Chapter 10
Modern Physics

INTRODUCTION

The subject matter of the previous chapters was developed in the 17th, 18th, and 19th centuries, but as we delve into the physics of the very small, we enter the 20th century. Max Planck first proposed the idea of light being emitted as individual packets of constant energy, called **quanta**. This is where the name quantum mechanics comes from. The particle nature of light was pioneered by Einstein in 1905, when he showed light transferred energy like a particle, and Arthur Compton in 1923, when he showed light has momentum and can undergo elastic collisions.

BLACKBODY RADIATION

Planck first proposed the idea that energy is quantized to explain blackbody radiation. A blackbody is an idealized object that absorbs all incident radiation. Blackbody radiation describes the electromagnetic waves that the blackbody radiates. Blackbodies emit a continuous spectrum of electromagnetic radiation in which the intensity varies for different wavelengths. The wavelength that is emitted with the highest intensity, λ_{max}, depends on the temperature, T, of the blackbody, which is described by **Wien's Law**,

Equation Sheet

$$\lambda_{max} = \frac{b}{T}$$

in which b is a proportionality constant. As the temperature of a blackbody increases, the maximum wavelength of its emission decreases. In addition, the total power, P, of the emitted radiation increases with temperature as well, as described by the **Stefan-Boltzmann Law**,

Equation Sheet

$$P = A\sigma T^4$$

in which A is the surface area of the object, and σ is the Stefan-Boltzmann constant.

Although a blackbody is an idealized object, we can observe this general trend in objects around us. For example, consider the heating element of an electric stove. When it is cold, we do not see it emitting any light because its λ_{max} is in the infrared. As it heats up, it starts to glow with a dark red color. As it gets hotter, the color turns reddish orange. As the temperature increases, the peak wavelength of light we see decreases from infrared to red to reddish orange.

Although it's not obvious from this conceptual description, it was unclear why blackbodies behaved in this way based on classical physics. Planck was only able to explain the behavior of blackbodies when he considered that energy was quantized.

PHOTONS AND THE PHOTOELECTRIC EFFECT

Planck realized that electromagnetic energy is quantized. It was Einstein who characterized how this occurs. A quantum of electromagnetic energy is known as a **photon**. Light behaves like a stream of photons, and this is illustrated by the **photoelectric effect**.

When a piece of metal is illuminated by electromagnetic radiation (specifically visible light, ultraviolet light, or X-rays), the energy absorbed by electrons near the surface of the metal can liberate them from their bound state, and these electrons can fly off. The released electrons are known as **photoelectrons**. In this case, the classical, wave-only theory of light would predict three results:

(1) There would be a significant time delay between the moment of illumination and the ejection of photoelectrons. This is because the metal would have to heat up to the point that the thermal energy was enough to overcome the binding energy.

(2) Increasing the intensity of the light would cause the electrons to leave the metal surface with greater kinetic energy. This is because the energy carried by a wave is related to its intensity.

(3) Photoelectrons would be emitted regardless of the frequency of the incident energy, as long as the intensity was high enough.

Surprisingly, none of these predictions was observed. Photoelectrons were ejected within just a few billionths of a second after illumination, disproving prediction (1). Secondly, increasing the intensity of the light did not cause photoelectrons to leave the metal surface with greater kinetic energy. Although more electrons were ejected as the intensity was increased, there was a maximum photoelectron kinetic energy; prediction (2) was false. And, for each metal, there was a certain **threshold frequency**, f_0: if light of frequency lower than f_0 were used to illuminate the metal surface, *no* photoelectrons were ejected, regardless of how intense the incident radiation was; prediction (3) was also false. Clearly, something was wrong with the wave-only theory of light.

Einstein explained these observations by borrowing from Planck's idea that light came in individual quanta of energy, which were later called photons by Gilbert Lewis. The energy of a photon is proportional to the frequency of the wave,

$$E = hf$$

Equation Sheet

where h is **Planck's constant** (about $6.63 \cdot 10^{-34}$ J·s). A certain amount of energy had to be imparted to an electron on the metal surface in order to liberate it; this was known as the metal's **work function**, or ϕ. If an electron absorbed a photon whose energy E was greater than ϕ, it would leave the metal with a maximum kinetic energy equal to $E - \phi$. This process could occur *very* quickly, which accounts for the rapidity with which photoelectrons are produced after illumination. This explains why prediction (1) was incorrect.

In this view, increasing the intensity (and therefore the energy) just means increasing the number of photons and results in the ejection of more photoelectrons—but since the energy of each incident photon is fixed by the equation $E = hf$, the value of K_{max} will still be $E - \phi$. This can be expressed as

Equation Sheet

$$K_{max} = hf - \phi$$

This accounts for the observation that disproved prediction (2).

Finally, if the incoming photon's energy was less than the work function (or, $E = hf < \phi$), the photon energy would not be enough to liberate electrons. Blasting the metal surface with more photons (that is, increasing the intensity of the incident beam) would also do nothing; none of the photons would have enough energy to eject electrons. This accounts for the observation of a threshold frequency, which we now know is ϕ/h. This can be expressed as

$$f_o = \phi/h$$

and is why prediction (3) was incorrect. Before we get to some examples, it's worthwhile to introduce a new unit of energy. The SI unit for energy is the joule, but it's too large to be convenient in the domains we're studying now. We'll use a much smaller unit, the **electronvolt** (abbreviated **eV**). The eV is equal to the energy gained (or lost) by an electron accelerated through a potential difference of one volt. Using the equation $\Delta U_E = qV$, we find that

$$1 \text{ eV} = (1 \text{ e})(1 \text{ V}) = (1.6 \times 10^{-19} \text{ C})(1 \text{ V}) = 1.6 \times 10^{-19} \text{ J}$$

In terms of electronvolts, the value of Planck's constant is 4.14×10^{-15} eV·s.

Example 1 The work function, ϕ, for aluminum is 4.08 eV.

(a) What is the threshold frequency required to produce photoelectrons from aluminum?

(b) Classify the electromagnetic radiation that can produce photoelectrons.

(c) If light of frequency $f = 4.00 \times 10^{15}$ Hz is used to illuminate a piece of aluminum,

(i) what is K_{max}, the maximum kinetic energy of ejected photoelectrons?

(ii) what's the maximum speed of the photoelectrons? (Electron mass = 9.11×10^{-31} kg)

(d) If the light described in part (b) were increased by a factor of 2 in intensity, what would happen to the value of K_{max}?

Solution.

(a) We know from the statement of the question that, in order for a photon to be successful in liberating an electron from the surface of the aluminum, its energy cannot be less than 4.08 eV. Therefore, the minimum frequency of the incident light—the threshold frequency—must be

$$f_0 = \frac{\phi}{h} = \frac{4.08 \text{ eV}}{4.14 \times 10^{-15} \text{ eV} \cdot \text{s}} = 9.86 \times 10^{14} \text{ Hz}$$

(b) Based on the electromagnetic spectrum given in the previous chapter, the electromagnetic radiation used to produce photoelectrons from aluminum must be at least in the ultraviolet region of the EM spectrum.

(c) (i) The maximum kinetic energy of photoelectrons is found from the equation

$$\begin{aligned} K_{\text{max}} &= hf - \phi \\ &= (4.14 \times 10^{-15} \text{ eV} \cdot \text{s})(4.00 \times 10^{15} \text{ Hz}) - (4.08 \text{ eV}) \\ &= 12.5 \text{ eV} \end{aligned}$$

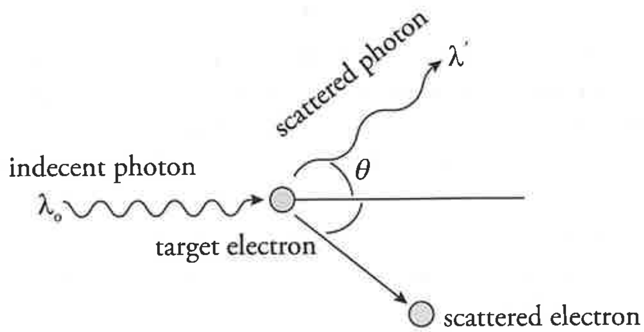
(ii) Using the above result and $K = \frac{1}{2}mv^2$, we can find v_{max}:

$$\begin{aligned} v_{\text{max}} &= \sqrt{\frac{2}{m_e} K_{\text{max}}} \\ &= \sqrt{\frac{2}{9.11 \times 10^{-31} \text{ kg}} \left(12.5 \text{ eV} \cdot \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \right)} \\ &= 2.1 \times 10^6 \text{ m/s} \end{aligned}$$

(d) Nothing will happen. Increasing the intensity of the illuminating radiation will cause more photons to impinge on the metal surface, thereby ejecting more photoelectrons, but their maximum kinetic energy will remain the same. The only way to increase K_{max} would be to increase the frequency of the incident energy.

COMPTON SCATTERING

Compton provided further evidence that light behaves like a particle. Like the photoelectric effect, Compton scattering involves light interacting with electrons. Unlike the photoelectric effect, the photon is not absorbed. Instead, the light is scattered. In other words, an incident photon, with an incident wavelength λ_0, interacts with an electron and moves off with a longer wavelength λ' at an angle θ relative to its original direction.



The resulting Compton shift, $\Delta\lambda$, depends only on the scattered angle:

Equation Sheet

$$\Delta\lambda = \lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$$

This equation follows from applying conservation of momentum and energy to the collision of the photon and electron, just like it would for the elastic collision of any two objects. This result provides another example of how light can behave like a particle.

Example 2 A coherent light source with a wavelength of 4×10^{-12} m is shone through a thin sheet of graphite.

(a) At what scattering angle would you expect to detect light with a wavelength of 5×10^{-12}?

(b) How much energy would the electron that the light scattered off of at this angle receive?

Solution.

(a) From the Compton shift equation,

$$\lambda' - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta)$$

so

$$\begin{aligned}\theta &= \cos^{-1}\left(1-\frac{m_e c}{h}(\lambda'-\lambda_0)\right)\\ &= \cos^{-1}\left(1-\frac{(9.11\times10^{-31}\ \text{kg})(3\times10^{8}\ \text{m/s})}{6.63\times10^{-34}\ \text{J}\cdot\text{s}}(5\times10^{-12}\ \text{m}-4\times10^{-12}\ \text{m})\right)\\ &\approx 54^\circ\end{aligned}$$

(b) The energy given to the electron must be the energy lost by the light during the scattering. Because the energy of a photon is given by $E = hf$ and, from the wave equation, $\lambda = c/f$, we have

$$\begin{aligned}\Delta E &= E_f - E_i = hf_f - hf_i = \frac{hc}{\lambda'}-\frac{hc}{\lambda_0}\\ &= hc\left(\frac{1}{\lambda'}-\frac{1}{\lambda_0}\right)\\ &= (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3\times10^{8}\ \text{m/s})\left(\frac{1}{5\times10^{-12}\ \text{m}}-\frac{1}{4\times10^{-12}\ \text{m}}\right)\\ &= -9.9\times10^{-15}\ \text{J}\end{aligned}$$

so the photon loses 9.9×10^{-15} J during scattering which is how much energy the electron will gain.

THE BOHR MODEL OF THE ATOM

In the years immediately following Rutherford's announcement of his nuclear model of the atom, a young physicist, Niels Bohr, would add an important piece to the atomic puzzle. Rutherford told us where the positive charge of the atom was located; Bohr would tell us about the electrons.

For 50 years, it had been known that atoms in a gas discharge tube emit and absorb light only at specific wavelengths. The light from a glowing gas, passed through a prism to disperse the beam into its component wavelengths, produces patterns of sharp lines called **atomic spectra**. The visible wavelengths that appear in the emission spectrum of hydrogen had been summarized by the *Balmer formula*:

$$\frac{1}{\lambda_n} = R\left(\frac{1}{2^2}-\frac{1}{n^2}\right)$$

where R is the *Rydberg constant* (about 1.1×10^7 m^{-1}). The formula worked—that is, it fit the observational data—but it had no theoretical basis. So, the question was, *why* do atoms emit (or absorb) radiation only at certain discrete wavelengths?

Chemistry
Modern physics begins to explore atoms, which bridges into chemistry. Fortunately, you will not need to know much about chemistry for the AP Physics 2 Exam—just a basic understanding of some topics.

Bohr's model of the atom explains the spectroscopists' observations. Using the simplest atom, hydrogen (with only one electron), Bohr postulated that the electron would orbit the nucleus only at certain discrete radii. When the electron is in one of these special orbits, it does not radiate away energy (as the classical theory would predict). However, if the electron absorbs a certain amount of energy, it is **excited** to a higher orbit, one with a greater radius. After a short time in this excited state, it returns to a lower orbit, emitting a photon in the process. Since each allowed orbit—or **energy level**—has a specific radius (and corresponding energy), the photons emitted in each jump have only specific wavelengths.

When an excited electron drops from energy level $n = j$ to a lower one, $n = i$, the transition causes a photon of energy to be emitted, and the energy of the photon is the difference between the two energy levels. When an electron absorbs a photon and transitions from a lower energy level, $n = i$, to a higher energy level, $n = j$, the energy of the absorbed photon is the difference between the two energy levels.

$$E_{\text{emitted or absorbed photon}} = |\Delta E| = E_j - E_i$$

The wavelength of this photon is

Speed of Light
Photons travel at the speed of light.

$$\lambda = \frac{c}{f} = \frac{c}{|E_{\text{photon}}|/h} = \frac{hc}{|E_j - E_i|}$$

Example 3 The first five energy levels of an atom are shown in the diagram below:

−3 eV ____________ $n = 5$

−4 eV ____________ $n = 4$

−7 eV ____________ $n = 3$

−15 eV ____________ $n = 2$

−62 eV ____________ $n = 1$ ground state

(a) If the atom begins in the $n = 3$ level, what photon energies could be emitted as it returns to its ground state?

(b) What could happen if this atom, while in its ground state, were bombarded with a photon of energy 10 eV?

Solution.

(a) If the atom is in the $n = 3$ level, it could return to ground state by a transition from $3 \rightarrow 1$, or from $3 \rightarrow 2$ and then $2 \rightarrow 1$. The energy emitted in each of these transitions is simply the difference between the energies of the corresponding levels:

$$E_{3\rightarrow1} = E_3 - E_1 = (-7 \text{ eV}) - (-62 \text{ eV}) = 55 \text{ eV}$$

$$E_{3\rightarrow2} = E_3 - E_2 = (-7 \text{ eV}) - (-15 \text{ eV}) = 8 \text{ eV}$$

$$E_{2\rightarrow1} = E_2 - E_1 = (-15 \text{ eV}) - (-62 \text{ eV}) = 47 \text{ eV}$$

(b) The ground state has an energy level of −62 eV, and since there is no level with an energy of −52 eV, the atom could not absorb a 10 eV photon. In its ground state, this atom would be transparent to light of energy 10 eV.

Example 4 The ground state of hydrogen is −13.6 eV. The first excited state is −3.4 eV. The second excited state is −1.5 eV. The third excited state is −0.85 eV.

(a) How much energy must a ground state electron in a hydrogen atom absorb to be excited to the $n = 4$ energy level?

(b) With the electron in the $n = 4$ level, what wavelengths are possible for the photon emitted when the electron drops to a lower energy level? In what regions of the EM spectrum do these photons lie?

Solution.

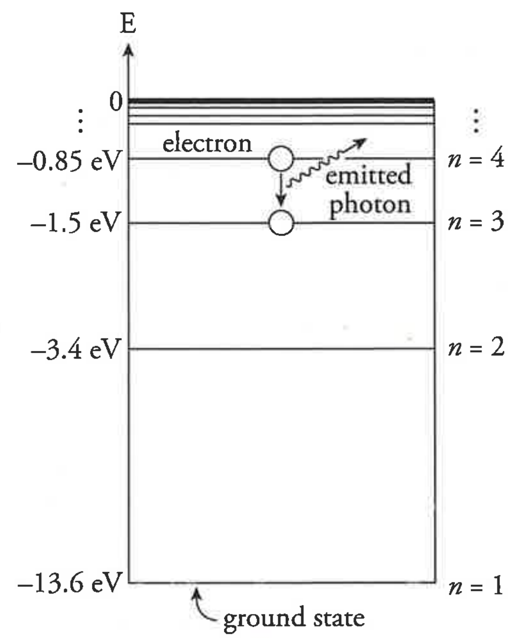
(a) The ground-state energy level ($n = 1$) for hydrogen is −13.6 eV, and

$E_4 = -0.85$ eV.

Therefore, in order for an electron to make the transition from E_1 to E_4, it must absorb energy in the amount $E_4 - E_1 = (-0.85 \text{ eV}) - (-13.6 \text{ eV}) = 12.8$ eV.

(b) An electron in the $n = 4$ energy level can make several different transitions: it can drop to $n = 3$, $n = 2$, or all the way down to the ground state, $n = 1$.

The following diagram shows the electron dropping from $n = 4$ to $n = 3$:



There are three possible values for the energy of the emitted photon, $E_{4\to3}$, $E_{4\to2}$, or $E_{4\to1}$:

$$E_{4\to3} = E_4 - E_3 = (-0.85 \text{ eV}) - (-1.5 \text{ eV}) = 0.65 \text{ eV}$$
$$E_{4\to2} = E_4 - E_2 = (-0.85 \text{ eV}) - (-3.4 \text{ eV}) = 2.55 \text{ eV}$$
$$E_{4\to1} = E_4 - E_1 = (-0.85 \text{ eV}) - (-13.6 \text{ eV}) = 12.8 \text{ eV}$$

From the equation $E = hf = hc/\lambda$, we get $\lambda = hc/E$, so

$$\lambda_{4\to3} = \frac{hc}{E_{4\to3}} = \frac{(4.14\times10^{-15} \text{ eV}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{0.65 \text{ eV}} = 1910 \text{ nm}$$
$$\lambda_{4\to2} = \frac{hc}{E_{4\to2}} = \frac{(4.14\times10^{-15} \text{ eV}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{2.55 \text{ eV}} = 487 \text{ nm}$$
$$\lambda_{4\to1} = \frac{hc}{E_{4\to1}} = \frac{(4.14\times10^{-15} \text{ eV}\cdot\text{s})(3.00\times10^{8} \text{ m/s})}{12.8 \text{ eV}} = 97 \text{ nm}$$

Note that $\lambda_{4\to2}$ is in the visible spectrum; this wavelength corresponds to the color blue-green; $\lambda_{4\to1}$ is an ultraviolet wavelength, and $\lambda_{4\to3}$ is infrared.

WAVE-PARTICLE DUALITY

Light and other electromagnetic waves exhibit wave-like characteristics through interference and diffraction. However, as we saw in the photoelectric effect, light also behaves as if its energy were granular, composed of particles. This is **wave-particle duality**: electromagnetic radiation propagates like a wave but exchanges energy like a particle.

Since an electromagnetic wave can behave like a particle, can a particle of matter behave like a wave? In 1924, the French physicist Louis de Broglie proposed that the answer is "yes." His conjecture, which has since been supported by experiment, is that a particle of mass m and speed v—and thus, linear momentum p—has an associated wavelength, which is called its **de Broglie wavelength**:

$$\lambda = \frac{h}{p}$$

Equation Sheet

Particles in motion can display wave characteristics and behave as if they had a wavelength.

Since the value of h is so small, ordinary macroscopic objects do not display wave-like behavior. For example, a baseball (mass = 0.15 kg) thrown at a speed of 40 m/s has a de Broglie wavelength of

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(0.15\ \text{kg})(40\ \text{m/s})} = 1.1\times10^{-34}\ \text{m}$$

This is much too small to measure. However, with subatomic particles, the wave nature is clearly evident. The 1937 Nobel Prize in physics was awarded for experiments by C. J. Davisson and G. P. Thomson that revealed that a stream of electrons exhibited diffraction patterns when scattered by crystals—a behavior that's characteristic of waves.

Example 5 Electrons in a diffraction experiment are accelerated through a potential difference of 200 V. What is the de Broglie wavelength of these electrons?

Solution. By definition, the kinetic energy of these electrons is 200 eV. Since the relationship between linear momentum and kinetic energy is $p = \sqrt{2mK}$,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\ 200\ \text{eV}\cdot\dfrac{1.6\times10^{-19}\ \text{J}}{1\ \text{eV}}}} = 8.7\times10^{-11}\ \text{m} = 0.087\ \text{nm}$$

This wavelength is characteristic of X-rays.

Interpreting Wave-Function Graphs
Wave-function graphs will often have values that are positive as well as negative. When interpreting the graph, only the absolute value is important in ranking relative likelihoods of experimental outcomes (technically the likelihood is proportional to Ψ^2, but the absolute value will give correct interpretations and is easier to read from a graph).

THE WAVE FUNCTION

There are several major differences between classical physics and modern physics. One of the most important is that the mathematics that underpin modern physics (which are too complicated for treatment here) no longer specify a definite position at a definite time for a particle such as an electron. Instead, the mathematics only give the *probability* that a particle will be measured to be at a particular position when the position is measured. That probability is related to a new physical parameter called the *wave function*, Ψ.

In one interpretation of that mathematics, the act of measuring the position changes the wave function, Ψ, so that it has a single position with a probability of 100%, and that location corresponds to the result of the measurement (this is referred to as "collapsing the wave-function"). However, before the position is measured (or after a measurement has taken place and some time has passed so the position is no longer certain), there are a range of probable locations that can be represented as a graph of Ψ versus x. To interpret such a graph, the position with the largest absolute value is the most probable location for the result of a measurement. Any position where the graph has a height of 0, it is certain that the position will not be observed.

NUCLEAR PHYSICS

The nucleus of the atom is composed of particles called **protons** and **neutrons**, which are collectively called **nucleons**. The number of protons in a given nucleus is called the atom's **atomic number**, and is denoted Z, and the number of neutrons (the **neutron number**) is denoted N. The total number of nucleons, $Z + N$, is called the **mass number** (or **nucleon number**), and is denoted A. The number of protons in the nucleus of an atom defines the element. For example, the element chlorine (abbreviated Cl) is characterized by the fact that the nucleus of every chlorine atom contains 17 protons, so the atomic number of chlorine is 17; but, different chlorine atoms may contain different numbers of neutrons. In fact, about three-fourths of all naturally occurring chlorine atoms have 18 neutrons in their nuclei (mass number = 35), and most of the remaining one-fourth contain 20 neutrons (mass number = 37).

Nuclei that contain the same numbers of protons but different numbers of neutrons are called **isotopes**. The notation for a **nuclide**—the term for a nucleus with specific numbers of protons and neutrons—is to write Z and A before the chemical symbol of the element:

$$^{A}_{Z}\text{X}$$

The isotopes of chlorine mentioned above would be written as follows:

$$^{35}_{17}\text{Cl} \quad \text{and} \quad ^{37}_{17}\text{Cl}$$

Example 6 How many protons and neutrons are contained in the nuclide $^{63}_{29}\text{Cu}$?

Solution. The subscript (the atomic number, Z) gives the number of protons, which is 29. The superscript (the mass number, A) gives the total number of nucleons. Since $A = 63 = Z + N$, we find that $N = 63 - 29 = 34$.

Example 7 The element neon (abbreviated Ne, atomic number 10) has several isotopes. The most abundant isotope contains 10 neutrons, and two others contain 11 and 12. Write symbols for these three nuclides.

Solution. The mass numbers of these isotopes are 10 + 10 = 20, 10 + 11 = 21, and 10 + 12 = 22. So, we'd write them as follows:

$$^{20}_{10}\text{Ne}, \ ^{21}_{10}\text{Ne}, \ \text{and} \ ^{22}_{10}\text{Ne}$$

Another common notation—which we also use—is to write the mass number after the name of the element. These three isotopes of neon would be written as neon-20, neon-21, and neon-22.

The Nuclear Force

Why wouldn't any nucleus that has more than one proton be unstable? After all, protons are positively charged and would therefore experience a repulsive Coulomb force from each other. Why don't these nuclei explode? And what holds neutrons—which have no electric charge—in the nucleus? These issues are resolved by the presence of another fundamental force, the **strong nuclear force**, which binds neutrons and protons together to form nuclei. Although the strength of the Coulomb force can be expressed by a simple mathematical formula (it's inversely proportional to the square of their separation), the nuclear force is much more complicated; no simple formula can be written for the strength of the nuclear force.

Binding Energy

The masses of the proton and neutron are listed below.

$$\text{proton: } m_p = 1.6726 \times 10^{-27} \text{ kg}$$

$$\text{neutron: } m_n = 1.6749 \times 10^{-27} \text{ kg}$$

Because these masses are so tiny, a much smaller mass unit is used. With the most abundant isotope of carbon (carbon-12) as a reference, the **atomic mass unit** (abbreviated **amu** or simply **u**) is defined as 1/12 the mass of a ^{12}C atom. The conversion between kg and u is 1 u = 1.6605 $\times 10^{-27}$ kg. In terms of atomic mass units,

$$m_p = 1.00728 \text{ u}$$

$$m_n = 1.00867 \text{ u}$$

Now consider the **deuteron**, the nucleus of **deuterium**, an isotope of hydrogen that contains 1 proton and 1 neutron. The mass of a deuteron is 2.01356 u, which is a little *less* than the sum of the individual masses of the proton and neutron. The difference between the mass of any bound nucleus and the sum of the masses of its constituent nucleons is called the **mass defect**, Δm. In the case of the deuteron (symbolized **d**), the mass defect is

$$\begin{aligned}\Delta m &= (m_p + m_n) - m_d \\ &= (1.00728 \text{ u} + 1.00867 \text{ u}) - (2.01356 \text{ u}) \\ &= 0.00239 \text{ u}\end{aligned}$$

What happened to this missing mass? It was converted to energy when the deuteron was formed. In 1905, Einstein gave us the famous equation:

Equation Sheet

$$E = mc^2$$

which tells us how much energy mass contains. Because of this, the mass-difference gives us the amount of energy needed to break the deuteron into a separate proton and neutron. Since this tells us how strongly the nucleus is bound, it is called the **binding energy** of the nucleus.

$$E_B = (\Delta m)c^2$$

Using $E = mc^2$, the energy equivalent of 1 atomic mass unit is

$$\begin{aligned}E &= (1.6605 \times 10^{-27} \text{ kg})(2.9979 \times 10^8 \text{ m/s})^2 \\ &= 1.4924 \times 10^{-10} \text{ J} \\ &= 1.4924 \times 10^{-10} \text{ J} \cdot \frac{1 \text{ eV}}{1.6022 \times 10^{-19} \text{ J}} \\ &= 9.31 \times 10^8 \text{ eV} \\ &= 931 \text{ MeV}\end{aligned}$$

In terms of electronvolts, then, the binding energy of the deuteron is

$$E_B \text{ (deuteron)} = 0.00239 \text{ u} \times \frac{931 \text{ MeV}}{1 \text{ u}} = 2.23 \text{ MeV}$$

Since the deuteron contains 2 nucleons, the **binding-energy-per-nucleon** is

$$\frac{2.23 \text{ MeV}}{2 \text{ nucleons}} = 1.12 \text{ MeV/nucleon}$$

This is the lowest value of all nuclides. The highest, 8.8 MeV/nucleon, is for an isotope of nickel, ^{62}Ni. Typically, when nuclei smaller than nickel are fused to form a single nucleus, the binding energy per nucleon increases, which tells us that energy is released in the process. On the other hand, when nuclei *larger* than nickel are *split*, binding energy per nucleon again increases, releasing energy. This is the basis of nuclear fission.

Example 8 What is the maximum wavelength of EM radiation that could be used to photodisintegrate a deuteron?

Solution. The binding energy of the deuteron is 2.23 MeV, so a photon would need to have at least this much energy to break the deuteron into a proton and neutron. Since $E = hf$ and $f = c/\lambda$,

$$E = \frac{hc}{\lambda} \longrightarrow \lambda_{\text{max}} = \frac{hc}{E_{\text{min}}} = \frac{(4.14 \times 10^{-15}\ \text{eV}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})}{2.23 \times 10^{6}\ \text{eV}} = 5.57 \times 10^{-13}\ \text{m}$$

Example 9 The atomic mass of $^{27}_{13}$Al is 26.9815 u. What is its nuclear binding energy per nucleon? (Mass of electron = 0.0005486 u)

Solution. The nuclear mass of $^{27}_{13}$Al is equal to its atomic mass minus the mass of its electrons. Since an aluminum atom has 13 protons, it must also have 13 electrons. So,

$$\begin{aligned}\text{nuclear mass of } ^{27}_{13}\text{Al} &= (\text{atomic mass of } ^{27}_{13}\text{Al}) - 13m_e \\ &= 26.9815\ \text{u} - 13(0.0005486\ \text{u}) \\ &= 26.9744\ \text{u}\end{aligned}$$

Now, the nucleus contains 13 protons and 27 – 13 = 14 neutrons, so the total mass of the individual nucleons is

$$\begin{aligned}M &= 13m_p + 14m_n \\ &= 13(1.00728\ \text{u}) + 14(1.00867\ \text{u}) \\ &= 27.2160\ \text{u}\end{aligned}$$

and the mass defect of the aluminum nucleus is

$$\Delta m = M - m = 27.2160\ \text{u} - 26.9744\ \text{u} = 0.2416\ \text{u}$$

Converting this mass to energy, we can see that

$$E_B = 0.2416\ \text{u} \times \frac{931\ \text{MeV}}{1\ \text{u}} = 225\ \text{MeV}$$

so the binding energy per nucleon is

$$\frac{225\ \text{MeV}}{27} = 8.3\ \text{MeV/nucleon}$$

NUCLEAR REACTIONS

Matter Cannot be Created or Destroyed
But it *can* undergo other forms—which does not break this rule. The quickest way to determine whether something undergoes alpha, beta, or gamma decay is to find the amount of nucleons that have escaped and then to determine which kind of decay happened.

Natural radioactive decay provides one example of a nuclear reaction. Other examples of nuclear reactions include the bombardment of target nuclei with subatomic particles to artificially induce radioactivity, such as the emission of a particle or the splitting of the nucleus (this is **nuclear fission**), and the **nuclear fusion** of small nuclei at extremely high temperatures. In all cases of nuclear reactions that we'll study, nucleon number and charge must be conserved. In order to balance nuclear reactions, we write ${}^{1}_{1}\text{p}$ or ${}^{1}_{1}\text{H}$ for a proton and ${}^{1}_{0}\text{n}$ for a neutron. Gamma-ray photons can also be produced in nuclear reactions; they have no charge or nucleon number and are represented as ${}^{0}_{0}\gamma$.

Alpha Decay

When a nucleus undergoes alpha decay, it emits an alpha particle, which consists of two protons and two neutrons and is the same as the nucleus of a helium-4 atom. An alpha particle can be represented as

$$\alpha,\ {}^{4}_{2}\alpha,\ \text{or}\ {}^{4}_{2}\text{He}$$

Very large nuclei can shed nucleons quickly by emitting one or more alpha particles; for example, radon-222 (${}^{222}_{86}\text{Rn}$) is radioactive and undergoes alpha decay.

$${}^{222}_{86}\text{Rn} \rightarrow {}^{218}_{84}\text{Po} + {}^{4}_{2}\alpha$$

This reaction illustrates two important features of a nuclear reaction.

(1) Mass number is conserved.
(2) Charge is conserved.

The decaying nuclide is known as the **parent**, and the resulting nuclide is known as the **daughter**. (Here, radon-222 is the parent nuclide and polonium-218 is the daughter.) Alpha decay decreases the mass number by 4 and the atomic number by 2. Therefore, alpha decay looks like the following:

$${}^{A}_{Z}\text{X} \rightarrow {}^{A-4}_{Z-2}\text{X}' + {}^{4}_{2}\alpha$$

Beta Decay

There are three subcategories of **beta** (β) decay, called β^-, β^+, and **electron capture** (EC).

β^- Decay When the neutron-to-proton ratio is too large, the nucleus undergoes β^- decay, which is the most common form of beta decay. β^- decay occurs when a neutron transforms into a proton and releases an electron. The expelled electron is called a **beta particle.** The transformation of a neutron into a proton and an electron (and another particle, the **electron-antineutrino,** $\overline{\nu}_e$) is caused by the action of the **weak nuclear force,** another of nature's fundamental forces. A common example of a nuclide that undergoes β^- decay is carbon-14, which is used to date archaeological artifacts.

$$^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + e^- + \overline{\nu}_e$$

The reaction is balanced, since 14 = 14 + 0 and 6 = 7 + (–1).

β^+ Decay When the neutron-to-proton ratio is too small, the nucleus will undergo β^+ decay. In this form of beta decay, a proton is transformed into a neutron and a **positron,** ${}^{0}_{+1}e$ (the electron's **antiparticle**), plus another particle, the **electron-neutrino,** ν_e, which are then both ejected from the nucleus. An example of a positron emitter is fluorine-17.

$$^{17}_{9}\text{F} \rightarrow {}^{17}_{8}\text{O} + e^+ + \nu_e$$

Electron Capture Another way in which a nucleus can increase its neutron-to-proton ratio is to capture an orbiting electron and then cause the transformation of a proton into a neutron. Beryllium-7 undergoes this process.

$$^{7}_{4}\text{Be} + e^- \rightarrow {}^{7}_{3}\text{Li} + \nu_e$$

Gamma Decay

In each of the decay processes defined on the previous pages, the daughter was a different element than the parent. Radon becomes polonium as a result of α decay, carbon becomes nitrogen as a result of β^- decay, fluorine becomes oxygen from β^+ decay, and beryllium becomes lithium from electron capture. By contrast, gamma decay does not alter the identity of the nucleus; it just allows the nucleus to relax and shed energy. Imagine that potassium-42 undergoes β^- decay to form calcium-42.

$$^{42}_{19}\text{K} \rightarrow {}^{42}_{20}\text{Ca}^* + e^- + \overline{\nu}_e$$

The asterisk indicates that the daughter calcium nucleus is left in a high-energy, excited state. For this excited nucleus to drop to its ground state, it must emit energy in the form of a photon, a **gamma ray,** symbolized by γ .

$$^{42}_{20}\text{Ca}^* \rightarrow {}^{42}_{20}\text{Ca} + \gamma$$

Let's sum up the three types of radiation:

Type of Radiation		What Happens?	Charge
Alpha		Particle—a helium nucleus, containing 2 protons and 2 neutrons, is released from the nucleus.	Positive 2+
Beta	Beta-minus	Particle—a highly energetic, massless electron is released from the nucleus after a neutron divides into a proton and an electron.	Negative 1–
	Beta-plus	Particle—a highly energetic, massless positron is released from the nucleus after a proton divides into a neutron and a positron.	Positive 1+
Gamma		Wave—energy is emitted in the form of a photon.	No Charge

Example 10 A mercury-198 nucleus is bombarded by a neutron, which causes a nuclear reaction:

$$ {}_{0}^{1}\text{n} + {}_{80}^{198}\text{Hg} \rightarrow {}_{79}^{197}\text{Au} + {}_{?}^{?}\text{X} $$

What's the unknown product particle, X?

Solution. In order to balance the superscripts, we must have 1 + 198 = 197 + A, so A = 2, and the subscripts are balanced if 0 + 80 = 79 + Z, so Z = 1:

$$ {}_{0}^{1}\text{n} + {}_{80}^{198}\text{Hg} \rightarrow {}_{79}^{197}\text{Au} + {}_{1}^{2}\text{X} $$

Therefore, X must be a deuteron, ${}_{1}^{2}\text{H}$ (or just d).

DISINTEGRATION ENERGY

Nuclear reactions not only produce new nuclei and other subatomic product particles, but they also involve the absorption or emission of energy. Nuclear reactions must conserve total energy, so changes in mass are accompanied by changes in energy according to Einstein's equation

$$ \Delta E = (\Delta m)c^2 $$

A general nuclear reaction is written

$$ \text{A} + \text{B} \rightarrow \text{C} + \text{D} + Q $$

where Q denotes the **disintegration energy**. If Q is positive, the reaction is **exothermic** (or **exoergic**) and the reaction can occur spontaneously; if Q is negative, the reaction is **endothermic** (or **endoergic**) and the reaction cannot occur spontaneously. The energy Q is calculated as follows:

$$Q = [(m_A + m_B) - (m_C + m_D)]c^2 = (\Delta m)c^2$$

For spontaneous reactions—ones that liberate energy—most of the energy is revealed as kinetic energy of the least massive product nuclei.

Example 11 The process that powers the Sun—and upon which all life on Earth is dependent—is the fusion reaction:

$$4\,{}^1_1\text{H} \rightarrow {}^4_2\alpha + 2e^+ + 2\nu_e + \gamma$$

(a) Show that this reaction releases energy.

(b) How much energy is released per proton?

Use the fact that $m_\alpha = 4.0015$ u and ignore the mass of the electron-neutrino, ν_e.

Solution.

(a) We need to find the mass difference between the reactants and products:

$$\begin{aligned}\Delta m &= 4m_p - (m_\alpha + 2m_e)\\ &= 4(1.00728\text{ u}) - [4.0015\text{ u} + 2(0.0005486\text{ u})]\\ &= 0.02652\text{ u}\end{aligned}$$

Since Δm is positive, the reaction is exothermic: energy is released.

(b) Converting the mass difference to energy gives

$$Q = 0.02652\text{ u} \times \frac{931\text{ MeV}}{1\text{ u}} = 24.7\text{ MeV}$$

Since four protons went into the reaction, the energy liberated per proton is

$$\frac{24.7\text{ MeV}}{4\text{ p}} = 6.2\text{ MeV/proton}$$

Example 12 Can the following nuclear reaction occur spontaneously?

$${}^{4}_{2}\alpha + {}^{14}_{7}\text{N} \rightarrow {}^{17}_{8}\text{O} + {}^{1}_{1}\text{H}$$

(The mass of the nitrogen nucleus is 13.9992 u, and the mass of the oxygen nucleus is 16.9947 u.)

Solution. We first figure out the mass equivalent of the disintegration energy:

$$\begin{aligned}\Delta m &= (m_\alpha + m_\text{N}) - (m_\text{O} + m_\text{p}) \\ &= (4.0015\text{ u} + 13.9992\text{ u}) - (16.9947\text{ u} + 1.00728\text{ u}) \\ &= -0.00128\text{ u}\end{aligned}$$

Since Δm is negative, this reaction is nonspontaneous; energy must be *supplied* in order for this reaction to proceed. But how much?

$$|Q| = 0.00128\text{ u} \times \frac{931\text{ MeV}}{1\text{ u}} = 1.19\text{ MeV}$$

Want Even More Physics Review?
Check out *High School Physics Unlocked*, our skill-building guide to mastering complex physics concepts.

Chapter 10 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review

According to the theory put forth by Louis de Broglie, all matter has wave-like properties such as interference, but these properties are seen only at a microscopic scale. Why are these properties not typically observed at a macroscopic scale?

(A) The wavelength of matter is typically too large to observe this interference.

(B) Interference of matter occurs only when these waves interact with other objects comparable to their wavelength and those things are microscopic.

(C) There are no energy level transitions available to allow for this interference to be observed.

(D) At the macroscopic scale, the interference is always destructive so it cannot be observed.

2 Mark for Review

An experiment is conducted of the photoelectric effect. A metal is struck with a beam of light with a certain brightness and wavelength. A photoelectron is ejected from the surface of the metal and is found to have a certain stopping potential. Which of the following changes could be made so that the photoelectron would not be ejected from the surface of the metal?

(A) Decrease the brightness of the light

(B) Decrease the wavelength of the light

(C) Increase the wavelength of the light

(D) Increase the stopping potential

3 Mark for Review

An atom with one electron has an ionization energy of 25.0 eV. An electron in this atom makes a transition from an excited energy level, where $E = -16.0$ eV, to the ground state. What is the wavelength of the emitted photon from this transition?

(A) 138 nm

(B) 112 nm

(C) 77.5 nm

(D) 49.6 nm

4 Mark for Review

The single electron in an atom has an energy of –40 eV when it's in the ground state, and the first excited state for the electron is at –10 eV. What will happen to this electron if the atom is struck by a stream of photons, each of energy 15 eV?

(A) The electron will absorb the energy of one photon and become excited halfway to the first excited state, then quickly return to the ground state, emitting a 15 eV photon in the process.

(B) The electron will absorb the energy of one photon and become excited halfway to the first excited state, then quickly absorb the energy of another photon to reach the first excited state.

(C) The electron will absorb two photons and be excited to the first excited state.

(D) Nothing will happen.

5 Mark for Review

The products of several radioactive decays are being studied. Each particle starts with the same speed and enters into a region with a uniform magnetic field directed perpendicular to the initial velocity of the particles. Which observation could be made?

(A) A neutron and an electron are deflected in the same direction, but with the electron turning with a larger radius.

(B) An alpha particle and an electron are deflected in the same direction, but with the alpha particle turning with a larger radius.

(C) An alpha particle and a neutron are both undeflected.

(D) An alpha particle is deflected and a neutron is undeflected.

6 Mark for Review

A partial energy-level diagram for an atom is shown below. What photon energies could this atom emit if it begins in the $n = 3$ state?

–3 eV ________ $n = 4$

–5 eV ________ $n = 3$

–8 eV ________ $n = 2$

–12 eV ________ $n = 1$ ground state

(A) 5 eV only

(B) 3 eV or 7 eV only

(C) 2 eV, 3 eV, or 7 eV

(D) 3 eV, 4 eV, or 7 eV

7 Mark for Review

Which of the following transitions between energy levels results in emission of the shortest wavelength photon?

(A) A large energy transition to a higher energy level

(B) A small energy transition to a higher energy level

(C) A large energy transition to a lower energy level

(D) A small energy transition to a lower energy level

8 Mark for Review

The de Broglie Hypothesis, that $\lambda = h/p$, was experimentally confirmed by Davisson, Germer, and Thompson. Which of the following describes this hypothesis?

(A) A photon carries a momentum that depends on the wavelength of that photon.

(B) Momentum will be conserved during quantum mechanical processes.

(C) Photons may behave like particles under certain circumstances.

(D) Electrons will undergo diffraction in certain circumstances.

9 Mark for Review

In nuclear reactions, all of the following are true about mass and mass defect EXCEPT

(A) the mass defect is directly proportional to the binding energy

(B) the mass of the unbound nucleons will be less than their mass when combined

(C) the energy associated with the mass defect is significantly greater than the energy levels for electrons in the atom

(D) the mass defect is significantly less than the rest mass of the nucleons

10 Mark for Review

What's the missing particle in the following nuclear reaction?

$$^{2}_{1}H + ^{63}_{29}Cu \longrightarrow ^{64}_{30}Zn + (?)$$

(A) Proton

(B) Neutron

(C) Electron

(D) Positron

11 Mark for Review

A particular isotope of platinum $^{175}_{78}Pt$ has a half-life of just over 2.5 seconds. It can decay either via alpha decay or beta(+) decay. What are the daughter nuclei from each of these processes?

(A) Alpha decay results in $^{171}_{76}Os$ and beta(+) decay results in $^{175}_{77}Ir$.

(B) Alpha decay results in $^{171}_{76}Os$ and beta(+) decay results in $^{175}_{79}Au$.

(C) Alpha decay results in $^{179}_{80}Hg$ and beta(+) decay results in $^{175}_{77}Ir$.

(D) Alpha decay results in $^{179}_{80}Hg$ and beta(+) decay results in $^{175}_{79}Au$.

Section II: Free Response

1 Mark for Review

An experiment is carried out with a series of different colored LED lights (which glow by emitting photoelectrons), which are connected one at a time to a variable voltage supply. When the voltage supply is set to 0 V, none of the LED bulbs light. As the voltage is increased, each LED turns on at a different voltage setting. As the voltage is further increased, the brightness of the LED increases.

Data is collected on the color wavelength (and corresponding frequency) for each LED and the smallest value of the voltage where the light is illuminated.

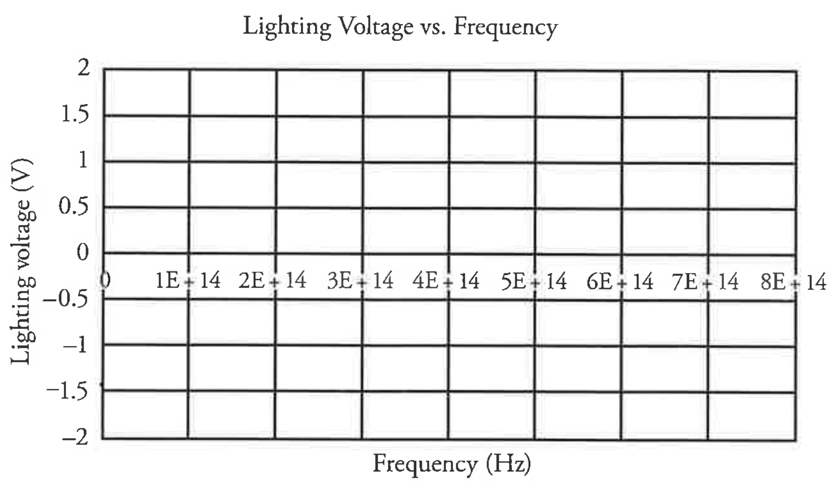
Table 1

Color	Wavelength (nm)	Frequency (10^{14} Hz)	Lighting Voltage (V)
Red	650	4.62	0.51
Yellow	580	5.17	0.74
Green	532	5.64	0.93
Blue	400	7.50	1.71

A. Explain why an increase of $\Delta V = 0.1$ V in the voltage after the light is illuminated causes the brightness to increase, but the bulb remains dark for any of the LEDs with the same voltage increase $\Delta V = 0.1$ V from 0 V to 0.1 V.

B. Plot the data on the axes in Figure 1. Then calculate the slope and the y-intercept of the plot.

Figure 1



C.

i. Explain how, if possible, this experiment could be repeated to get a plot with the same slope but a different intercept. If this is not possible, explain why.

ii. Explain how, if possible, this experiment could be repeated to get a plot with a different slope but the same intercept. If this is not possible, explain why.

Chapter 10 Summary

- The energy available to liberate electrons near the surface of a metal (the photoelectric effect) is proportional to the frequency of the incident photon. This idea is expressed in $E = hf$, where h is Planck's constant ($h = 6.63 \times 10^{-34}$ J·s).

- The work function (ϕ) indicates the amount of energy needed to liberate the electron. There is a minimum frequency needed to liberate the electrons: $f_0 = \frac{\phi}{h}$. The kinetic energy of the emitted electron is $K_{max} = hf - \phi$.

- Particles in motion have wavelike properties: $\lambda = \frac{h}{p}$, where p is the momentum of the particle. Combining this with $E = hf$ and $c = f\lambda$ yields $E = pc$.

- The standard notation for an element is ${}_{Z}^{A}X$, where A is the mass number, Z is the number of protons in the nucleus, and $A = Z + N$, where N is the number of neutrons in the nucleus.

- A nuclear reaction produces new nuclei, other subatomic particles, and the absorption or emission of energy. The change in mass between the reactants and the products tells how much energy is released (exothermic or $+Q$) or how much energy is needed to produce the reaction (endothermic or $-Q$) in the general equation $\text{A} + \text{B} \rightarrow \text{C} + \text{D} + Q$, where A and B are reactants and C and D are products. This energy released or absorbed is given by $\Delta E = (\Delta m)c^2$.