lost half of its gravitational potential energy, its kinetic energy at the halfway point is half of its kinetic energy at impact. Since K is proportional to v^2, if $K_{\text{at halfway point}}$ is equal to $\frac{1}{2}K_{\text{at impact}}$, then the rock's speed at the halfway point is $\sqrt{1/2} = 1/\sqrt{2}$ its speed at impact.

10. **D** Using the equation $P = Fv$, find that $P = (200\text{ N})(2\text{ m/s}) = 400\text{ W}$.

Section II: Free Response

1. A. Applying Conservation of Energy,

$$K_A + U_A = K_{\text{at } H/2} + U_{\text{at } H/2}$$
$$0 + mgH = \frac{1}{2}mv^2 + mg(\frac{1}{2}H)$$
$$\frac{1}{2}mgH = \frac{1}{2}mv^2$$
$$v = \sqrt{gH}$$

B. Applying Conservation of Energy again,

$$K_A + U_A = K_B + U_B$$
$$0 + mgH = \frac{1}{2}mv_B^2 + 0$$
$$v_B = \sqrt{2gH}$$

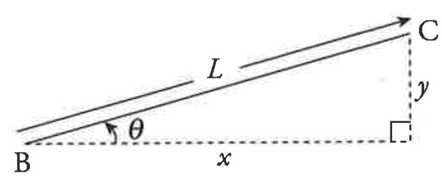
C. By the Work–Energy Theorem, you want the work done by friction to be equal (but opposite) to the kinetic energy of the box at Point B:

$$W = \Delta K = \frac{1}{2}m(v_C^2 - v_B^2) = -\frac{1}{2}mv_B^2 = -\frac{1}{2}m(\sqrt{2gH})^2 = -mgH$$

Therefore,

$$W = -mgH \quad \Rightarrow \quad -F_f x = -mgH \quad \Rightarrow \quad -\mu_k mgx = -mgH \quad \Rightarrow \quad \mu_k = H/x$$

D. Apply Conservation of Energy (including the negative work done by friction as the box slides up the ramp from B to C):



$$K_B + U_B + W_f = K_C + U_C$$
$$\frac{1}{2}m(\sqrt{2gH})^2 + 0 - F_f L = 0 + mgy$$
$$mgH + 0 - F_f L = 0 + mgy$$
$$mg(H - y) - (\mu_k mg\cos\theta)(L) = 0$$
$$\mu_k = \frac{H - y}{L\cos\theta} = \frac{H - y}{x}$$

E. The result of part B reads $v_B = \sqrt{2gH}$. Therefore, by Conservation of Mechanical Energy (with the work done by the frictional force on the slide included), you get

$$K_A + U_A + W_f = K'_B + U_B$$
$$0 + mgH + W_f = \frac{1}{2}m\left(\frac{1}{2}v_B\right)^2 + 0$$
$$mgH + W_f = \frac{1}{2}m\left(\frac{1}{2}\sqrt{2gH}\right)^2$$
$$mgH + W_f = \frac{1}{4}mgH$$
$$W_f = -\frac{3}{4}mgH$$

2. A. Using Conservation of Energy, $K_i + U_i = K_f + U_f$ and $v_i = 0$, and this becomes $U_i = K_f + U_f$ or $K_f = U_i - U_f$. This is equivalent to $\frac{1}{2}mv^2 = mgh_i - mgh_f$, which simplifies to $\frac{1}{2}v^2 - gh_i = -gh_f$ or $h_f = h_i - \frac{v^2}{2g}$. Now fill in the table.

Time (s)	Velocity (m/s)	Height (m)
0.00	0.00	1.5
0.05	1.41	1.4
0.10	2.45	1.2
0.15	3.74	0.8
0.20	3.74	0.8
0.25	3.46	0.9
0.30	3.16	1.0
0.35	2.83	1.1
0.40	3.46	0.9
0.45	4.24	0.6
0.50	4.47	0.5

B. The greatest acceleration would occur where there is the greatest change in velocity. This occurs between 0.00 and 0.05 seconds. The acceleration during that time interval is given by $a = \frac{v_f - v_i}{t_f - t_i} \Rightarrow \frac{1.41 - 0}{0.05 - 0.00}$ or $a = 28\ \text{m/s}^2$.

C. Changing the mass does not affect the time spent falling or the velocity of the object. Thus, a change in mass will not affect the results.

3. A. Use the Work–Energy Theorem:

$$W_{total} = \Delta K$$

The force doing work during the motion is provided by the force of friction:

$$W = F_f \cdot d \cdot \cos\theta = \Delta K$$

$$\mu_k F_N \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

As the force of friction is antiparallel to the direction of the displacement, $\theta = 180°$.

$$\mu_k F_N \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

$$\mu_k mg \cdot d \cdot \cos\theta = \frac{1}{2}m(v^2 - v_0^2)$$

$$\mu_k g \cdot d \cdot \cos\theta = \frac{(v^2 - v_0^2)}{2}$$

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta} = \frac{(0 - 10^2)}{2(0.2) \cdot 10 \cdot \cos 180°} = \frac{-100}{-4} = 25 \text{ m}$$

The skidding distance is 25 m.

B. The final equation for the skidding distance is

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta}$$

Since the final velocity is 0, the stopping distance is proportional to the initial speed. So if the initial speed is doubled, the skidding distance is quadrupled. This can also be determined by plugging in an initial velocity of 20 m/s:

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta} = \frac{(0 - 20^2)}{2(0.2) \cdot 10 \cdot \cos 180°} = \frac{-400}{-4} = 100 \text{ m}$$

C. Look back at the equation for the skidding distance:

$$d = \frac{(v^2 - v_0^2)}{2\mu_k \cdot g \cdot \cos\theta}$$

This equation does not include mass, so mass does not affect the skidding distance. (Note: While doubling the mass doubles the initial kinetic energy of the car, it also doubles the normal force and thus the frictional force acting on the car.)

CHAPTER 8 REVIEW QUESTIONS

Section I: Multiple Choice

1. **C** The magnitude of the object's linear momentum is $p = mv$. If $p = 6$ kg · m/s and $m = 2$ kg, then $v = 3$ m/s. Therefore, the object's kinetic energy is $K = \frac{1}{2}mv^2 = \frac{1}{2}(2\text{ kg})(3\text{ m/s})^2 = 9$ J.

2. **B** The impulse delivered to the ball, $J = F\Delta t$, equals its change in momentum. Since the ball started from rest, you get:

$$F\Delta t = mv \quad \Rightarrow \quad \Delta t = \frac{mv}{F} = \frac{(0.5\text{ kg})(4\text{ m/s})}{20\text{ N}} = 0.1\text{ s}$$

3. **D** The impulse delivered to the box, $J = \bar{F}\Delta t$, equals its change in momentum. Thus,

$$\bar{F}\Delta t = \Delta p = p_{fi} - p = m(v_{fi} - v) \quad \Rightarrow \quad \bar{F} = \frac{m(v_{fi} - v)}{\Delta t} = \frac{(2\text{ kg})(8\text{ m/s} - 4\text{ m/s})}{0.5\text{ s}} = 16\text{ N}$$

4. **C** The impulse delivered to the ball is equal to its change in momentum. The momentum of the ball was $m\mathbf{v}$ before hitting the wall and $m(-\mathbf{v})$ after. Therefore, the change in momentum is $m(-\mathbf{v}) - m\mathbf{v} = -2m\mathbf{v}$, so the magnitude of the momentum change (and the impulse) is $2mv$.

5. **B** By definition of *perfectly inelastic*, the objects move off together with one common velocity, $\mathbf{v}'$, after the collision. By Conservation of Linear Momentum,

$$\begin{aligned} m_1\mathbf{v}_1 + m_2\mathbf{v}_2 &= (m_1 + m_2)\mathbf{v}' \\ \mathbf{v}' &= \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \\ &= \frac{(3\text{ kg})(2\text{ m/s}) + (5\text{ kg})(-2\text{ m/s})}{3\text{ kg} + 5\text{ kg}} \\ &= 0.5\text{ m/s} \end{aligned}$$

6. **D** First, apply Conservation of Linear Momentum to calculate the speed of the combined object after the (perfectly inelastic) collision:

$$\begin{aligned} m_1v_1 + m_2v_2 &= (m_1 + m_2)v' \\ v' &= \frac{m_1v_1 + m_2v_2}{m_1 + m_2} \\ &= \frac{m_1v_1 + (2m_1)(0)}{m_1 + 2m_1} \\ &= \frac{1}{3}v_1 \end{aligned}$$

Therefore, the ratio of the kinetic energy after the collision to the kinetic energy before the collision is:

$$\frac{K'}{K} = \frac{\frac{1}{2}m'v'^2}{\frac{1}{2}m_1v_1^2} = \frac{\frac{1}{2}(m_1 + 2m_1)(\frac{1}{3}v_1)^2}{\frac{1}{2}m_1v_1^2} = \frac{1}{3}$$

7. **C** Total linear momentum is conserved in a collision during which the net external force is zero. If kinetic energy is lost, then by definition, the collision is not elastic.

8. **D** Because the two carts are initially at rest, the initial momentum is zero. Therefore, the final total momentum must be zero.

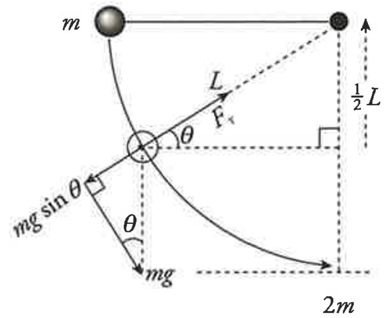
9. **C** The linear momentum of the bullet must have the same magnitude as the linear momentum of the block in order for their combined momentum after impact to be zero. The block has momentum MV to the left, so the bullet must have momentum MV to the right. Since the bullet's mass is m, its speed must be $v = MV/m$.

10. **C** In a perfectly inelastic collision, kinetic energy is never conserved; some of the initial kinetic energy is always lost to heat and some is converted to potential energy in the deformed shapes of the objects as they lock together.

11. **B** Total linear momentum is conserved in the absence of external forces. If the final speed of both objects is 0, that means the total linear momentum after the collision is 0, which then implies that the total linear momentum before the collision is also 0. As Object 2 has half the mass of Object 1, Object 2 must have twice the initial speed of Object 1 (and must be traveling in the opposite direction) in order for the total linear momentum to equal 0.

Section II: Free Response

1. A. First, draw a free-body diagram:



The net force toward the center of the steel ball's circular path provides the centripetal force. From the geometry of the diagram, you get:

$$F_T - mg\sin\theta = \frac{mv^2}{L} \qquad (*)$$

In order to determine the value of mv^2, use Conservation of Mechanical Energy:

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + mgL &= \frac{1}{2}mv^2 + mg(\frac{1}{2}L) \\ \frac{1}{2}mgL &= \frac{1}{2}mv^2 \\ mgL &= mv^2 \end{aligned}$$

Substituting this result into Equation (*), you get:

$$\begin{aligned} F_T - mg\sin\theta &= \frac{mgL}{L} \\ F_T &= mg(1+\sin\theta) \end{aligned}$$

Now, from the free-body diagram, $\sin\theta = \frac{1}{2}L/L = \frac{1}{2}$, so:

$$F_T = mg(1+\frac{1}{2}) = \frac{3}{2}mg$$

B. Apply Conservation of Energy to find the speed of the ball just before impact:

$$\begin{aligned} K_i + U_i &= K_f + U_f \\ 0 + mgL &= \frac{1}{2}mv^2 + 0 \\ v &= \sqrt{2gL} \end{aligned}$$

Now, use Conservation of Linear Momentum and Conservation of Kinetic Energy for the elastic collision to derive the expressions for the speeds of the ball, v_1, and the block, v_2, immediately after the collision. Applying Conservation of Linear Momentum yields:

$$m\sqrt{2gL} = mv_1 + 4mv_2$$

$$\sqrt{2gL} = v_1 + 4v_2$$

$$v_1 = \sqrt{2gL} - 4v_2$$

Applying Conservation of Kinetic Energy for the elastic collision yields:

$$\frac{1}{2}m(\sqrt{2gL})^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}(4m)v_2^2$$

$$2gL = v_1^2 + (4m)v_2^2$$

Plugging in the expression for v_1 from the Conservation of Linear Momentum yields:

$$2gL = (\sqrt{2gL} - 4v_2)^2 + 4v_2^2$$

$$2gL = 2gL - 8\sqrt{2gL}v_2 + 16v_2^2 + 4v_2^2$$

$$0 = 20v_2^2 - 8\sqrt{2gL}v_2$$

$$0 = 2v_2(10v_2 - 4\sqrt{2gL})$$

$$0 = 10v_2 - 4\sqrt{2gL}$$

$$10v_2 = 4\sqrt{2gL}$$

$$v_2 = \frac{2\sqrt{2}}{5}\sqrt{gL}$$

C. Using the velocity of the block immediately after the collision, v_2, solve for the velocity of the ball immediately after the collision:

$$v_1 = \sqrt{2gL} - 4v_2$$

$$v_1 = \sqrt{2gL} - 4\left(\frac{2\sqrt{2}}{5}\sqrt{gL}\right)$$

$$v_1 = \sqrt{2gL} - \frac{4 \cdot 2}{5}\sqrt{2gL}$$

Factor out $\sqrt{2gL}$ to get:

$$v_1 = \sqrt{2gL}\left(1 - \frac{8}{5}\right)$$

$$v_1 = -\frac{3}{5}\sqrt{2gL}$$

Now, apply Conservation of Mechanical Energy to find:

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}mv_1'^2 + 0 = 0 + mgh$$
$$h = \frac{v_1'^2}{2g} = \frac{\left(\frac{3}{5}\sqrt{2gL}\right)^2}{2g} = \frac{9}{25}L$$

2. A. By Conservation of Linear Momentum, $mv = (m + M)v'$, so $v' = \dfrac{mv}{m+M}$.

Now, by Conservation of Mechanical Energy,

$$K_i + U_i = K_f + U_f$$
$$\frac{1}{2}(m+M)v'^2 + 0 = 0 + (m+M)gy$$
$$\frac{1}{2}v'^2 = gy$$
$$\frac{1}{2}\left(\frac{mv}{m+M}\right)^2 = gy$$
$$v = \frac{m+M}{m}\sqrt{2gy}$$

B. Use the result derived in part A to compute the kinetic energy of the block and bullet immediately after the collision:

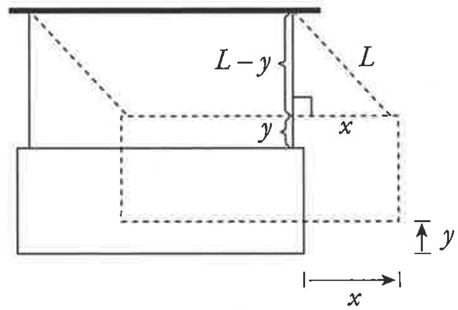
$$K' = \frac{1}{2}(m+M)v'^2 = \frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}\frac{m^2v^2}{m+M}$$

Since $K = \frac{1}{2}mv^2$, the difference is:

$$\Delta K = K' - K = \frac{1}{2}\frac{m^2v^2}{m+M} - \frac{1}{2}mv^2$$
$$= \frac{1}{2}mv^2\left(\frac{m}{m+M} - 1\right)$$
$$= K\left(\frac{-M}{m+M}\right)$$

Therefore, the fraction of the bullet's original kinetic energy that was lost is $M/(m + M)$. This energy is manifested as heat (the bullet and block are warmer after the collision than before), and some was used to break the intermolecular bonds within the wooden block to allow the bullet to penetrate.

C. From the geometry of the diagram,



the Pythagorean Theorem implies that $(L-y)^2 + x^2 = L^2$. Therefore,

$$L^2 - 2Ly + y^2 + x^2 = L^2 \quad \Rightarrow \quad y = \frac{x^2}{2L}$$

(where you have used the fact that y^2 is small enough to be neglected). Substituting this into the result of part A, derive the following equation for the speed of the bullet in terms of x and L instead of y:

$$v = \frac{m+M}{m}\sqrt{2gy} = \frac{m+M}{m}\sqrt{2g\frac{x^2}{2L}} = \frac{m+M}{m}x\sqrt{\frac{g}{L}}$$

D. No; momentum is conserved only when the net external force on the system is zero (or at least negligible). In this case, the block and bullet feel a net nonzero force that causes it to slow down as it swings upward. Since its speed is decreasing as it swings upward, its linear momentum cannot remain constant.

CHAPTER 9 REVIEW QUESTIONS

Section I: Multiple Choice

1. **C** The acceleration of a simple harmonic oscillator is not constant, since the restoring force—and consequently, the acceleration—depends on the position, so eliminate (A). The restoring force is proportional to the displacement from the equilibrium, so at the equilibrium position, the restoring force is zero, eliminating (B). Choice (C) describes a defining characteristic of simple harmonic motion, so (C) is correct. Furthermore, since period is the reciprocal of frequency, period must also be independent of amplitude, eliminating (D).

2. **B** The acceleration of the block has its maximum magnitude at the points where its displacement from equilibrium has the maximum magnitude (since $a = F/m = kx/m$). At the endpoints of the oscillation region, the potential energy is maximized and the kinetic energy (and hence the speed) is zero.

3. **D** By Conservation of Mechanical Energy, $K + U_S$ is a constant for the motion of the block. At the endpoints of the oscillation region, the block's displacement, x, is equal to $\pm A$. Since $K = 0$ here, all the energy is in the form of potential energy of the spring, $\frac{1}{2}kA^2$. Because $\frac{1}{2}kA^2$ gives the total energy at these positions, it also gives the total energy at any other position.

   Using the equation $U_S(x) = \frac{1}{2}kx^2$, find that, at $x = \frac{1}{2}A$.

$$K + U_S = \frac{1}{2}kA^2$$
$$K + \frac{1}{2}k(\frac{1}{2}A)^2 = \frac{1}{2}kA^2$$
$$K = \frac{3}{8}kA^2$$

   Therefore,

$$K/E = \frac{\frac{3}{8}kA^2}{\frac{1}{2}kA^2} = \frac{3}{4}$$

4. **C** The maximum speed of the block is given by the equation $v_{max} = A\sqrt{k/m}$. Therefore, v_{max} is inversely proportional to $\sqrt{m}$. If m is increased by a factor of 2, then v_{max} will decrease by a factor of $\sqrt{2}$.

5. **D** The period of a spring-block simple harmonic oscillator is independent of the value of g. (Recall that $T = 2\pi\sqrt{m/k}$.) Therefore, the period will remain the same.

6. **D** The frequency of a spring-block simple harmonic oscillator is given by the equation $f = (1/2\pi)\sqrt{k/m}$. Squaring both sides of this equation, you get $f^2 = (k/4\pi^2)(1/m)$. Therefore, if f^2 is plotted versus $(1/m)$, then the graph will be a straight line with slope $k/4\pi^2$. (Note: The slope of the line whose equation is $y = ax$ is a.)

7. **C** For small angular displacements, the period of a simple pendulum is essentially independent of amplitude.

8. **D** Combining Hooke's Law with Newton's Second Law, you get:

$$F = kx = ma \Rightarrow a = \frac{kx}{m} = \frac{50\text{ N/m} \cdot 4\text{ m}}{20\text{ kg}} = 10\text{ m/s}^2$$

9. **B** By Conservation of Mechanical Energy, the energy of the block is the same throughout the motion. At the amplitude, the block has potential energy $U = \frac{1}{2}kA^2$ and zero kinetic energy. At the equilibrium position, the block has kinetic energy $K = \frac{1}{2}mv^2$ and zero potential energy. Applying Conservation of Mechanical Energy to these two points in the motion yields

$$\frac{1}{2}kA^2 + 0 = 0 + \frac{1}{2}mv^2$$

$$kA^2 = mv^2$$

$$k = \frac{mv^2}{A^2} = \frac{10\text{ kg} \cdot (4\text{ m/s})^2}{(2\text{ m})^2} = 40\text{ kg/s}^2$$

The period of the block can then be calculated using the following equation:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{10\text{ kg}}{40\text{ kg/s}^2}} = \pi\text{ s} \approx 3\text{ s}$$

10. **B** The frequency of a spring-block simple harmonic oscillator is independent of the amplitude. The equation for the frequency of a spring-block simple harmonic oscillator is $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. The frequency is inversely proportional to the square root of the mass, so decreasing the mass of the block by a factor of 4 would increase the frequency by a factor of 2.

Section II: Free Response

1. A. Since the spring is compressed to 3/4 of its natural length, the block's position relative to equilibrium is $x = -\frac{1}{4}L$. Therefore, from $F_S = -kx$, find:

$$a = \frac{F_S}{m} = \frac{-k(-\frac{1}{4}L)}{m} = \frac{kL}{4m}$$

B. Let v_1 denote the velocity of Block 1 just before impact, and let v_1' and v_2' denote, respectively, the velocities of Block 1 and Block 2 immediately after impact. By Conservation of Linear Momentum, write $mv_1 = mv_1' + mv_2'$, or:

$$v_1 = v_1' + v_2' \qquad (1)$$

The initial kinetic energy of Block 1 is $\frac{1}{2}mv_1^2$. If half is lost to heat, then $\frac{1}{4}mv_1^2$ is left to be shared by Block 1 and Block 2 after impact: $\frac{1}{4}mv_1^2 = \frac{1}{2}mv_1'^2 + \frac{1}{2}mv_2'^2$, or:

$$v_1^2 = 2v_1'^2 + 2v_2'^2 \qquad (2)$$

Square Equation (1) and multiply by 2 to give:

$$2v_1^2 = 2v_1'^2 + 4v_1'v_2' + 2v_2'^2 \qquad (1')$$

Then subtract Equation (2) from Equation (1′):

$$v_1^2 = 4v_1'v_2' \qquad (3)$$

Square Equation (1) again,

$$v_1^2 = v_1'^2 + 2v_1'v_2' + v_2'^2$$

and substitute into this the result of Equation (3):

$$\begin{aligned} 4v_1'v_2' &= v_1'^2 + 2v_1'v_2' + v_2'^2 \\ 0 &= v_1'^2 - 2v_1'v_2' + v_2'^2 \\ 0 &= (v_1' - v_2')^2 \\ v_1' &= v_2' \qquad (4) \end{aligned}$$

Thus, combining Equations (1) and (4), find that:

$$v_1' = v_2' = \frac{1}{2}v_1$$

C. When Block 1 reaches its new amplitude position, A', all of its kinetic energy is converted to elastic potential energy of the spring. That is,

$$K_1' \to U_S' \quad \Rightarrow \quad \frac{1}{2}mv_1'^2 = \frac{1}{2}kA'^2$$
$$A'^2 = \frac{m}{k}v_1'^2$$
$$A'^2 = \frac{m}{k}(\frac{1}{2}v_1)^2$$
$$A'^2 = \frac{mv_1^2}{4k} \qquad (1)$$

But the original potential energy of the spring, $U_S = \frac{1}{2}k(-\frac{1}{4}L)^2$, gave K_1:

$$U_S \to K_1 \quad \Rightarrow \quad \frac{1}{2}k(-\frac{1}{4}L)^2 = \frac{1}{2}mv_1^2 \quad \Rightarrow \quad mv_1^2 = \frac{1}{16}kL^2 \qquad (2)$$

Substituting this result into Equation (1) gives:

$$A'^2 = \frac{\frac{1}{16}kL^2}{4k} = \frac{L^2}{64} \quad \Rightarrow \quad A' = \frac{1}{8}L$$

D. The period of a spring-block simple harmonic oscillator depends only on the spring constant k and the mass of the block. Since neither of these changes, the period will remain the same; that is, $T' = T_0$.

E. As shown in part B, Block 2's velocity as it slides off the table is $\frac{1}{2}v_1$ (horizontally). The time required to drop the vertical distance H is found as follows (calling *down* the positive direction):

$$\Delta y = v_{0y}t + \frac{1}{2}gt^2 \quad \Rightarrow \quad H = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2H}{g}}$$

Therefore,

$$R = (\frac{1}{2}v_1)t = \frac{1}{2}v_1\sqrt{\frac{2H}{g}}$$

Now, from Equation (2) of part C, $v_1 = \sqrt{\frac{kL^2}{16m}}$, so:

$$R = \frac{1}{2}\sqrt{\frac{kL^2}{16m}}\sqrt{\frac{2H}{g}} = \frac{L}{8}\sqrt{\frac{2kH}{mg}}$$

2. A. By Conservation of Linear Momentum,

$$mv = (m+M)v' \quad \Rightarrow \quad v' = \frac{mv}{m+M}$$

B. When the block is at its amplitude position (maximum compression of the spring), the kinetic energy it (and the embedded bullet) had just after impact will become the potential energy of the spring:

$$K' \rightarrow U_S$$

$$\frac{1}{2}(m+M)\left(\frac{mv}{m+M}\right)^2 = \frac{1}{2}kA^2$$

$$A = \frac{mv}{\sqrt{k(m+M)}}$$

C. Since the mass on the spring is $m + M$, $f = \frac{1}{2\pi}\sqrt{\frac{k}{m+M}}$

3. A. By Conservation of Mechanical Energy, $K + U = E$, so:

$$\frac{1}{2}Mv^2 + \frac{1}{2}k(\frac{1}{2}A)^2 = \frac{1}{2}kA^2$$

$$\frac{1}{2}Mv^2 = \frac{3}{8}kA^2$$

$$v = A\sqrt{\frac{3k}{4M}}$$

B. Since the clay ball delivers no horizontal linear momentum to the block, horizontal linear momentum is conserved. Thus,

$$Mv = (M+m)v'$$

$$v' = \frac{Mv}{M+m} = \frac{MA}{M+m}\sqrt{\frac{3k}{4M}} = \frac{A}{M+m}\sqrt{\frac{3kM}{4}}$$

C. Applying the general equation for the period of a spring-block simple harmonic oscillator,

$$T = 2\pi\sqrt{\frac{M+m}{k}}$$

D. The total energy of the oscillator after the clay hits is $\frac{1}{2}kA'^2$, where A' is the new amplitude. Just after the clay hits the block, the total energy is:

$$K' + U_S = \frac{1}{2}(M+m)v'^2 + \frac{1}{2}k(\frac{1}{2}A)^2$$

Substitute for v' from part B, set the resulting sum equal to $\frac{1}{2}kA'^2$, and solve for A'.

$$\frac{1}{2}(M+m)\left(\frac{A}{M+m}\sqrt{\frac{3kM}{4}}\right)^2+\frac{1}{2}k(\frac{1}{2}A)^2=\frac{1}{2}kA'^2$$

$$\frac{A^2\cdot 3kM}{8(M+m)}+\frac{1}{8}kA^2=\frac{1}{2}kA'^2$$

$$A^2\left(\frac{3M}{M+m}+1\right)=4A'^2$$

$$A'=\frac{1}{2}A\sqrt{\frac{3M}{M+m}+1}$$

E. No, because the period depends only on the mass and the spring constant k.

F. Yes. For example, if the clay had landed when the block was at $x = A$, the speed of the block would have been zero immediately before the collision and immediately after. No change in the block's speed would have meant no change in K, so no change in E, so no change in $A = \sqrt{2E/k}$.

CHAPTER 10 REVIEW QUESTIONS

Section I: Multiple Choice

1. **B** For a disc rotating at 8 rev/s, the time for 1 revolution is 1/8 s. Speed is distance divided by time, so for one rotation of a point on the rim,

$$v = \frac{2\pi r}{T} = \frac{2\pi(0.05\text{ m})}{(1/8\text{ s})} = 2.5\text{ m/s}$$

2. **B** Use Big 5 #3 for rotational motion:

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2 \Rightarrow \alpha = \frac{2\theta}{t^2} = \frac{2(90\text{ rad})}{(15\text{ s})^2} = 0.80\text{ rad/s}^2$$

3. **B** Apply the rotational definition for torque:

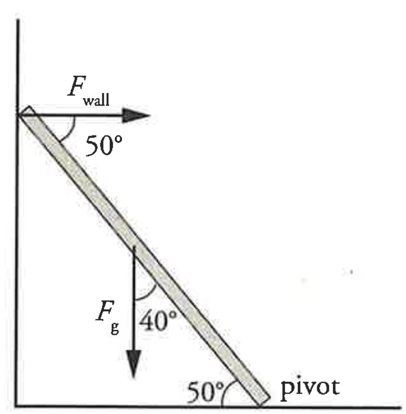
$$\tau = I\alpha = I\frac{\omega - \omega_0}{t} = (0.5\text{ kg}\cdot\text{m}^2)\frac{0-12\text{ rad/s}}{0.4\text{ s}} = 15\text{ N}\cdot\text{m}$$

4. **A** To maintain a constant angular velocity, the angular acceleration must equal zero. Therefore, the net torque on the grinding wheel also equals zero.

$$\Sigma\tau = \text{I}\alpha = 0$$

$$\tau_{\text{motor}} = \tau_{\text{knife}} = rF_f\sin 90° = r\mu_k F_N = (0.12\text{ m})(0.28)(16\text{ N}) = 0.54\text{ N·m}$$

5. **A** Since the ladder isn't moving, it's in equilibrium, so $\Sigma F = 0$ and $\Sigma\tau = 0$. The ladder experiences a normal force and a frictional force where it touches the ground, both of which are unknown. Therefore, select that point as the pivot, and apply the torque condition for equilibrium.



$$\Sigma\tau = 0 \Rightarrow \tau_g + \tau_{wall} = 0 \Rightarrow r_g F_g \sin 40° = r_{wall} F_{wall} \sin 50°$$

$$F_{wall} = \frac{r_g mg \sin 40°}{r_{wall} \sin 50°} = \frac{\left(\frac{L}{2}\right)(20\text{ kg})(9.8\text{ m/s}^2)\sin 40°}{L \sin 50°}$$

$$= 82\text{ N}$$

By Newton's Third Law, if the wall exerts an 82 N force against the ladder, the ladder exerts an 82 N force against the wall.

6. **A** By Conservation of Momentum,

$$L_i = L_f \Rightarrow I_i\omega_i = I_f\omega_f \Rightarrow \omega_f = \frac{I_i\omega_i}{I_f} = \frac{I_i\omega}{4I_i} = \frac{\omega}{4}$$

7. **D** First, convert the initial angular velocity to units of rad/s:

$$20\frac{\text{rev}}{\text{min}} \times \frac{1\text{ min}}{60\text{ s}} \times \frac{2\pi\text{ rad}}{1\text{ rev}} = 2.09\text{ rad/s}$$

Since the child and merry-go-round are rotating without friction, angular momentum is conserved. The rotational inertia of the system is the sum of the rotational inertias of the child and the merry-go-round.

$$I_i\omega_i = I_f\omega_f$$

$$(I_{child,\,i} + I_m)\omega_i = (I_{child,\,f} + I_m)\omega_f$$

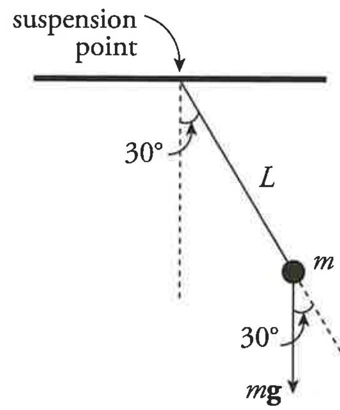
$$(mr_i^2 + I_m)\omega_i = (mr_f^2 + I_m)\omega_f$$

$$\omega_f = \frac{(mr_i^2 + I_m)\omega_i}{(mr_f^2 + I_m)}$$

$$\omega_f = \frac{((25\text{ kg})(2.0\text{ m})^2 + 1200\text{ kg}\cdot\text{m}^2)(2.1\text{ rad/s})}{((25\text{ kg})(0.5\text{ m})^2 + 1200\text{ kg}\cdot\text{m}^2)} = 2.25\text{ rad/s}$$

8. **C** The torque is $\tau = rF = (0.20\text{ m})(20\text{ N}) = 4\text{ N}\cdot\text{m}$.

9. **D** From the diagram,



calculate:

$$\tau = rF\sin\theta = Lmg\sin\theta$$
$$= (0.80\text{ m})(0.50\text{ kg})(10\text{ N/kg})(\sin 30°)$$
$$= 2.0\text{ N}\cdot\text{m}$$

Section II: Free Response

1. A. i. The wheel will rotate according to the equation $\tau = I\alpha$. The torque is provided by the tension on the string that the mass hangs from, which acts at a distance to the rotation axis equal to the radius of the wheel. The tension depends on the mass of the hanging mass and the acceleration of the hanging mass. The acceleration of the hanging mass can be calculated from its starting height and the time it takes for the mass to hit the ground. The angular acceleration of the wheel is related to the tangential acceleration at its outer rim, which is the same as the acceleration of the hanging mass. Therefore, the quantities to be measured are the mass of the hanging object, *m*, which can be measured with any type of scale; the radius, *R*, of the wheel, which can be measured with a meter stick; the initial height, *h*, of the hanging mass, which can be measured with a meter stick; and the time it takes for the hanging mass to reach the ground, *t*, which can be measured with a stopwatch.

ii. Start with $\tau = I\alpha$. Torque is given by $\tau = r_{\perp}F = RT$, where *T* is the tension in the string, and angular acceleration is given by $\alpha = a / R$, where *a* is the acceleration of the hanging mass. Solving for the rotational inertia gives:

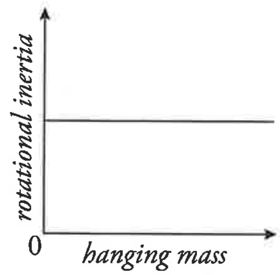
$$I = \frac{\tau}{\alpha} = \frac{RT}{a / R} = \frac{R^2T}{a}$$

Apply Newton's Second Law on the hanging mass to solve for *T*: $mg - T = ma \rightarrow$ $T = m(g - a)$. Substitute into the expression for *I*: $I = R^2m\left(\frac{g}{a} - 1\right)$. Apply kinematics to express *a* as a function of *h* and *t*, assuming that we drop the mass at rest: $h = \frac{1}{2}at^2 \rightarrow a = \frac{2h}{t^2}$. Finally, substitute into the expression for *I* in terms of the experimentally measured values:

$$I = R^2m\left(\frac{gt^2}{2h} - 1\right)$$

B. Mount the wheel on a fixed horizontal axis. Cut a length of string long enough to wrap around the wheel and reach the ground. Fix one end of the string on the edge of the wheel, and wind the string around the wheel. Tie the free end of the string to the hanging mass. Turn the wheel to wind the string so that the string is taut and the mass is hanging from one side of the wheel. From rest, release the system, allowing the mass to drop and the wheel to spin.

C. The rotational inertia of the wheel is independent of the hanging mass, so changing the value of the hanging mass should lead to approximately the same value of the rotational inertia:



2. A. Newton's Second Law for rotational motion states that $\tau_{\text{net}} = I\alpha$. In the absence of friction, the net torque is the torque applied by the cyclist, $\tau_{\text{app}} = I\alpha$. The angular acceleration is given by $\alpha = \frac{\omega - \omega_0}{t}$. The wheel starts from rest, so $\omega_0 = 0$. The final angular velocity is $\omega = \frac{v}{r} = \frac{2v}{D}$. Therefore, $\tau_{\text{app}} = I\left(\frac{\omega - 0}{t}\right) = \frac{I}{t}\left(\frac{2v}{D}\right) = \frac{2Iv}{tD}$.

B. The kinetic friction force applies a frictional torque, $\tau_f = r_{\perp}F = sf$. Apply Newton's Second Law for rotational motion to solve for α, $\tau_{\text{app}} - \tau_f = I\alpha \rightarrow \alpha = \frac{\tau_{\text{app}} - \tau_f}{I}$. The same rotational kinematic equations apply as before, with t' replacing t, $\alpha = \frac{2v / D}{t'}$. Set the two equations for α equal to each other, and solve for t', $\frac{\tau_{\text{app}} - \tau_f}{I} = \frac{2v / D}{t'} \rightarrow t' = \frac{2vI}{D\left(\tau_{\text{app}} - \tau_f\right)}$.

3. The balls roll without slipping, so no work is done by kinetic friction. Therefore, the total mechanic energy of each ball is conserved, $\Delta K = -\Delta U$. The balls experience the same change in height, so as they roll off the first ramp, they've experienced the same change in gravitational potential energy in their respective ball-Earth systems. Therefore, they also experience the same change in kinetic energy, so their kinetic energies are the same at the bottom of the ramp, as balls start from rest. However, kinetic energy has both translational, $K_{\text{t}} = \frac{1}{2}mv^2$, and rotational, $K_{\text{r}} = \frac{1}{2}I\omega^2$, contributions. For spherical objects that roll without slipping, $\omega = \frac{v}{R}$, so the total kinetic energy is $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left(m + \frac{I}{R^2}\right)v^2$. The objects have the same mass and radius, so for Ball A to have a greater translational speed, it must have a lower rotational inertia. On the second incline, the total kinetic energy is converted back into gravitational potential energy. The total kinetic energies of the balls on the horizontal segment is the same, so the final gravitational potential energy, and therefore the final height, is the same.

CHAPTER 11 REVIEW QUESTIONS

Section I: Multiple Choice

1. **D** Since Point X is 5 m below the surface of the water, the pressure due to the water at X, P_X, is $\rho g h_X = \rho g(5\text{ m})$, where ρ is the density of water. Because Point Y is 4 m below the surface of the water, the pressure due to the water at Y, P_Y, is $\rho g(4\text{ m})$. Therefore,

$$\frac{P_X}{P_Y} = \frac{\rho g(5\text{ m})}{\rho g(4\text{ m})} = \frac{5}{4} \quad \Rightarrow \quad 4P_X = 5P_Y$$

2. **C** Because the top of the box is at a depth of $D - z$ below the surface of the liquid, the pressure on the top of the box is:

$$P = P_{atm} + \rho g h = P_{atm} + \rho g(D - z)$$

The area of the top of the box is $A = xy$, so the force on the top of the box is:

$$F = PA = [P_{atm} + \rho g(D - z)]xy$$

3. **B** The scale reading is the difference between the weight (which is constant) and the buoyant force. The buoyant force is calculated from $F_{buoy} = \rho_{fluid} V_{sub} g$ and will increase as the volume submerged is increased. As the amount the cube is lowered increases, the scale reading will drop at a constant rate, since V is directly proportional to h for a cube.

4. **B** The independent variable and dependent variable are not changed because those are determined by the experimental setup, so you can eliminate (C) and (D). Changing the cube to a sphere makes V no longer directly proportional to h, so the graphs will not be the same, so you can eliminate (A).

5. **B** The buoyant force on the Styrofoam block is $F_{buoy} = \rho_L V g$, and the weight of the block is $F_g = m_S g = \rho_S V g$. Because $\rho_L > \rho_S$, the net force on the block is upward and has magnitude:

$$F_{net} = F_{buoy} - F_g = (\rho_L - \rho_S)Vg$$

Therefore, by Newton's Second Law, you have:

$$a = \frac{F_{net}}{m} = \frac{(\rho_L - \rho_S)Vg}{\rho_S V} = \left(\frac{\rho_L}{\rho_S} - 1\right) g$$

6. **A** If the object weighs 100 N less when completely submerged in water, the buoyant force must be 100 N; therefore,

$$F_{buoy} = \rho_{water} V_{sub} g = \rho_{water} Vg = 100 \text{ N} \Rightarrow V = \frac{100 \text{ N}}{\rho_{water} g} = \frac{100 \text{ N}}{(1000 \frac{\text{kg}}{\text{m}^3})(10 \frac{\text{N}}{\text{kg}})} =$$

Now that you know the volume of the object, you can figure out its weight:

$$F_g = mg = \rho_{object} Vg = (2000 \frac{\text{kg}}{\text{m}^3})(10^{-2} \text{ m}^3)(10 \frac{\text{N}}{\text{kg}}) = 200 \text{ N}$$

7. **D** The upward force on the ball is the buoyant force. The downward forces are gravity and tension. Gravity will remain constant, so you can eliminate (C). There is no change in the water density, so you can eliminate (A). The change in tension results from a change in the buoyant force and not because of flow speed, so you can eliminate (B).

8. **D** Apply Bernoulli's Equation to a point at the pump (Point 1) and at the nozzle (the exit point, Point 2). Choose the level of Point 1 as the horizontal reference level; this makes $y_1 = 0$ and $y_2 = 1$ m. Now, because the cross-sectional diameter decreases by a factor of 10 between Points 1 and 2, the cross-sectional area decreases by a factor of $10^2 = 100$, so flow speed must increase by a factor of 100; that is, $v_2 = 100v_1 = 100(0.4 \text{ m/s}) = 40$ m/s. Because Point 2 is exposed to the air, the pressure there is P_{atm}. Bernoulli's Equation becomes

$$P_1 + \frac{1}{2}\rho v_1^2 = P_{atm} + \rho g y_2 + \frac{1}{2}\rho v_2^2$$

Therefore,

$$\begin{aligned} P_1 - P_{atm} &= \rho g y_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 \\ &= (1000 \frac{\text{kg}}{\text{m}^3})(10 \text{ m/s}^2)(1 \text{ m}) + \frac{1}{2}(1000 \frac{\text{kg}}{\text{m}^3})(40 \text{ m/s})^2 - \frac{1}{2}(1000 \frac{\text{kg}}{\text{m}^3})(0.4 \text{ m/s})^2 \\ &\approx (1000 \frac{\text{kg}}{\text{m}^3})(10 \text{m/s}^2)(1 \text{ m}) + \frac{1}{2}(1000 \frac{\text{kg}}{\text{m}^3})(40 \text{ m/s})^2 \\ &= (10{,}000 \text{ Pa}) + (800{,}000 \text{ Pa}) \\ &= 810{,}000 \text{ Pa} \\ &= 810 \text{ kPa} \end{aligned}$$

9. **A** The cross-sectional diameter at Y is 3 times the cross-sectional diameter at X, so the cross-sectional area at Y is $3^2 = 9$ times that at X. The Continuity Equation states that the flow speed, v, is inversely proportional to the cross-sectional area, A. So, if A is 9 times greater at Point Y than it is at X, then the flow speed at Y is 1/9 the flow speed at X; that is, $v_Y = (1/9)v_X = (1/9)(6 \text{ m/s}) = 2/3 \text{ m/s}$.

10. **D** Each side of the rectangle at the bottom of the conduit is 1/4 the length of the corresponding side at the top. Therefore, the cross-sectional area at the bottom is $(1/4)^2 = 1/16$ the cross-sectional area at the top. The Continuity Equation states that the flow speed, v, is inversely proportional to the cross-sectional area, A. So, if A at the bottom is 1/16 the value of A at the top, then the flow speed at the bottom is 16 times the flow speed at the top.

Section II: Free Response

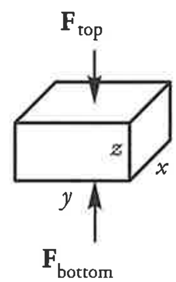
1. A. The pressure at the top surface of the block is $P_{top} = P_{atm} + \rho_L g h$. Since the area of the top of the block is $A = xy$, the force on the top of the block has magnitude:

$$F_{top} = P_{top}A = (P_{atm} + \rho_L g h)xy$$

The pressure at the bottom of the block is $P_{bottom} = P_{atm} + \rho_L g(h + z)$. Since the area of the bottom face of the block is also $A = xy$, the force on the bottom surface of the block has magnitude:

$$F_{bottom} = P_{bottom}A = [P_{atm} + \rho_L g(h + z)]xy$$

These forces are sketched below:



B. Each of the other four faces of the block (left and right, front and back) is at an average depth of $h + \frac{1}{2}z$, so the average pressure on each of these four sides is:

$$\bar{P}_{sides} = P_{atm} + \rho_L g(h + \frac{1}{2}z)$$

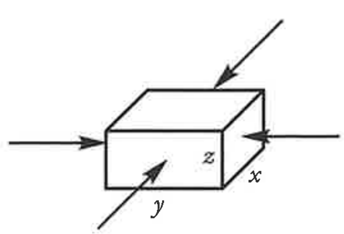
The left and right faces each have area $A = xz$, so the magnitude of the average force on this pair of faces is:

$$\bar{F}_{left\ and\ right} = \bar{P}_{sides}A = [P_{atm} + \rho_L g(h + \frac{1}{2}z)]xz$$

The front and back faces each have area $A = yz$, so the magnitude of the average force on this pair of faces is:

$$\bar{F}_{front\ and\ back} = \bar{P}_{sides}A = [P_{atm} + \rho_L g(h + \frac{1}{2}z)]yz$$

These forces are sketched below:



C. The four forces sketched in part B add up to zero, so the total force on the block due to the pressure is the sum of F_{top} and F_{bottom}; because $F_{bottom} > F_{top}$, this total force points upward and its magnitude is:

$$F_{bottom} - F_{top} = [P_{atm} + \rho_L g(h+z)]xy - (P_{atm} + \rho_L gh)xy = \rho_L gxyz$$

D. By Archimedes' Principle, the buoyant force on the block is upward and has magnitude:

$$F_{buoy} = \rho_L V_{sub} g = \rho_L V g = \rho_L xyzg$$

This is the same as the result you found in part C.

E. The weight of the block is:

$$F_g = mg = \rho_B V g = \rho_B xyzg$$

If F_T is the tension in the string, then the total upward force on the block, $F_T + F_{buoy}$, must balance the downward force, F_g; that is, $F_T + F_{buoy} = F_g$, so:

$$F_T = F_g - F_{buoy} = \rho_B xyzg - \rho_L xyzg = xyzg(\rho_B - \rho_L)$$

2. A. Applying Bernoulli's Equation to a point on the surface of the water in the tank (Point 1) and a point at the hole (Point 2), the assumption that $v_1 \approx 0$ leads to the result:

$$P_1 + \rho g h_1 + 0 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

The external pressures P_1 and P_2 are both P_{atm} so they cancel. Then all the ρ terms cancel. Identifying the difference in the hole heights as h results in:

$$v_2 = \sqrt{2gh}$$

B. The initial velocity of the water, as it emerges from the hole, is horizontal. Since there's no initial vertical velocity, the time t required to drop the distance $y = D - h$ to the ground is found as follows:

$$y = \frac{1}{2}gt^2 \quad \Rightarrow \quad D - h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2(D-h)}{g}}$$

Therefore, the horizontal distance the water travels is:

$$x = v_x t = \sqrt{2gh} \cdot \sqrt{\frac{2(D-h)}{g}} = 2\sqrt{h(D-h)}$$

C. The second hole would be at a depth of $h/2$ below the surface of the water, so the horizontal distance it travels—from the edge of the tank to the point where it hits the ground—is given by the same formula you found in part B except it will have $h/2$ in place of h; that is,

$$x_2 = 2\sqrt{\frac{1}{2}h(D - \frac{1}{2}h)}$$

If both streams land at the same point, then the value of x from part B is the same as x_2:

$$2\sqrt{h(D-h)} = 2\sqrt{\frac{1}{2}h(D - \frac{1}{2}h)}$$

$$h(D-h) = \frac{1}{2}h(D - \frac{1}{2}h)$$

$$D - h = \frac{1}{2}(D - \frac{1}{2}h)$$

$$4D - 4h = 2D - h$$

$$-3h = -2D$$

$$h = \frac{2}{3}D$$

D. Once again, apply Bernoulli's Equation to a point on the surface of the water in the tank (Point 1) and to a point at the hole (Point 2). Choose the ground level as the horizontal reference level; then, $y_1 = D$ and $y_2 = D - h$. If v_1 is the flow speed of Point 1—that is, the speed with which the water level in the tank drops—and v_2 is the efflux speed from the hole, then, by the Continuity Equation, $A_1v_1 = A_2v_2$, where A_1 and A_2 are the cross-sectional areas at Points 1 and 2, respectively. Therefore, $v_1 = (A_2/A_1)v_2$. Bernoulli's Equation then becomes:

$$P_1 + \rho g D + \frac{1}{2}\rho v_1^2 = P_2 + \rho g(D - h) + \frac{1}{2}\rho v_2^2$$

Since $P_1 = P_2 = P_{atm}$, these terms cancel out; and substituting $v_1 = (A_2/A_1)v_2$, you have:

$$\rho g D + \frac{1}{2}\rho\left(\frac{A_2}{A_1}v_2\right)^2 = \rho g(D-h) + \frac{1}{2}\rho v_2^2$$

$$\frac{1}{2}\rho v_2^2\left[\left(\frac{A_2}{A_1}\right)^2 - 1\right] = -\rho g h$$

$$v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Now, since $A_1 = \pi R^2$ and $A_2 = \pi r^2$, this final equation can be written as:

$$v_2 = \sqrt{\frac{2gh}{1 - \left(\frac{r}{R}\right)^4}}$$

[Note that if $r << R$, then $(r/R)^4 \approx 0$, and the equation above reduces to $v_2 = \sqrt{2gh}$, as in part A.]

3. A. Point X is at a depth of h_1 below Point 1, where the pressure is P_1. Therefore, the hydrostatic pressure at X is $P_X = P_1 + \rho_F g h_1$.

B. Point Y is at a depth of $h_2 + d$ below Point 2, where the pressure is P_2. The column of static fluid above Point Y is comprised of two parts. For the depth h_2, the fluid has density ρ_F. For the depth d, the fluid has density ρ_V. Therefore, the hydrostatic pressure at Y is $P_Y = P_2 + \rho_F g h_2 + \rho_V g d$.

C. First, notice that Points 1 and 2 are at the same horizontal level; therefore, the heights y_1 and y_2 are the same, and the terms $\rho_F g y_1$ and $\rho_F g y_2$ will cancel out of the equation. Bernoulli's Equation then becomes:

$$P_1 + \frac{1}{2}\rho_F v_1^2 = P_2 + \frac{1}{2}\rho_F v_2^2$$

By the Continuity Equation, you have $A_1 v_1 = A_2 v_2$, so $v_1 = (A_2/A_1)v_2$. Therefore,

$$\begin{aligned} P_1 - P_2 &= \frac{1}{2}\rho_F v_2^2 - \frac{1}{2}\rho_F v_1^2 \\ &= \frac{1}{2}\rho_F v_2^2 - \frac{1}{2}\rho_F \left(\frac{A_2}{A_1}v_2\right)^2 \\ &= \frac{1}{2}\rho_F v_2^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] \end{aligned}$$

D. In parts A and B above, you found that $P_X = P_1 + \rho_F g h_1$ and $P_Y = P_2 + \rho_F g h_2 + \rho_V g d$. Since $P_X = P_Y$, you have:

$$P_1 + \rho_F g h_1 = P_2 + \rho_F g h_2 + \rho_V g d$$

so:

$$\begin{aligned} P_1 - P_2 &= \rho_F g(h_2 - h_1) + \rho_V g d \\ &= \rho_F g(-d) + \rho_V g d \\ &= (\rho_V - \rho_F) g d \end{aligned}$$

E. In parts C and D, you found two expressions for $P_1 - P_2$. Setting them equal to each other gives:

$$\begin{aligned} \frac{1}{2}\rho_F v_2^2 \left[1 - \left(\frac{A_2}{A_1}\right)^2\right] &= (\rho_V - \rho_F) g d \\ v_2^2 &= \frac{\rho_V - \rho_F}{\rho_F} \cdot \frac{2gd}{1 - \left(\frac{A_2}{A_1}\right)^2} \\ v_2 &= \sqrt{\frac{2gd\left(\frac{\rho_V}{\rho_F} - 1\right)}{1 - \left(\frac{A_2}{A_1}\right)^2}} \end{aligned}$$

The flow rate in the pipe is:

$$f = A_2 v_2 = A_2 \sqrt{\frac{2gd\left(\frac{\rho_V}{\rho_F} - 1\right)}{1 - \left(\frac{A_2}{A_1}\right)^2}}$$

Since

$$f = A_2 \sqrt{\frac{2g\left(\frac{\rho_V}{\rho_F} - 1\right)}{1 - \left(\frac{A_2}{A_1}\right)^2}} \cdot \sqrt{d}$$

you see that f is proportional to $\sqrt{d}$, as desired.