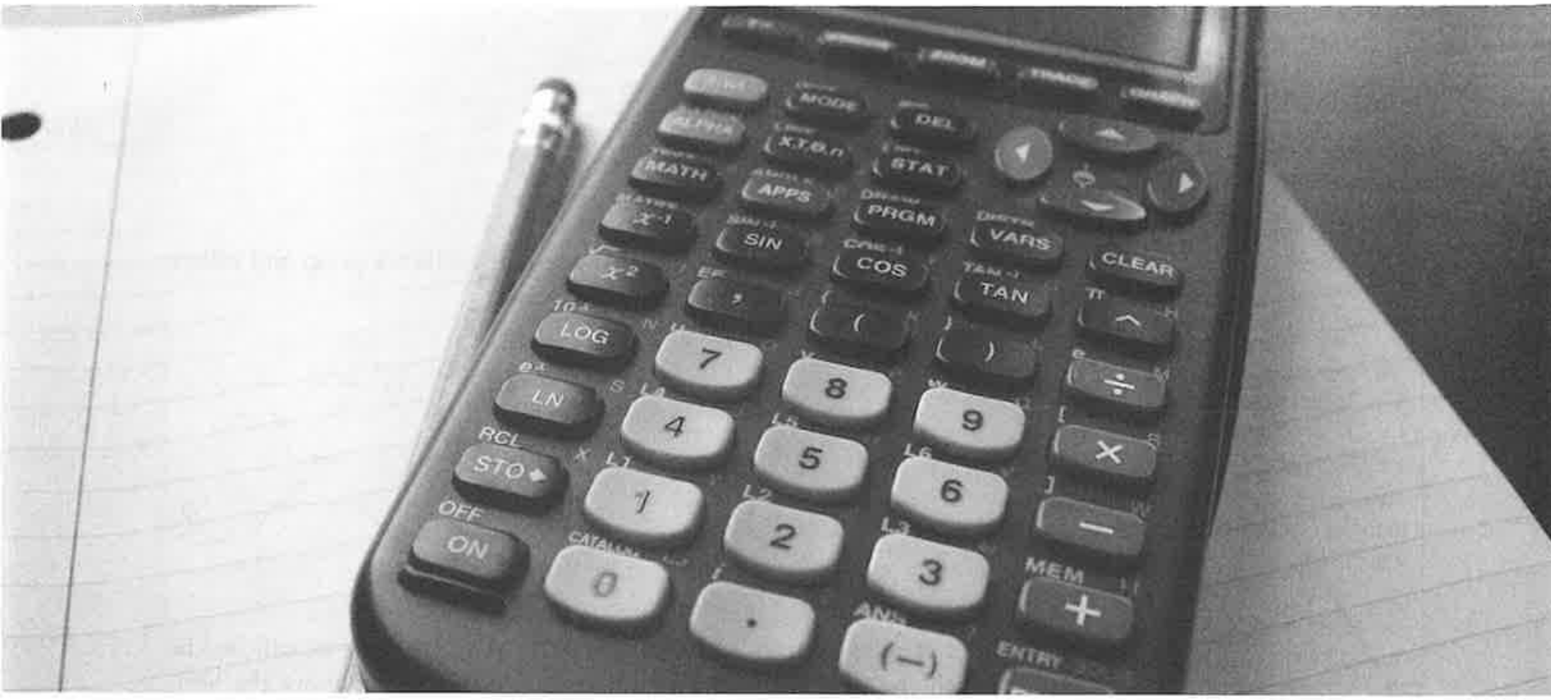
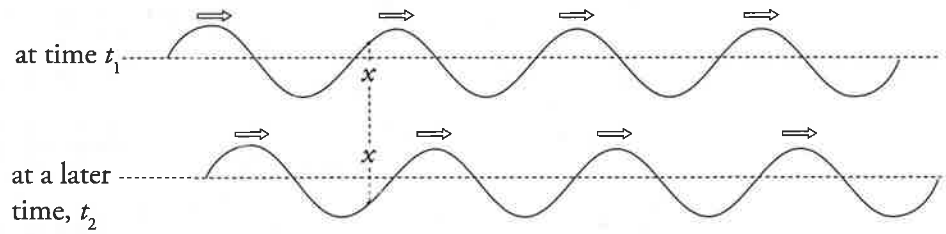


Chapter 9
Mechanical Waves, Sound, and Physical Optics

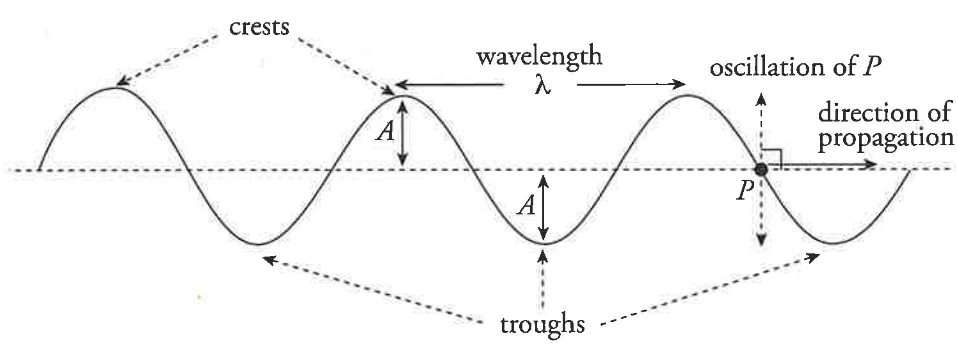
TRANSVERSE TRAVELING WAVES

Let's take a look at a long rope. Someone standing near the system would see peaks and valleys actually moving along the rope, in what's called a **traveling wave.**



Transverse Wave
A transverse wave travels (propagates) in a direction perpendicular to the direction in which the medium is vibrating. Basically, the wave oscillates perpendicular to its direction of travel.

What features of the wave can we see in this point of view? Well, we can see the points at which the rope has its maximum vertical displacement above the horizontal; these points are called **crests.** The points at which the rope has its maximum vertical displacement below the horizontal are called **troughs**. These crests and troughs repeat themselves at regular intervals along the rope, and the distance between two adjacent crests (or two adjacent troughs) is the length of one wave, and is called the **wavelength** (λ, *lambda*). Also, the maximum displacement from the horizontal equilibrium position of the rope is also measurable; this is known as the **amplitude** (A) of the wave.



Future Physicist?
If you're planning to pursue physics beyond this course, pay particular attention to this chapter. Waves are a topic that will come up in one way or another for just about every physics course you take. Forming a strong foundation now will serve you well.

Consider the point P on the rope, as shown in the figure. As the wave **propagates** (or travels) to the right, point P oscillates up and down. Since the direction in which the rope oscillates (vertically) is perpendicular to the direction in which the wave propagates (horizontally), this wave is transverse.

The time it takes for one complete vertical oscillation of a point on the rope is called the period, T, of the wave, and the number of cycles it completes in one second is called its frequency, f. The period and frequency are established by the source of the wave and, of course, $T = 1/f$.

In addition to the amplitude, period, frequency, and wavelength, another important characteristic of a traveling wave is its speed, v. Consider point P in the second figure on the previous page moving from its crest position, down to its trough position, and then back up to the crest position. The wave took a time period of T to move a distance of one wavelength, signified by λ. Therefore, the equation *distance* = *rate* × *time* becomes

$$\lambda = vT$$

$$\lambda = v\left(\frac{1}{f}\right)$$

$$\lambda = v / f$$

The simple equation $\lambda = v/f$ shows how the wave speed, wavelength, and frequency are interconnected. It's the most basic equation in wave theory.

Example 1 A traveling wave on a rope has a frequency of 2.5 Hz. If the speed of the wave is 1.5 m/s, what are its period and wavelength?

Solution. The period is the reciprocal of the frequency:

$$T = \frac{1}{f} = \frac{1}{2.5 \text{ Hz}} = 0.4 \text{ s}$$

and the wavelength can be found from the equation $\lambda = v/f$:

$$\lambda = \frac{v}{f} = \frac{1.5 \text{ m/s}}{2.5 \text{ Hz}} = 0.6 \text{ m}$$

Example 2 The period of a traveling wave is 0.5 s, its amplitude is 10 cm, and its wavelength is 0.4 m. What are its frequency and wave speed?

Solution. The frequency is the reciprocal of the period: $f = 1/T = 1/(0.5 \text{ s}) = 2$ Hz. The wave speed can be found from the equation $\lambda = v/f$:

$$v = \lambda f = (0.4 \text{ m})(2 \text{ Hz}) = 0.8 \text{ m/s}$$

Note that the frequency, period, wavelength, and wave speed have nothing to do with the amplitude.

WAVE SPEED ON A STRETCHED STRING

We can also derive an equation for the speed of a transverse wave on a stretched string or rope. Let the mass of the string be m and its length be L. If the tension in the string is F_T, then the speed of a traveling transverse wave on this string is given by

$$v = \sqrt{\frac{F_T}{m/L}}$$

Note that v depends only on the physical characteristics of the string: its tension and linear density. So, because $\lambda = v/f$ for a given stretched string, varying f will create different waves that have different wavelengths, but v will not vary.

Big Wave Rules

Notice that the equation for the wave speed on a stretched rope shows that v does not depend on f (or λ). While this may seem to contradict the first equation, $\lambda = v/f$, it really doesn't. The speed of the wave depends on the characteristics of the rope: how tense it is and what it's made of. We can wiggle the end at any frequency we want, and the speed of the wave we create will be a constant. However, because $\lambda = v/f$ must always be true, a higher f will mean a shorter λ, and a lower f will mean a longer λ. Thus, changing f doesn't change v: it changes λ. This brings up our first big wave rule:

> Wave Rule #1: The speed of a wave is determined by the type of wave and the characteristics of the medium, not by the frequency.

Rule #1 deals with a wave in a single medium. What travels to you faster, a yell or a whisper? Neither does; they both travel to you at the speed of sound. In a single medium, the speed of a wave is constant and is represented by an inverse relationship between wavelength and frequency: $\lambda_1 f_1 = \lambda_2 f_2$.

Notice that two different types of waves can move at different speeds through the same medium. For example, sound and light move through air at very different speeds.

Our second wave rule addresses what happens when a wave passes from one medium into another. Because wave speed is determined by the characteristics of the medium, a change in the medium causes a change in wave speed, but the frequency won't change.

> Wave Rule #2: When a wave passes into another medium, its speed changes, but its frequency does not.

Example 3 A horizontal rope with linear mass density μ = 0.5 kg/m sustains a tension of 60 N. The non-attached end is oscillated vertically with a frequency of 4 Hz.

(a) What are the speed and wavelength of the resulting wave?

(b) How would you answer these questions if *f* were increased to 5 Hz?

Solution.

(a) Wave speed is established by the physical characteristics of the rope:

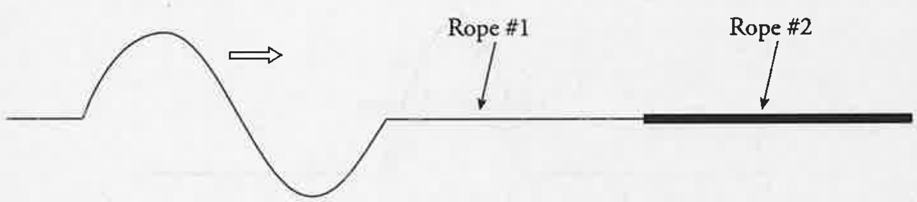
$$v = \sqrt{\frac{F_T}{\mu}} = \sqrt{\frac{60 \text{ N}}{0.5 \text{ kg/m}}} = 11 \text{ m/s}$$

With v, we can find the wavelength: $\lambda = v/f = (11 \text{ m/s})/(4 \text{ Hz}) = 2.8$ m.

(b) If *f* were increased to 5 Hz, then v would not change, but λ would; the new wavelength would be

$$\lambda' = v/f' = (11 \text{ m/s})/(5 \text{ Hz}) = 2.2 \text{ m}$$

Example 4 Two ropes of unequal linear densities are connected, and a wave is created in the rope on the left, which propagates to the right, toward the interface with the heavier rope.

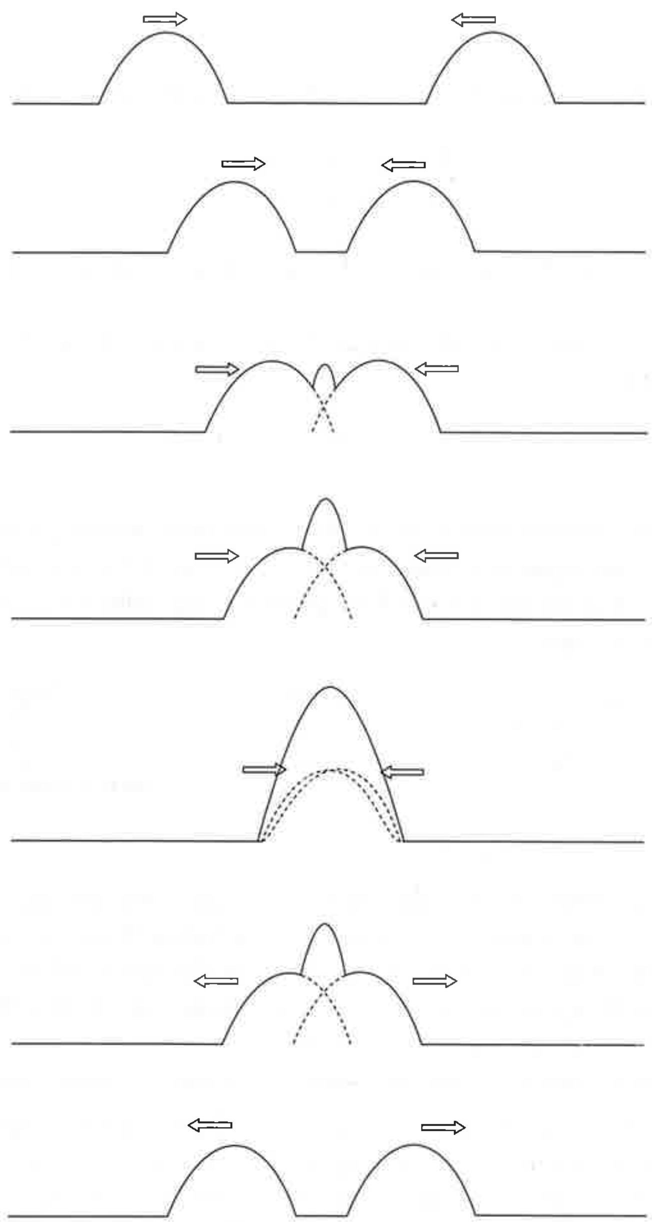


When a wave strikes the boundary to a new medium (in this case, the heavier rope), some of the wave's energy is reflected and some is transmitted. The frequency of the transmitted wave is the same, but the speed and wavelength are not. How do the speed and wavelength of the transmitted wave compare to the speed and wavelength of the incident wave?

Solution. Since the wave enters a new medium, it will have a new wave speed. Because Rope #2 has a greater linear mass density than Rope #1, and because v is inversely proportional to the square root of the linear mass density, the speed of the wave in Rope #2 will be less than the speed of the wave in Rope #1. Since $\lambda = v/f$ must always be satisfied and f does not change, the fact that v changes means that λ must change too. In particular, since v decreases upon entering Rope #2, so will λ.

SUPERPOSITION OF WAVES

When two or more waves meet, the displacement at any point of the medium is equal to the algebraic sum of the displacements due to the individual waves: This is **superposition**. The figure below shows two wave pulses traveling toward each other along a stretched string. Note that when they meet and overlap (**interfere**), the displacement of the string is equal to the sum of the individual displacements, but after they pass, the wave pulses continue, unchanged by their meeting.

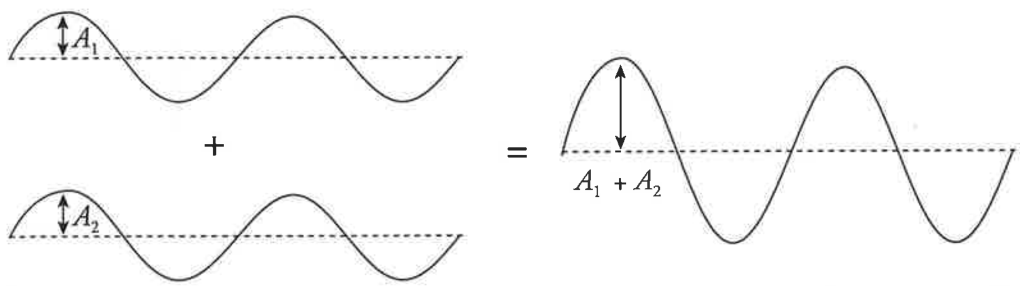


Constructive = Add

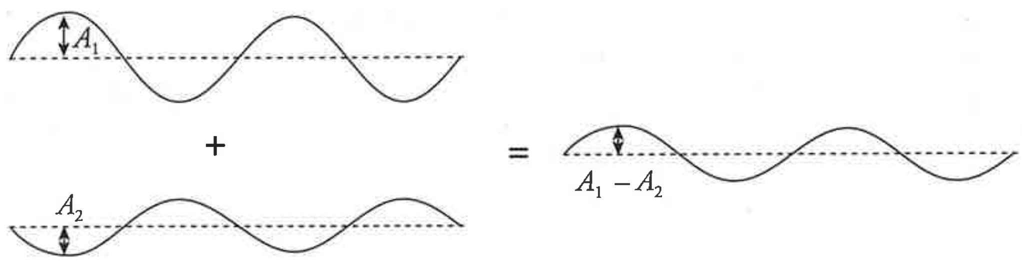
Destructive = Subtract

If the two waves have displacements of the same sign when they overlap, the combined wave will have a displacement of greater magnitude than either individual wave; this is called **constructive interference**. Similarly, if the waves have opposite displacements when they meet, the combined waveform will have a displacement of smaller magnitude than either individual wave; this is called **destructive interference**. If the waves travel in the same direction, the amplitude of the combined wave depends on the relative phase of the two waves. If the waves are exactly **in phase**—that is, if crest meets crest and trough meets trough—then the waves will constructively interfere completely, and the amplitude of the combined wave will be the sum of the individual amplitudes. However, if the waves are exactly **out of phase**—that is, if crest meets trough and trough meets crest—then they will destructively interfere completely, and the amplitude of the combined wave will be the difference between the individual amplitudes. In general, the waves will be somewhere in between exactly in phase and exactly out of phase.

In Phase



Out of Phase

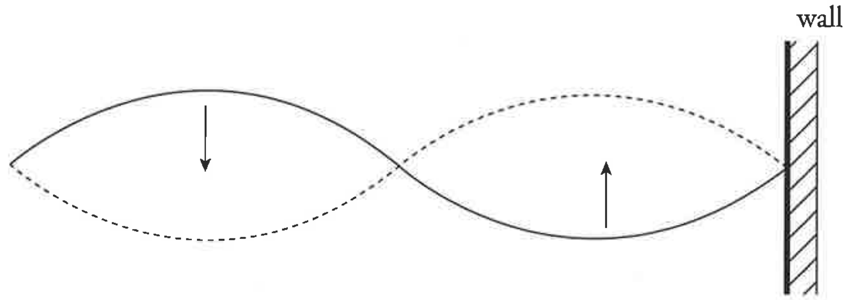


Example 5 Two waves, one with an amplitude of 8 cm and the other with an amplitude of 3 cm, travel in the same direction on a single string and overlap. What are the maximum and minimum possible amplitudes of the string while these waves overlap?

Solution. The maximum amplitude occurs when the waves are exactly in phase; the amplitude of the combined waveform will be 8 cm + 3 cm = 11 cm. The minimum amplitude occurs when the waves are exactly out of phase; the amplitude of the combined waveform will then be 8 cm – 3 cm = 5 cm. Without more information about the relative phase of the two waves, all we can say is that the amplitude will be at least 5 cm and no greater than 11 cm.

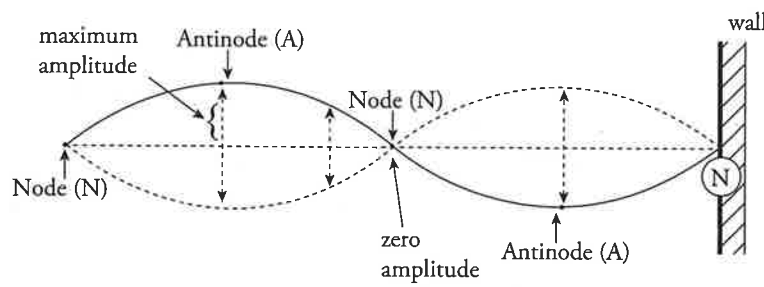
STANDING WAVES

When our prototype traveling wave on a string strikes the wall, the wave will reflect and travel back toward us. The string now supports two traveling waves: the wave we generated at our end, which travels toward the wall, and the reflected wave. What we actually see on the string is the superposition of these two oppositely directed traveling waves, which have the same frequency, amplitude, and wavelength. If the length of the string is just right, the resulting pattern will oscillate vertically and remain fixed. The crests and troughs no longer travel down the length of the string. This is a **standing wave.**



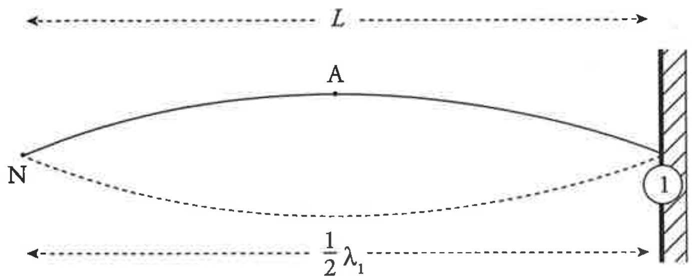
The right end is fixed to the wall, and the left end is oscillated through a negligibly small amplitude so that we can consider both ends to be essentially fixed (no vertical oscillation). The interference of the two traveling waves results in complete destructive interference at some points (marked N in the figure on the next page), and complete constructive interference at other points (marked A in the figure). Other points have amplitudes between these extremes. Note another difference between a traveling wave and a standing wave: while every point on the string had the same amplitude as the traveling wave went by, each point on a string supporting a standing wave has an individual amplitude. The points marked N are called **nodes**, and those marked A are called **antinodes.**

Tip
You can remember nodes as areas of "**no d**isplac**e**ment." Antinodes are the opposite of that.

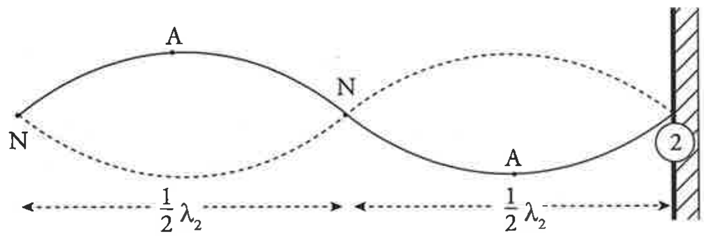


Nodes and antinodes always alternate, they're equally spaced, and the distance between two successive nodes (or antinodes) is equal to $\frac{1}{2}\lambda$. This information can be used to determine how standing waves can be generated. The following figures show the three simplest standing waves that our string can support. The first standing wave has one antinode, the second has two, and the third has three. The length of the string in all three diagrams is L.

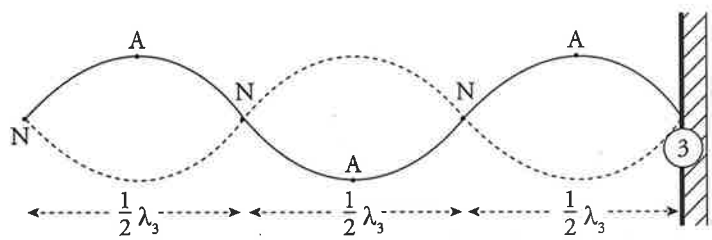
1st Harmonic



2nd Harmonic



3rd Harmonic



For the first standing wave, notice that L is equal to $1(\frac{1}{2}\lambda)$. For the second standing wave, L is equal to $2(\frac{1}{2}\lambda)$, and for the third, $L = 3(\frac{1}{2}\lambda)$. A pattern is established: a standing wave can form only when the length of the string is a multiple of $\frac{1}{2}\lambda$:

$$L = n(\frac{1}{2}\lambda)$$

Solving this for the wavelength, we get

$$\lambda_n = \frac{2L}{n}$$

These are called the **harmonic** (or **resonant**) **wavelengths**, and the integer n is known as the **harmonic number**.

Since we typically have control over the frequency of the waves we create, it's more helpful to figure out the *frequencies* that generate a standing wave. Because $\lambda = v/f$, and because v is fixed by the physical characteristics of the string, the special λs found above correspond to equally special frequencies. From $f_n = v/\lambda_n$, we get

$$f_n = \frac{nv}{2L}$$

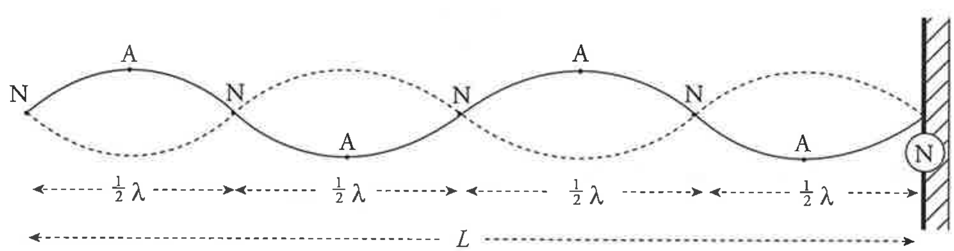
These are the **harmonic** (or **resonant**) **frequencies.** A standing wave will form on a string if we create a traveling wave whose frequency is the same as a resonant frequency. The first standing wave, the one for which the harmonic number, n, is 1, is called the **fundamental standing wave.** From the equation for the harmonic frequencies, we see that the nth harmonic frequency is simply n times the fundamental frequency:

$$f_n = nf_1$$

Similarly, the nth harmonic wavelength is equal to λ_1 divided by n. Therefore, by knowing the fundamental frequency (or wavelength), all the other resonant frequencies and wavelengths can be determined.

Example 6 A string of length 12 m that's fixed at both ends supports a standing wave with a total of 5 nodes. What are the harmonic number and wavelength of this standing wave?

Solution. First, draw a picture.



This shows that the length of the string is equal to $4(\frac{1}{2}\lambda)$, so

$$L = 4(\frac{1}{2}\lambda) \quad \Rightarrow \quad \lambda = \frac{2L}{4}$$

This is the fourth-harmonic standing wave, with wavelength λ_4 (because the expression above matches $\lambda_n = 2L/n$ for $n = 4$). Since $L = 12$ m, the wavelength is

$$\lambda_4 = \frac{2(12 \text{ m})}{4} = 6 \text{ m}$$

Example 7 A string of length 10 m and mass 300 g is fixed at both ends, and the tension in the string is 40 N. What is the frequency of the standing wave for which the distance between a node and the closest antinode is 1 m?

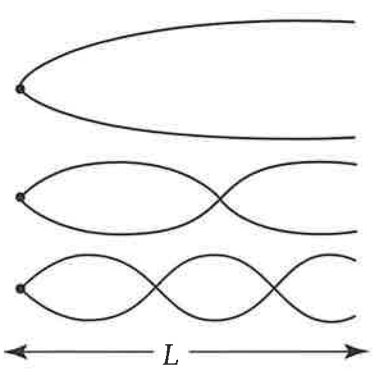
Solution. Because the distance between two successive nodes (or successive antinodes) is equal to $\frac{1}{2}\lambda$, the distance between a node and the closest antinode is half this, or $\frac{1}{4}\lambda$. Therefore, $\frac{1}{4}\lambda = 1$ m, so $\lambda = 4$ m. Since the harmonic wavelengths are given by the equation $\lambda_n = 2L/n$, we can find that

$$4 \text{ m} = \frac{2(10 \text{ m})}{n} \quad \Rightarrow \quad n = 5$$

The frequency of the fifth harmonic is

$$f_5 = \frac{5v}{2L} = \frac{5}{2L}\sqrt{\frac{F_T}{\mu}} = \frac{5}{2L}\sqrt{\frac{F_T}{m/L}} = \frac{5}{2(10 \text{ m})}\sqrt{\frac{40 \text{ N}}{(0.3 \text{ kg}/10 \text{ m})}} = 9.1 \text{ Hz}$$

If you attached a rope or string to a ring that could slide up and down a pole without friction, you would make a rope that is fixed at one end but free at another end. This would create nodes at the closed end and antinodes at the open end. Below are some possible examples of this.



$$L = \frac{1}{4}\lambda \quad \text{or} \quad \lambda_1 = \frac{4L}{1}$$

$$L = \frac{3}{4}\lambda \quad \text{or} \quad \lambda_3 = \frac{4L}{3}$$

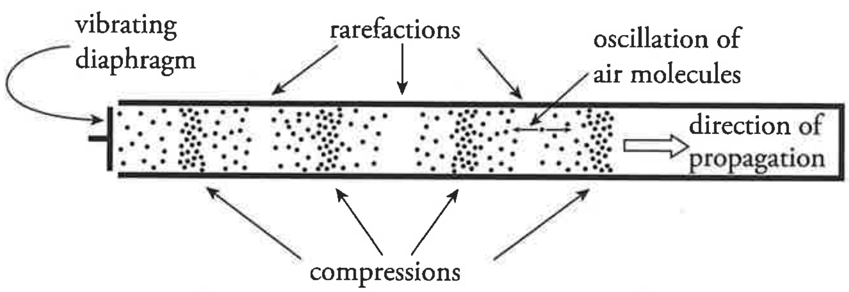
$$L = \frac{5}{4}\lambda \quad \text{or} \quad \lambda_5 = \frac{4L}{5}$$

If one end of the rope is free to move, standing waves can form for wavelengths of $\lambda_n = \frac{4L}{n}$ or by frequencies of $f_n = \frac{nv}{4L}$, where L is the length of the rope, v is the speed of the wave, and n must be an odd integer.

SOUND WAVES

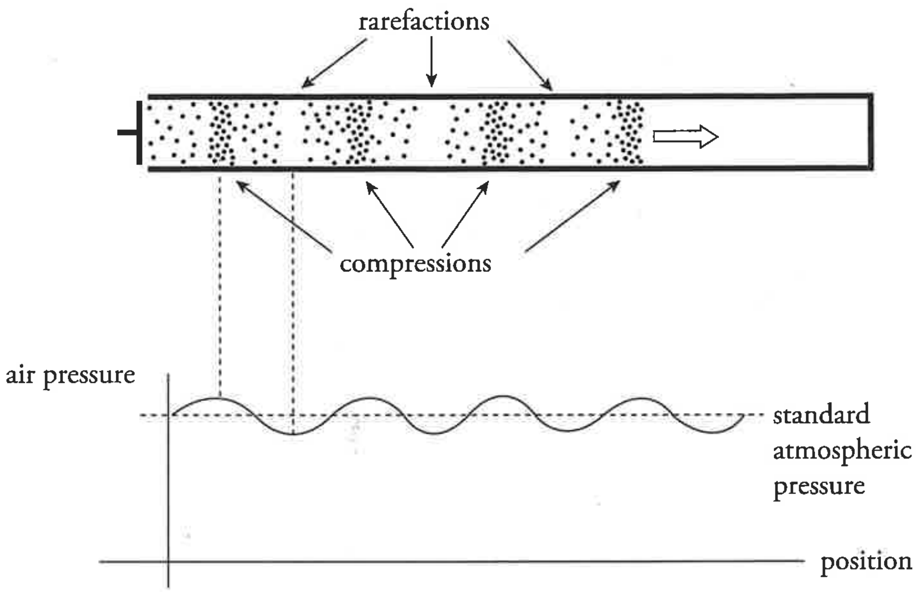
Speed of Sound
At sea level, approximately 340 m/s.

Sound waves are produced by the vibration of an object, such as your vocal cords, a plucked string, or a jackhammer. The vibrations cause pressure variations in the conducting medium (which can be gas, liquid, or solid), and if the frequency is between 20 Hz and 20,000 Hz, the vibrations may be detected by human ears and perceived as sound. The variations in the conducting medium can be positions at which the molecules of the medium are bunched together (where the pressure is above normal), which are called **compressions**, and positions where the pressure is below normal, called **rarefactions**. In the figure below, a vibrating diaphragm sets up a sound wave in an air-filled tube. Each dot represents many, many air molecules:



Longitudinal Waves
A longitudinal wave travels and oscillates in the same direction.

An important difference between sound waves and the waves we've been studying on stretched strings is that the molecules of the medium transmitting a sound wave move *parallel* to the direction of wave propagation, rather than perpendicular to it. For this reason, sound waves are said to be **longitudinal**. Despite this difference, all of the basic characteristics of a wave—amplitude, wavelength, period, frequency—apply to sound waves as they did for waves on a string. Furthermore, the all-important equation $\lambda = v/f$ also holds true. However, because it's very difficult to draw a picture of a longitudinal wave, an alternate method is used: we can graph the pressure as a function of position:



The speed of a sound wave depends on the medium through which it travels. At room temperature (approximately 20°C) and normal atmospheric pressure, sound travels at 343 m/s. This value increases as air warms.

Example 8 A sound wave with a frequency of 300 Hz travels through the air.
(a) What is its wavelength?
(b) If its frequency increased to 600 Hz, what are the wave's speed and wavelength?

Solution.

(a) Using v = 343 m/s for the speed of sound through air, we find that

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{300 \text{ Hz}} = 1.14 \text{ m}$$

(b) Unless the ambient pressure or the temperature of the air changed, the speed of sound would not change. Wave speed depends on the characteristics of the medium, not on the frequency, so v would still be 343 m/s. However, a change in frequency would cause a change in wavelength. Since f increased by a factor of 2, λ would decrease by a factor of 2, to $\frac{1}{2}$(1.14 m) = 0.57 m.

Example 9 A sound wave traveling through water has a frequency of 500 Hz and a wavelength of 3 m. How fast does sound travel through water?

Solution. $v = \lambda f$ = (3 m)(500 Hz) = 1500 m/s.

Beats

If two sound waves whose frequencies are close but not identical interfere, the resulting sound modulates in amplitude, becoming loud, then soft, then loud, then soft. This is due to the fact that as the individual waves travel, they are in phase, then out of phase, then in phase again, and so on. Therefore, by superposition, the waves interfere constructively, then destructively, then constructively, and so on. When the waves interfere constructively, the amplitude increases, and the sound is loud; when the waves interfere destructively, the amplitude decreases, and the sound is soft. Each time the waves interfere constructively, producing an increase in sound level, we say that a **beat** has occurred. The number of beats per second, known as the **beat frequency**, is equal to the difference between the frequencies of the two combining sound waves:

$$f_{\text{beat}} = |f_1 - f_2|$$

If frequencies f_1 and f_2 match, then the combined waveform doesn't waver in amplitude, and no beats are heard.

Example 10 A piano tuner uses a tuning fork to adjust the key that plays the A note above middle C (whose frequency should be 440 Hz). The tuning fork emits a perfect 440 Hz tone. When the tuning fork and the piano key are struck, beats of frequency 3 Hz are heard.

(a) What is the frequency of the piano key?

(b) If it's known that the piano key's frequency is too high, should the piano tuner tighten or loosen the wire inside the piano in order to tune it?

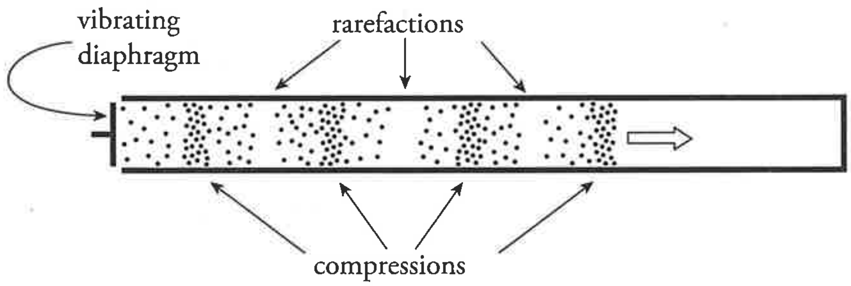
Solution.

(a) Since f_{beat} = 3 Hz, the tuning fork and the piano string are off by 3 Hz. Since the fork emits a tone of 440 Hz, the piano string must emit a tone of either 437 Hz or 443 Hz. Without more information, we can't determine which.

(b) If we know that the frequency of the tone emitted by the out-of-tune string is too high (that is, it's 443 Hz), we need to find a way to lower the frequency. Remember that the resonant frequencies for a stretched string fixed at both ends are given by the equation $f = nv/2L$, and that $v = \sqrt{F_T/\mu}$. Since f is too high, v must be too high. To lower v, we must reduce F_T. The piano tuner should loosen the string and listen for beats again, adjusting the string until the beats disappear.

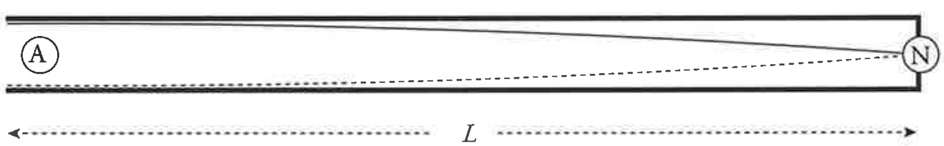
RESONANCE FOR SOUND WAVES

Just as standing waves can be set up on a vibrating string, standing *sound* waves can be established within an enclosure. In the figure below, a vibrating source at one end of an air-filled tube produces sound waves that travel the length of the tube.



These waves reflect off the far end, and the superposition of the forward and reflected waves can produce a standing wave pattern if the length of the tube and the frequency of the waves are related in a certain way.

Notice that air molecules at the far end of the tube can't oscillate horizontally because they're up against a wall. So, the far end of the tube is a displacement node. But the other end of the tube (where the vibrating source is located) is a displacement antinode. A standing wave with one antinode (A) and one node position (N) can be depicted as follows:

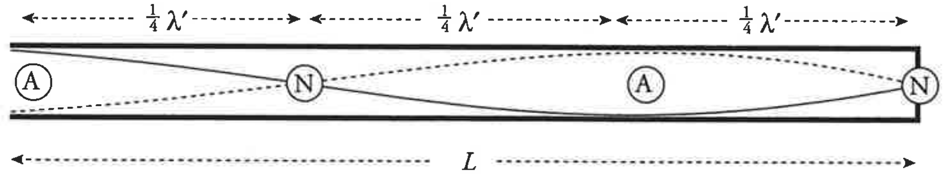


Although sound waves in air are longitudinal, here we'll show the wave as transverse so that it's easier to determine the wavelength. Since the distance between an antinode and an adjacent node is always $\frac{1}{4}$ of the wavelength, the length of the tube, L, in the figure above is $\frac{1}{4}$ the wavelength. This is the longest standing wavelength that can fit in the tube, so it corresponds to the lowest standing wave frequency, the fundamental:

$$L = \frac{\lambda_1}{4} \quad \Rightarrow \quad \lambda_1 = 4L \quad \Rightarrow \quad f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

Our condition for resonance was a node at the closed end and an antinode at the open end. Therefore, the next higher-frequency standing wave that can be supported in this tube must have two antinodes and two nodes:

Make sure you're comfortable with drawing these standing wave diagrams in addition to understanding the equations. Free-response questions often require you to produce images.

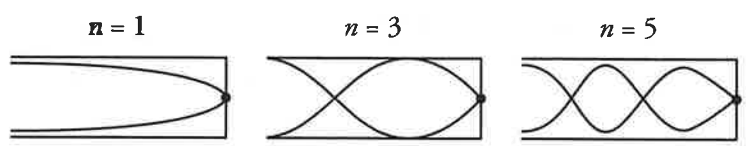


In this case, the length of the tube is equal to $3(\frac{1}{4}\lambda')$, so

$$L = \frac{3\lambda'}{4} \Rightarrow \lambda' = \frac{4L}{3} \Rightarrow f' = \frac{v}{\lambda'} = \frac{3v}{4L}$$

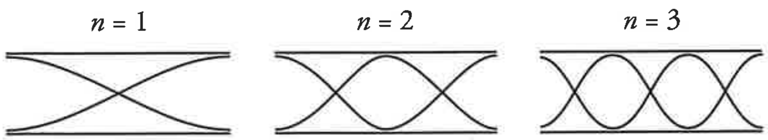
Here's the pattern: standing sound waves can be established in a tube that's closed at one end if the tube's length is equal to an *odd* multiple of $\frac{1}{4}\lambda$. The resonant wavelengths and frequencies are given by the following equations:

For a Closed Tube, $\left.\begin{aligned}\lambda_n &= \frac{4L}{n}\\ f_n &= n\frac{v}{4L}\end{aligned}\right\}$ for any *odd* integer n



If the far end of the tube is not sealed, standing waves can still be established in the tube, because sound waves can be reflected from the open air. A closed end is a displacement node, but an open end is a displacement antinode. In this case, then, the standing waves will have two displacement antinodes (at the ends of the tube), and the resonant wavelengths and frequencies will be given by

For an Open Tube, $\left.\begin{aligned} \lambda_n &= \frac{2L}{n} \\ f_n &= n\frac{v}{2L} \end{aligned}\right\}$ for any integer n



Note that, while an open-end tube can support any harmonic, a closed-end tube can support only odd harmonics.

Example 11 A closed-end tube resonates at a fundamental frequency of 440.0 Hz. The air in the tube is at a temperature of 20°C, and it conducts sound at a speed of 343 m/s.

(a) What is the length of the tube?
(b) What is the next higher harmonic frequency?
(c) Answer the questions posed in (a) and (b) assuming that the tube is open at its far end.

Solution.

(a) For a closed-end tube, the harmonic frequencies obey the equation $f_n = nv/(4L)$. The fundamental corresponds to $n = 1$, so

$$f_1 = \frac{v}{4L} \quad \Rightarrow \quad L = \frac{v}{4f_1} = \frac{343 \text{ m/s}}{4(440.0 \text{ Hz})} = 0.195 \text{ m} = 19.5 \text{ cm}$$

(b) Since a closed-end tube can support only *odd* harmonics, the next higher harmonic frequency (the first **overtone**) is the *third* harmonic, f_3, which is

$$3f_1 = 3(440.0 \text{ Hz}) = 1320 \text{ Hz}$$

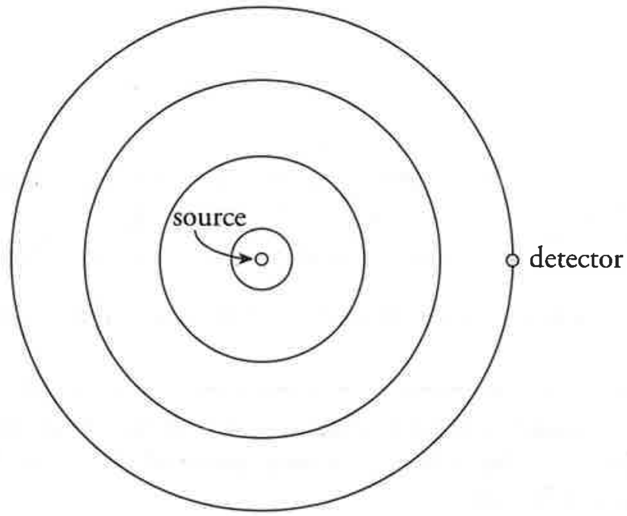
(c) For an open-end tube, the harmonic frequencies obey the equation $f_n = nv/(2L)$. The fundamental corresponds to $n = 1$, so

$$f_1 = \frac{v}{2L'} \quad \Rightarrow \quad L' = \frac{v}{2f_1} = \frac{343 \text{ m/s}}{2(440.0 \text{ Hz})} = 0.390 \text{ m} = 39.0 \text{ cm}$$

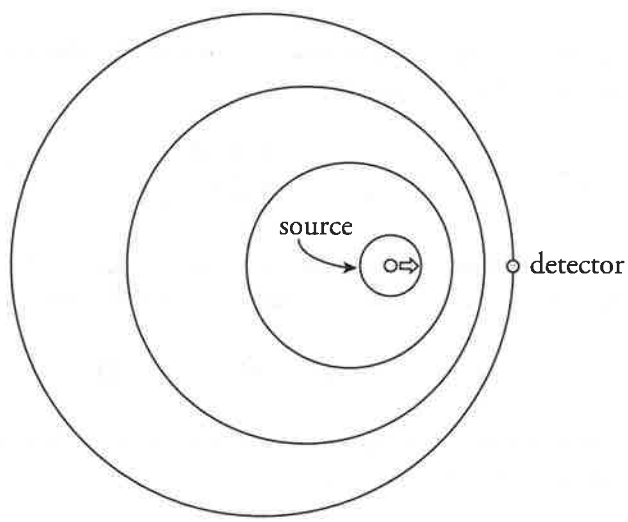
And, since an open-end tube can support any harmonic, the first overtone would be the second harmonic, $f_2 = 2f_1 = 2(440.0 \text{ Hz}) = 880.0 \text{ Hz}$.

THE DOPPLER EFFECT

When a source of sound waves and a detector are not in relative motion, the frequency that the source emits matches the frequency that the detector receives.

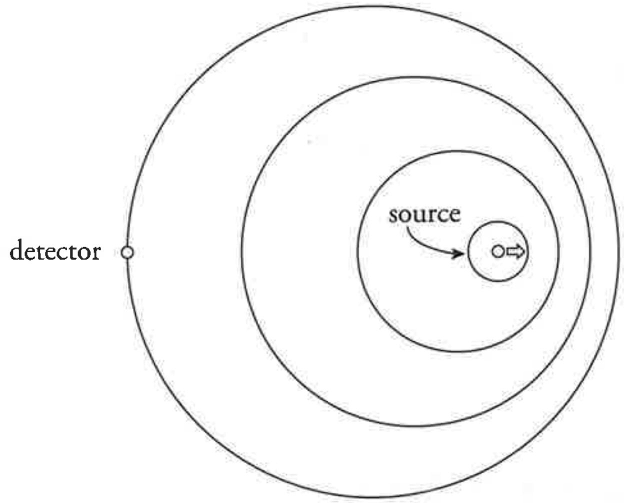


However, if there *is* relative motion between the source and the detector, then the waves that the detector receives are different in frequency (and wavelength). For example, if the detector moves toward the source, then the detector intercepts the waves at a rate higher than the one at which they were emitted; the detector hears a higher frequency than the source emitted. In the same way, if the source moves toward the detector, the wavefronts pile up, and this results in the detector receiving waves with shorter wavelengths and higher frequencies:



You're almost done with this chapter! If you haven't taken a break in a while, consider putting this book aside to take a walk, get some fresh air, or unwind by listening to music or reading a book (not this one!). Mental breaks are essential to test prep!

Conversely, if the detector is moving away from the source or if the source is moving away from the detector,



While equations for solving the changes due to Doppler effect exist, you don't need to know them for the AP Exam. Only a qualitative understanding is required.

then the detected waves have a longer wavelength and a lower frequency than they had when they were emitted by the source. The shift in frequency and wavelength that occurs when the source and detector are in relative motion is known as the **Doppler effect**. In general, relative motion *toward* each other results in a higher perceived frequency, and relative motion *away* from each other results in a lower perceived frequency.

Example 12 A ambulance blares a siren at a constant frequency, f_0, describe how the frequency detected would differ from f_0 for

(a) A person standing by the road before the ambulance passes them.

(b) A person standing by the road after the ambulance passes them.

(c) A car behind the ambulance going faster than the ambulance and accelerating.

Solution. The frequency detected is higher if the detector and source have relative motion toward each other, and lower if the detector and source have relative motion away from each other, so

(a) The ambulance is approaching the person by the road, so they would detect a frequency higher than f_0.

(b) The ambulance is now leaving the person by the road, so they would detect a frequency lower than f_0.

(c) The car is catching up to the ambulance, so they would detect a frequency higher than f_0, and the detected frequency would increase even more as the car sped up.

PHYSICAL OPTICS

Light (or visible light) makes up only a small part of the entire spectrum of electromagnetic waves, which ranges from radio waves to gamma rays. Most waves, including those we've discussed thus far, require a material medium for their transmission, but electromagnetic waves can propagate through empty space. Electromagnetic waves consist of time-varying electric and magnetic fields that oscillate perpendicular to each other and to the direction of propagation of the wave. Through a vacuum, all electromagnetic waves travel at a fixed speed:

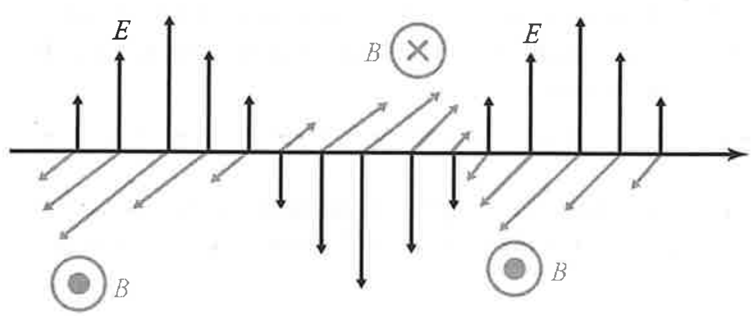
$$c = 3.00 \cdot 10^8 \text{ m/s}$$

regardless of their frequency. Like all waves, $\lambda = v/f$ for electromagnetic waves.

ELECTROMAGNETIC WAVES

We saw previously that if we oscillate one end of a long rope, we generate a wave that travels down the rope and has the same frequency as that of the oscillation.

You can think of an electromagnetic wave in a similar way: an oscillating electric charge generates an electromagnetic (EM) wave, which is composed of oscillating electric and magnetic fields. These fields oscillate with the same frequency at which the electric charge that created the wave oscillated. The fields oscillate in phase with each other, perpendicular to each other and the direction of propagation. For this reason, electromagnetic waves are transverse waves. The direction in which the wave's electric field oscillates is called the direction of polarization of the wave.

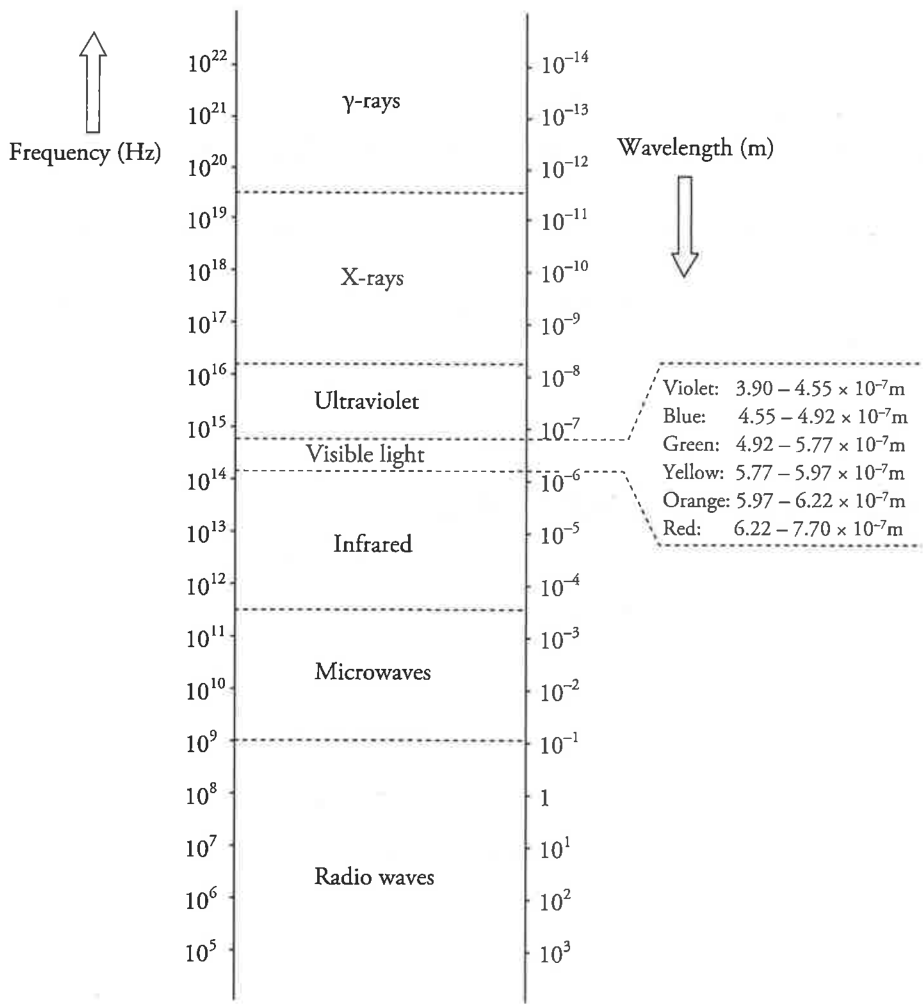


THE ELECTROMAGNETIC SPECTRUM

Electromagnetic waves can be categorized by their frequency (or wavelength); the full range of waves is called the **electromagnetic** (or **EM**) **spectrum**. Types of waves include **radio waves, microwaves, infrared, visible light, ultraviolet, X-rays**, and **γ-rays (gamma rays)** and, although they've been delineated in the spectrum below, there's no universal agreement on all the boundaries, so many of these bands overlap. You should be familiar with the names of the major categories, and, in particular, memorize the order of the colors within the visible spectrum (which, as you can see, accounts for only a tiny sliver of the full EM spectrum). In order of increasing wave frequency, the colors are red, orange, yellow, green, blue, and violet, which is commonly remembered as ROYGBV ("roy-gee-biv"). The wavelengths of the colors in the visible spectrum are usually expressed in nanometers (nm). For example, electromagnetic waves whose wavelengths are between 577 nm and 597 nm are seen as yellow light.

Electromagnetic Wave Speed
All electromagnetic waves, regardless of frequency, travel through a vacuum at this speed. The most important equation for waves, $\lambda = v/f$, is also true for electromagnetic waves. For EM waves traveling through a vacuum, $v = c$, so the equation becomes $\lambda f = c$.

ELECTROMAGNETIC SPECTRUM



Visible Spectrum
White light is not colorless. White light is actually composed of a combination of all of the colors of the visible spectrum.

Example 13 What's the frequency range for green light?

Solution. According to the spectrum, light is green if its wavelength is between 4.92×10^{-7} m and 5.77×10^{-7} m. Using the equation $\lambda = v/f$, we find that the upper end of this wavelength range corresponds to a frequency of

$$f_1 = \frac{v}{\lambda_1} = \frac{3.00 \times 10^8 \text{ m/s}}{5.77 \times 10^{-7} \text{ m}} = 5.20 \times 10^{14} \text{ Hz}$$

while the lower end corresponds to

$$f_2 = \frac{v}{\lambda_2} = \frac{3.00 \times 10^8 \text{ m/s}}{4.92 \times 10^{-7} \text{ m}} = 6.10 \times 10^{14} \text{ Hz}$$

So, the frequency range for green light is

$$5.20 \times 10^{14} \text{ Hz} \le f_{\text{green}} \le 6.10 \times 10^{14} \text{ Hz}$$

Example 14 How would you classify electromagnetic radiation that has a wavelength of 1 cm?

Solution. According to the electromagnetic spectrum presented above, electromagnetic waves with $\lambda = 10^{-2}$ m are microwaves.

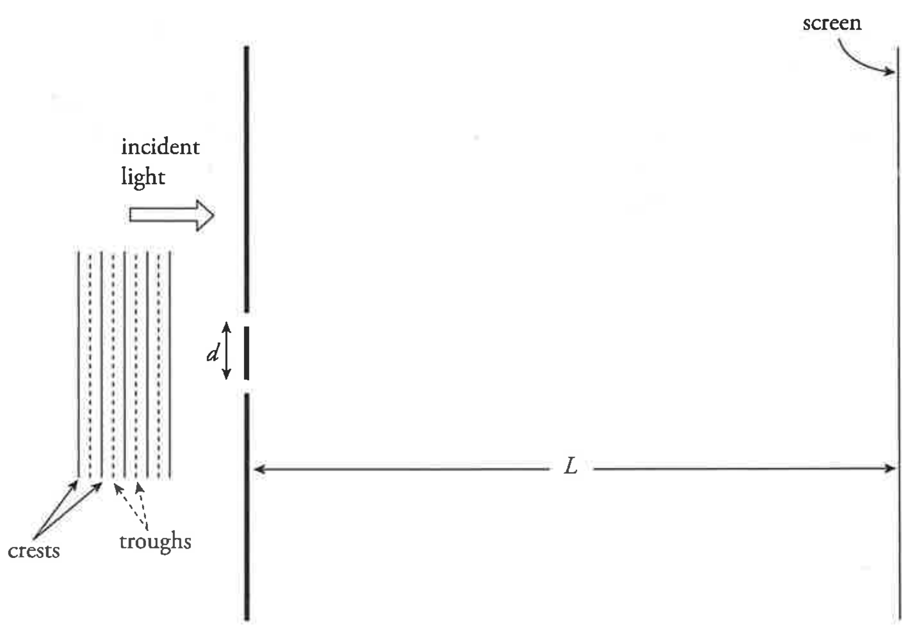
INTERFERENCE AND DIFFRACTION

As we learned previously, waves experience interference when they meet, and whether they interfere constructively or destructively depends on their relative phase. If they meet *in phase* (crest meets crest), they combine constructively, but if they meet *out of phase* (crest meets trough), they combine destructively. The key to the interference patterns we'll study in the next section rests on this observation. In particular, if waves that have the same wavelength meet, then the difference in the distances they've traveled determines whether the waves are in phase. Assuming that the waves are **coherent** (which means that their phase difference remains constant over time and does not vary), if the difference in their path lengths, ΔD, is a whole number of wavelengths—0, $\pm\lambda$, $\pm 2\lambda$, etc.—they'll arrive in phase at the meeting point. On the other hand, if this difference is a whole number plus one-half a wavelength—$\pm\frac{1}{2}\lambda$, $\pm(1+\frac{1}{2})\lambda$, $\pm(2+\frac{1}{2})\lambda$, etc.—then they'll arrive exactly out of phase. That is,

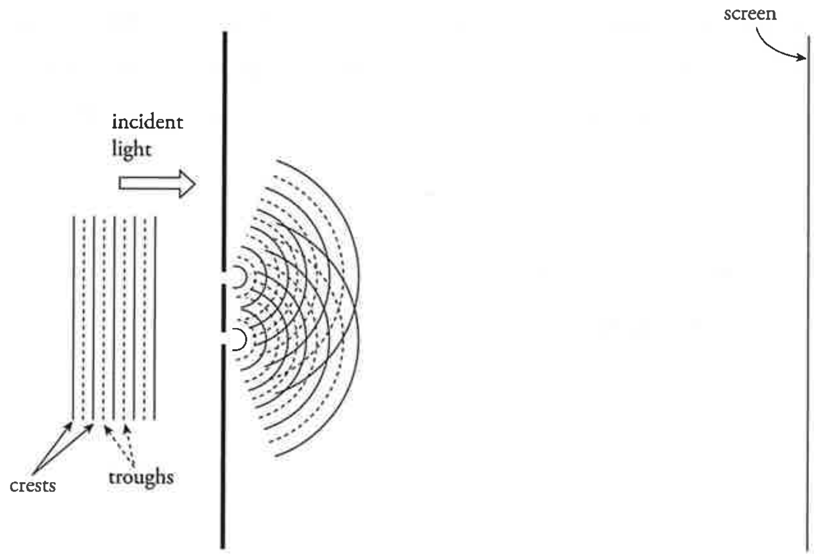
$$\left.\begin{array}{ll}\text{constructive interference:} & \Delta D = m\lambda \\ \text{destructive interference:} & \Delta D = (m+\frac{1}{2})\lambda\end{array}\right\} \text{where } m = 0, 1, 2\ldots$$

Young's Double-Slit Interference Experiment

The following figure shows incident light on a barrier that contains two narrow slits (perpendicular to the plane of the page), separated by a distance *d*. On the right is a screen whose distance from the barrier, *L*, is much greater than *d*. The question is, what will we see on the screen? You might expect that we'll just see two bright narrow strips of light, directly opposite the slits in the barrier. As reasonable as this may sound, it doesn't take into account the wave nature of light.



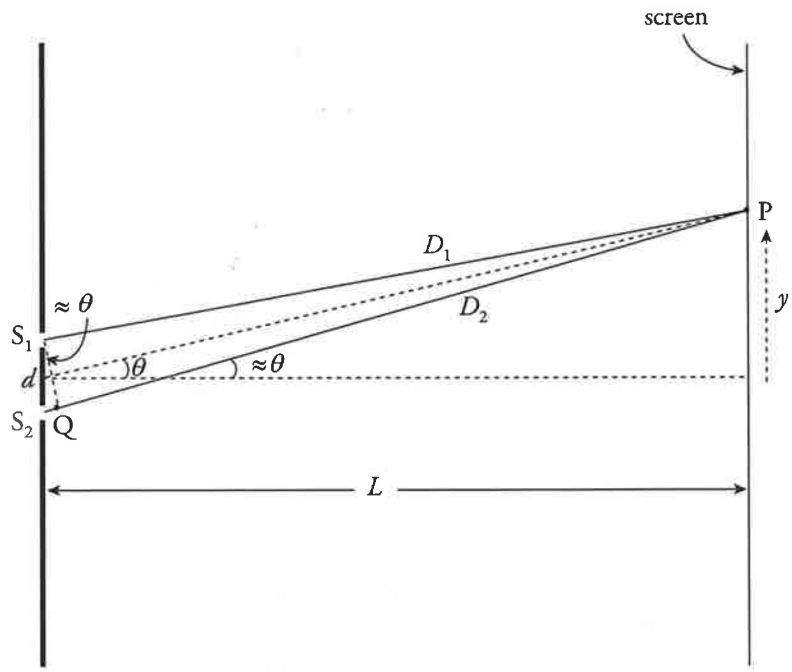
When a wave encounters an aperture whose width is comparable to its wavelength, the wave will fan out after it passes through. The alteration in the straight-line propagation of a wave when it encounters a barrier is called **diffraction**. In the setup above, the waves will diffract through the slits, and spread out and interfere as they travel toward the screen.



You can clearly see points of interference in the figure above. Look for solid lines intersecting other solid lines—these are points of constructive interference. Look for solid lines intersecting dashed lines—these are points of destructive interference.

The screen will show the results of this interference: there will be bright bands (bright **fringes**) centered at those points at which the waves interfere constructively, alternating with dark fringes, where the waves interfere destructively. Let's determine the locations of these fringes.

In the figure below, we've labeled the slits S_1 and S_2. A point P on the screen is selected, the path lengths from S_1 and S_2 to P are called D_1 and D_2, respectively, and the angle that the line from the midpoint of the slits to P makes with the horizontal is θ. Segment S_1Q is perpendicular to line S_2P. Because L is so much larger than d, the angle that line S_2P makes with the horizontal is also approximately θ, which tells us that $\angle S_2S_1Q$ is approximately θ and that the path difference, $\Delta D = D_2 - D_1$, is nearly equal to S_2Q.



From what we learned earlier about constructive interference, there will be an intensity maximum when the path difference is some multiple of the wavelength, λ. Because the path difference is $S_2Q = d\sin\theta$, we can write:

constructive interference
(intensity maximum
bright fringe on screen): $\Delta D = m\lambda = d\sin\theta$ where $m = 0, 1, 2...$

Equation Sheet

Similarly, there will be an intensity maximum when the path difference is half a wavelength off, so

destructive interference
(intensity minimum
dark fringe on screen): $\Delta D = (m + \frac{1}{2})\lambda = d\sin\theta$ where $m = 0, 1, 2...$

Equation Sheet

To locate the positions of the bright fringes on the screen, we use the fact that $\tan\theta = \frac{y}{L}$. If θ is small, then $\tan\theta \approx \sin\theta$, so we can write $\sin\theta = \frac{y}{L}$ (we can tell this from the figure). Since $d\sin\theta = m\lambda$ for bright fringes, we get

Equation Sheet

$$d\left(\frac{y_{max}}{L}\right) = m\lambda$$

Also, the intensity of the bright fringes decreases as m increases in magnitude. The bright fringe directly opposite the midpoint of the slits—the **central maximum**—corresponds to $m = 0$ and has the greatest intensity. The bright fringes with $m = 1$ will have a lower intensity, those with $m = 2$ will be fainter still, and so on. If more than two slits are cut in the barrier, the interference pattern becomes sharper, and the distinction between dark and bright fringes becomes more pronounced. Barriers containing thousands of tiny slits per centimeter—called **diffraction gratings**—are used precisely for this purpose.

Example 15 For the experimental setup we've been studying, assume that $d = 1.5$ mm, $L = 6.0$ m, and that the light used has a wavelength of 589 nm.

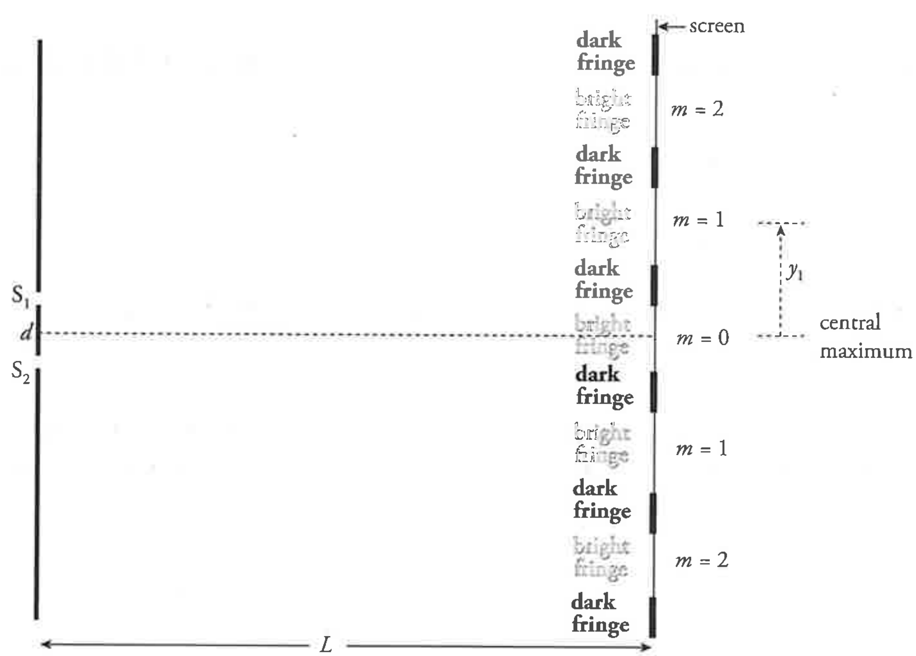
(a) How far above the center of the screen will the second brightest maximum appear?

(b) How far below the center of the screen is the third dark fringe?

(c) What would happen to the interference pattern if the slits were moved closer together?

Solution.

(a) The central maximum corresponds to $m = 0$ ($y = 0$). The first maximum above the central one is labeled y_1 (since $m = 1$). The other bright fringes on the screen are labeled accordingly.



The value of y_1 is

$$y_1 = \frac{1 \cdot \lambda L}{d} = \frac{(589\times10^{-9}\ \text{m})(6.0\ \text{m})}{1.5\times10^{-3}\ \text{m}} = 2.4\times10^{-3}\ \text{m} = 2.4\ \text{mm}$$

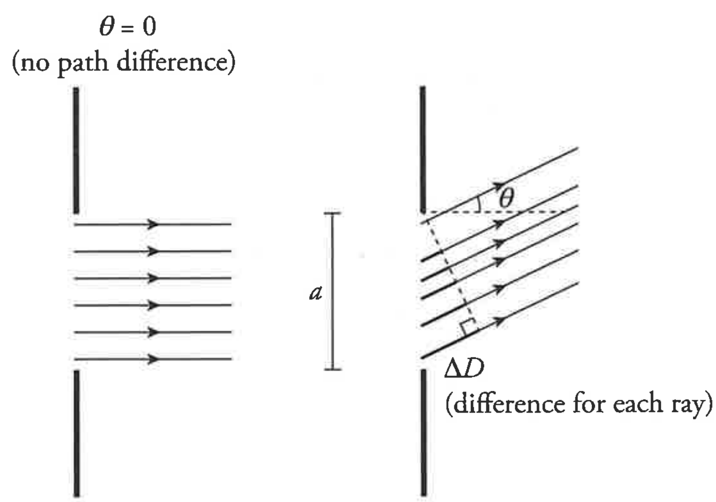
(b) The first dark fringe occurs when the path difference is 0.5λ, the second dark fringe occurs when the path difference is 1.5λ, and the third dark fringe occurs when the path difference is 2.5λ, so

$$y_{\substack{\text{3rd minimum} \\ \text{below central max}}} = \frac{(2+\frac{1}{2})\lambda L}{d} = \frac{(2+\frac{1}{2})(589\times10^{-9}\ \text{m})(6.0\ \text{m})}{1.5\times10^{-3}\ \text{m}} = 5.9\times10^{-3}\text{m} = 5.9\ \text{mm}$$

(c) Since $y = m\lambda L/d$, a decrease in d would cause an increase in y_{max}. That is, the fringes would become larger; the interference pattern would be more spread out.

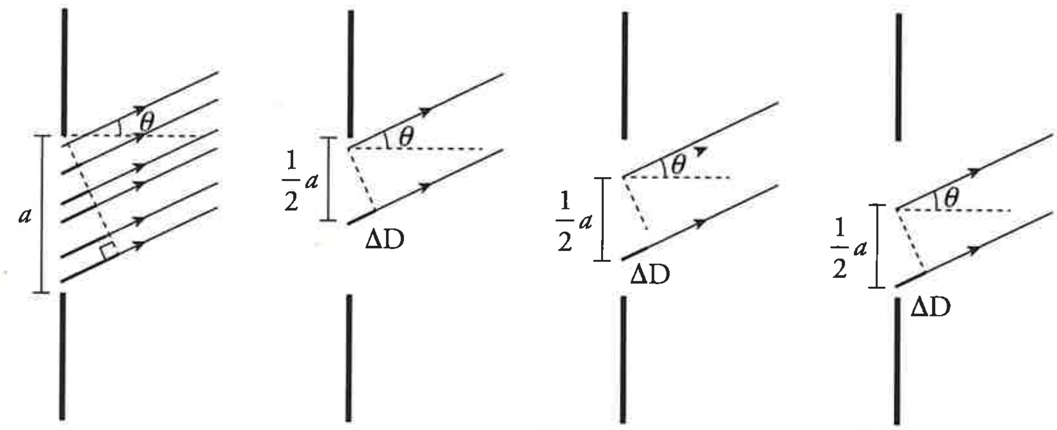
Single-Slit Diffraction

A diffraction pattern will also form on the screen if the barrier contains only one slit, as long as the size of the slit is roughly comparable to the wavelength. This occurs because, as the light passes through the slit, it can be treated as multiple light rays spread along the slit. The path difference for each ray can be found using a similar method to what we saw in the double-slit case:



In the case of $\theta = 0$, there is no path difference between the rays and there will be a very bright central maximum from constructive interference. When going at an angle, the path difference between the ray at the top and the bottom will, by the same argument from the double-slit case, be $a\sin\theta$. However, because each ray will have a different path difference, it becomes harder to see when they will all combine constructively or destructively; and, indeed, for most angles it will be a combination of constructive and destructive interference and it will show a dim light.

There is a case where the rays will combine to get near-perfect destructive interference. To see how, we can pair the rays off so that the vertical distance between them is always $a/2$:



Now the path difference, ΔD, is always the same for each pair, and it will be $\frac{1}{2}a\sin\theta$. When this path difference is exactly half a wavelength, the pairs combine destructively and we get a dark fringe on the screen. This is the first angle where we get a minimum. We can repeat this process by pairing up rays with a vertical distance of $a/4$, $a/6$, $a/8$, etc. (Note: It needs to be even to avoid having constructive interference.) When the path distance between these pairs is half a wavelength, there will be a minimum in the intensity:

First minimum: $\frac{1}{2}a\sin\theta = \frac{1}{2}\lambda$ or $a\sin\theta = \lambda$

Second minimum: $\frac{1}{4}a\sin\theta = \frac{1}{2}\lambda$ or $a\sin\theta = 2\lambda$

Third minimum: $\frac{1}{6}a\sin\theta = \frac{1}{2}\lambda$ or $a\sin\theta = 3\lambda$

The general pattern for a **single-slit minimum** is therefore

Equation Sheet

destructive interference (intensity minimum dark fringe on screen): $a\sin\theta = m\lambda$ where m = 1, 2, 3...

For a circular pinhole aperture, the diffraction pattern will consist of a central, bright, circular disk surrounded by rings of decreasing intensity. The equation above still gives the angles of the dark rings.

Example 16 A single-slit experiment is performed by shining a light through a pinhole with diameter of 2 μm. The screen is placed 5 m from the aperture, and the central maximum shows up as a bright disk with diameter of 2.5 m on the screen. What is the wavelength of the light?

Solution.

The center of the bright disk will correspond to when the angle, θ, is zero. The distance to the first dark fringe on the screen, y_1, is the distance to the edge of the disk and so will be 1.25 m, the radius of the disk. This occurs when $m = 1$.

$$a\sin\theta = \lambda$$

For small angles, we can use the approximation of $\sin\theta \approx \tan\theta = \frac{y_1}{L}$, so

$$\lambda = a\left(\frac{y_1}{L}\right) = (2\times10^{-6}\ \text{m})\frac{1.25\ \text{m}}{5\ \text{m}} = 5\times10^{-7}\ \text{m} = 500\ \text{nm}$$

This can be used in general, where the distance to the m^{th} dark fringe on the screen is given by

$$a\left(\frac{y_{\text{min}}}{L}\right) = m\lambda$$

Equation Sheet

Chapter 9 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review

What is the wavelength of a 5 Hz wave that travels with a speed of 10 m/s?

(A) 0.25 m

(B) 0.5 m

(C) 2 m

(D) 50 m

2 Mark for Review

A rope of length 5 m is stretched to a tension of 80 N. If its mass is 1 kg, at what speed would a 10 Hz transverse wave travel down the string?

(A) 2 m/s

(B) 5 m/s

(C) 20 m/s

(D) 200 m/s

3 Mark for Review

A transverse wave on a long horizontal rope with a wavelength of 8 m travels at 2 m/s. At $t = 0$, a particular point on the rope has a vertical displacement of $+A$, where A is the amplitude of the wave. At what time will the vertical displacement of this same point on the rope be $-A$?

(A) $t = \frac{1}{4}$ s

(B) $t = \frac{1}{2}$ s

(C) $t = 2$ s

(D) $t = 4$ s

4 Mark for Review

A string, fixed at both ends, supports a standing wave with a total of 4 nodes. If the length of the string is 6 m, what is the wavelength of the wave?

(A) 0.67 m

(B) 1.2 m

(C) 3 m

(D) 4 m

5 Mark for Review

A string, fixed at both ends, has a length of 6 m and supports a standing wave with a total of 4 nodes. If a transverse wave can travel at 40 m/s down the rope, what is the frequency of this standing wave?

(A) 6.7 Hz

(B) 10 Hz

(C) 20 Hz

(D) 26.7 Hz

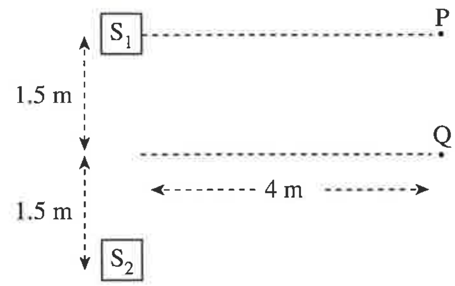
6 Mark for Review

A sound wave travels through a metal rod with wavelength λ and frequency *f*. Which of the following best describes the wave when it passes into the surrounding air?

	Wavelength	Frequency
(A)	Less than λ	Equal to *f*
(B)	Less than λ	Less than *f*
(C)	Greater than λ	Equal to *f*
(D)	Greater than λ	Less than *f*

7 Mark for Review

In the figure below, two speakers, S_1 and S_2, emit sound waves of wavelength 2 m, in phase with each other.



Let A_P be the amplitude of the resulting wave at Point P, and A_Q the amplitude of the resultant wave at Point Q. How does A_P compare to A_Q?

(A) $A_P < A_Q$

(B) $A_P = A_Q$

(C) $A_P > A_Q$

(D) $A_P < 0, A_Q > 0$

8 Mark for Review

An organ pipe that's closed at one end has a length of 17 cm. If the speed of sound through the air inside is 340 m/s, what is the pipe's fundamental frequency?

(A) 250 Hz

(B) 500 Hz

(C) 1000 Hz

(D) 1500 Hz

9 Mark for Review

A bat emits a 40 kHz "chirp" with a wavelength of 8.75 mm toward a tree and receives an echo 0.4 s later. How far is the bat from the tree?

(A) 35 m

(B) 70 m

(C) 105 m

(D) 140 m

10 Mark for Review

An X-ray with a frequency of 1.0×10^{18} Hz is an example of an electromagnetic wave. The equation for the electric field of such an X-ray could be given by $E = A\cos(\omega t)$ with

(A) A measured in N/C and $\omega = 1.0 \times 10^{18}$ Hz

(B) A measured in N · s/C and $\omega = 1.0 \times 10^{18}$ Hz

(C) A measured in N/C and $\omega = 2.0\pi \times 10^{18}$ Hz

(D) A measured in N · s/C and $\omega = 2.0\pi \times 10^{18}$ Hz

11 Mark for Review

In Young's double-slit interference experiment, what is the difference in path length of the light waves from the two slits to the center of the first bright fringe above the central maximum?

(A) $\frac{1}{4}\lambda$

(B) $\frac{1}{2}\lambda$

(C) λ

(D) $\frac{3}{2}\lambda$

12 Mark for Review

Three double-slit experiments are conducted and an interference pattern is observed in each. The second experiment has an identical setup to the original except a shorter wavelength of light is used. The third experiment has an identical setup to the original except the slits are closer together. What is true about the observed interference patterns for the experiments?

(A) Both the second and third experiments have closer fringes than the original.

(B) The second experiment has closer fringes than the original and the third has farther fringes.

(C) The second experiment has farther fringes than the original and the third has closer fringes.

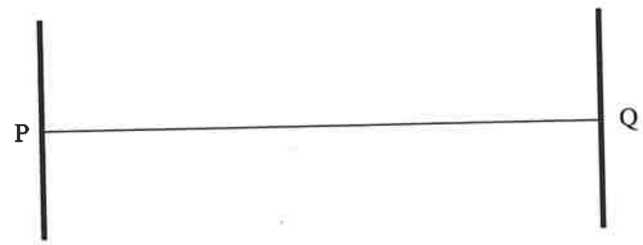
(D) Both the second and third experiments have farther fringes than the original.

Section II: Free Response

1 Mark for Review

A rope is stretched between two vertical supports. The points where it's attached (P and Q) are fixed. The linear density of the rope is 0.4 kg/m, and the speed of a transverse wave on the rope is 12 m/s.

Figure 1



A. What's the tension in the rope?

B. With what frequency must the rope vibrate to create a traveling wave with a wavelength of 2 m?

The rope can support standing waves of lengths 4 m and 3.2 m, whose harmonic numbers are consecutive integers.

C. Find

i. the length of the rope

ii. the mass of the rope

D. What is the harmonic number of the 4 m standing wave?

E. On the diagram above, draw a sketch of the 4 m standing wave, labeling the nodes and antinodes.

2 Mark for Review

A group of students performs a set of physics experiments using two tuning forks, one with a frequency of 400 Hz and the other 440 Hz.

A. What is the observed beat frequency when the two tuning forks are struck?

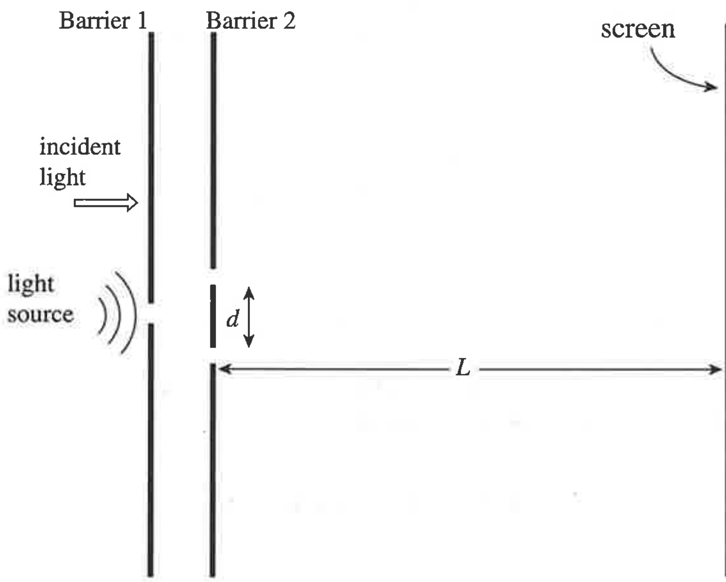
B. Describe the changes to the frequency, wavelength, and speed of the sound waves from the tuning forks as they travel from air into water.

C. One student strikes a tuning fork and then throws it straight up to a student on the second floor of a building. What happens to the frequency of the sound that the student on the second floor hears as the tuning fork travels upward?

3 Mark for Review

Two trials of a double-slit interference experiment are set up as follows. The slit separation is $d = 0.50$ mm, and the distance to the screen, L, is 4.0 m.

Figure 1



A. What is the purpose of the first (single-slit) barrier? Why not use two light sources, one at each slit at the second barrier? Explain briefly.

In the first trial, white light is used.

B. What is the vertical separation on the screen (in mm) between the first-order maxima for red light ($\lambda = 750$ nm) and violet light ($\lambda = 400$ nm)?

C. Locate the nearest point to the central maximum where an intensity maximum for violet light ($\lambda = 400$ nm) coincides with an intensity maximum for orange-yellow light ($\lambda = 600$ nm).

In the second trial, the entire region between the double-slit barrier and the screen is filled with a large slab of glass of refractive index $n = 1.5$, and monochromatic green light ($\lambda = 500$ nm in air) is used.

D. What is the separation between adjacent bright fringes on the screen?

Chapter 9 Summary

- The speed of a wave depends on the medium through which it travels. Speed can be determined by $v = f\lambda$, where f is the frequency ($f = \frac{\#\,cycles}{time}$) and λ is the wavelength. Because $f = \frac{1}{T}$, the speed can also be written $v = \frac{\lambda}{T}$.
- Changing any one of the period, the frequency, or the wavelength of a wave will affect the other two quantities, but will not affect the speed as long as the medium remains the same.
- If the wave travels through a stretched string, its speed can be determined by $v = \sqrt{\frac{F_T}{m/L}}$, where F_T is the tension in the spring, m is the mass of the string, and L is the length of the string.
- Superposition is what occurs when parts of waves interact so that they constructively or destructively interfere (e.g., create larger or smaller amplitudes, respectively).
- Two waves interfering at slightly different frequencies will form a beat frequency given by $f_{beat} = |f_1 - f_2|$.
- All electromagnetic waves travel at the speed of light ($c = 3.00 \times 10^8$ m/s). They obey the wave equation $\lambda = v/f$.

- When monochromatic coherent light goes through double slits (or a diffraction grating):
  - the path difference is given by $\Delta D = d\sin\theta$
  - there will be constructive interference when the path difference is $\Delta D = m\lambda$
  - there will be destructive interference when the path difference is $\Delta D = (m + \frac{1}{2})\lambda$
  - for small angles, we can say $d\left(\frac{y_{max}}{L}\right) = m\lambda$, where y_{max} is the distance to the m^{th} maximum, or bright fringe, from the central maximum and shows where constructive interference occurs.
- When monochromatic coherent light goes through a single slight or circular aperture:
  - there will be destructive interference when the path difference between pairs of rays is half a wavelength, which occurs when $a\sin\theta = m\lambda$
  - for small angles, we can say $a\left(\frac{y_{min}}{L}\right) = m\lambda$, where y_{min} is the distance to the m^{th} minimum, or dark fringe, from the central maximum and shows where destructive interference occurs.