Chapter 5
Electric Potential and Capacitance

INTRODUCTION

When an object moves in a gravitational field, it usually experiences a change in kinetic energy and in gravitational potential energy due to the work done on the object by gravity. Similarly, when a charge moves in an electric field, it generally experiences a change in kinetic energy and in electrical potential energy due to the work done on it by the electric field. By exploring the idea of electric potential, we can simplify our calculations of work and energy changes within electric fields.

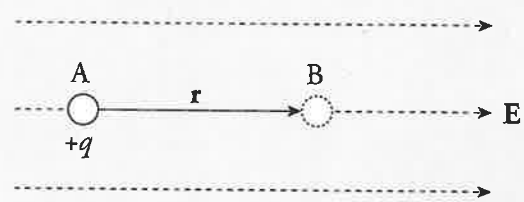
ELECTRICAL POTENTIAL ENERGY

When a charge moves in an electric field, unless its displacement is always perpendicular to the field, the electric force does work on the charge. If W_E is the work done by the electric force, then the change in the charge's **electrical potential energy** is defined by

$$\Delta U_E = -W_E$$

Notice that this is the same equation that defined the change in the gravitational potential energy of an object of mass m undergoing a displacement in a gravitational field ($\Delta U_G = -W_G$).

Example 1 A positive charge $+q$ moves from position A to position B in a uniform electric field **E**:



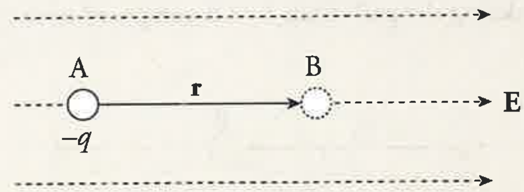
What is its change in electrical potential energy?

Solution. Since the field is uniform, the electric force that the charge feels, $\mathbf{F}_E = q\mathbf{E}$, is constant. Since q is positive, $\mathbf{F}_E$ points in the same direction as **E**, and, as the figure shows, they point in the same direction as the displacement, r. This makes the work done by the electric field equal to $W_E = F_E r = qEr$, so the change in the electrical potential energy is

$$\Delta U_E = -qEr$$

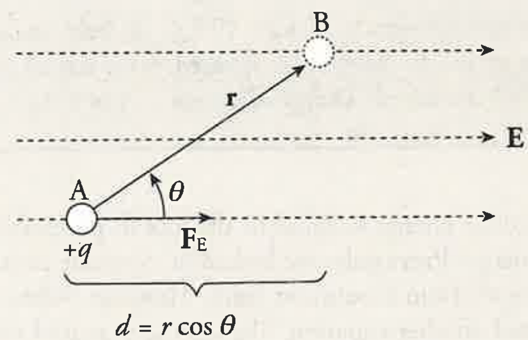
Note that the change in potential energy is negative, which means that potential energy has decreased; this always happens when the field does positive work. It's just like dropping a rock to the ground: gravity does positive work, and the rock loses gravitational potential energy.

Example 2 Do the previous problem, but consider the case of a negative charge, $-q$.



Solution. In this case, an outside agent must be pushing the charge to make it move, because the electric force *naturally* pushes negative charges against field lines. Therefore, we expect that the work done by the electric field is negative. The electric force, $\mathbf{F}_E = (-q)\mathbf{E}$, points in the direction opposite to the displacement, so the work it does is $W_E = -F_E r = -qEr = -qEr$. Thus, the change in electrical potential energy is positive: $\Delta U_E = -W_E = -(-qEr) = qEr$. Because the change in potential energy is positive, the potential energy increases; this always happens when the field does negative work. It's like lifting a rock off the ground: gravity does negative work, and the rock gains gravitational potential energy.

Example 3 A positive charge $+q$ moves from position A to position B in a uniform electric field **E**:



What is its change in electrical potential energy?

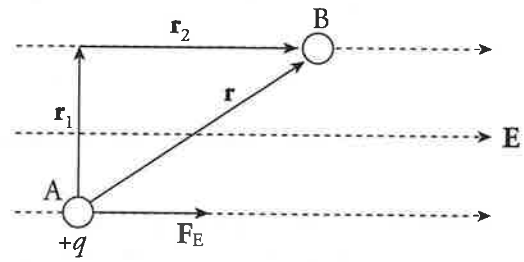
Solution. The electric force felt by the charge q is $\mathbf{F}_E = q\mathbf{E}$, and this force is parallel to **E** because q is positive. In this case, because $\mathbf{F}_E$ is not parallel to r (as it was in Example 1), we will use the more general definition of work for a constant force:

$$W_E = \mathbf{F}_E \cdot \mathbf{r} = F_E\, r \cos\theta = qEr\cos\theta$$

But $r\cos\theta = d$, so

$$W_E = qEd \text{ and } \Delta U_E = -W_E = -qEd$$

Because the electric force is a conservative force, which means that the work done does not depend on the path that connects the positions A and B, the work calculated above could have been figured out by considering the path from A to B composed of the segments $\mathbf{r}_1$ and $\mathbf{r}_2$:



Along $\mathbf{r}_1$, the electric force does no work since this displacement is perpendicular to the force. Thus, the work done by the electric field as q moves from A to B is simply equal to the work it does along $\mathbf{r}_2$. And since the length of $\mathbf{r}_2$ is $d = r \cos \theta$, we have $W_E = F_E d = qEd$, just as before.

Electric Potential Energy of a System of Two Point Charges

> **Example 4** A positive charge, $q_1 = +2 \cdot 10^{-6}$ C, is held stationary, while a negative charge, $q_2 = -1 \cdot 10^{-8}$ C, is released from rest at a distance of 10 cm from q_1. Find the kinetic energy of charge q_2 when it's 1 cm from q_1.

Solution. The gain in kinetic energy is equal to the loss in potential energy; you know this from Conservation of Energy. Previously, we looked at constant electric fields and were able to use the equation for work from a constant force. However, when the field (and therefore the force) changes, we need another equation. The electric potential energy of a system of two point charges separated by a distance r is given by

Equation Sheet

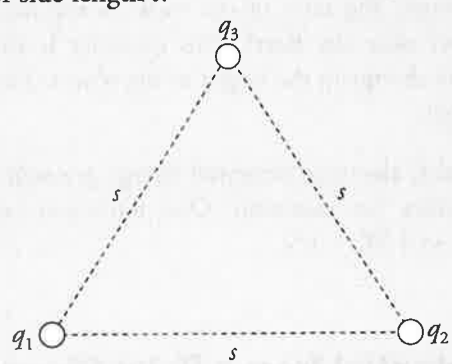
$$U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r} = k\frac{q_1 q_2}{r}$$

Therefore, if q_1 is fixed and q_2 moves from r_A to r_B, the change in potential energy is

$$\begin{aligned}\Delta U_E &= U_B - U_A \\ &= \frac{q_1}{4\pi\varepsilon_0}\left(\frac{q_2}{r_B} - \frac{q_2}{r_A}\right) \\ &= \frac{q_1 q_2}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \\ &= (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(+2\times10^{-6}\ \text{C})(-1\times10^{-8}\ \text{C})\left(\frac{1}{0.01\ \text{m}} - \frac{1}{0.10\ \text{m}}\right) \\ &= -0.016\ \text{J}\end{aligned}$$

Since q_2 lost 0.016 J of potential energy, the gain in kinetic energy is 0.016 J. Since q_2 started from rest (with no kinetic energy), this is the kinetic energy of q_2 when it's 1 cm from q_1.

Example 5 Two positive charges, q_1 and q_2, are held in the positions shown below. How much work would be required to bring (from infinity) a third positive charge, q_3, and place it as shown so that the three charges form the corners of an equilateral triangle of side length *s*?



Solution. An external agent would need to do positive work, equal in magnitude to the negative work done by the electric force on q_3 as it is brought into place, so let's first compute this quantity. Let's first compute the work done *by the electric force* as q_3 is brought in. Since q_3 is fighting against both q_1's and q_2's electric field, the total work done on q_3 by the electric force, W_E, is equal to the work done on q_3 by q_1 ($W_{1\text{-}3}$) plus the work done on q_3 by q_2 ($W_{2\text{-}3}$). Using the equation $W_E = -\Delta U_E$ and the one we gave above for ΔU_E, we have

$$\begin{aligned}W_{1\text{-}3} + W_{2\text{-}3} &= -\Delta U_{1\text{-}3} + -\Delta U_{2\text{-}3} \\ &= \left(-\frac{q_1 q_3}{4\pi\varepsilon_0}\frac{1}{s} - 0\right) + \left(-\frac{q_2 q_3}{4\pi\varepsilon_0}\frac{1}{s} - 0\right) \\ &= -\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{s}\end{aligned}$$

Therefore, the work that an external agent must do to bring q_3 into position is

$$-W_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_3}{s} + \frac{1}{4\pi\varepsilon_0}\frac{q_2 q_3}{s}$$

NOT the Same
Electric potential and electric potential energy, although very closely related and having similar names, are not the same thing. Even the units are measured differently—electric potential is measured in joules per coulomb and electric potential energy in joules.

Equation Sheet

ELECTRIC POTENTIAL

Let W_E be the work done by the electric field on a charge q as it undergoes a displacement. If another charge, say $2q$, were to undergo the same displacement, the electric force would be twice as great on this second charge, and the work done by the electric field would be twice as much, $2W_E$. Since the work would be twice as much in the second case, the change in electrical potential energy would be twice as great as well, but the ratio of the change in potential energy to the charge would be the same: $W_E/q = (2W_E)/2q$. This ratio says something about the work done by the field and the *displacement* but not the charge that made the move. The change in **electric potential**, ΔV, is defined as this ratio:

$$\Delta V = \frac{\Delta U_E}{q}$$

There is a similar concept with gravitational fields. The work done by gravity (near the Earth) on an object as it changes its position is $W = mgh$. If twice the mass were moved, the work would have to be $W = (2m)gh$. The ratio of the work to the mass moved is technically the "gravitational potential," but near the Earth, this quantity is simply the gravitational field strength, g, multiplied by the change in the height of the object. *Electric potential is the electrical equivalent of a change in height.*

Potential Difference
Finding the potential at a certain point in space is meaningless. With potential, what is more important is an initial position and final position. Just like in gravity, finding the potential energy at a certain point is meaningless if there is no relation to another point. Once we establish this potential difference, when we move a charge across it, we can generate energy. This will be key in the next chapter, when potential difference is needed in order to have an electrical circuit.

Electric potential is electrical potential energy *per unit charge*; the units of electric potential are joules per coulomb. One joule per coulomb is called one **volt** (abbreviated V), so 1 J/C = 1 V.

Electric Potential from a Point Charge

Consider the electric field created by a point source charge Q. If a charge q moves from a distance r_A to a distance r_B from Q, then the change in the potential energy is

$$U_B - U_A = \frac{Qq}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

The difference in electric potential between positions A and B in the field created by Q is

$$V_B - V_A = \frac{U_B - U_A}{q} = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

If we designate $V_A \to 0$ as $r_A \to \infty$ (an assumption that's stated on the AP Physics 2 Exam), then for a point charge Q, the electric potential at a distance r from Q is

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$$

Note that the potential depends on the strength of the source charge making the field and the distance from the source charge.

Example 6 Let $Q = 2 \cdot 10^{-9}$ C. What is the potential at a Point P that is 2 cm from Q?

Solution. Relative to $V = 0$ at infinity, we have

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \left(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{2\times10^{-9}\ \text{C}}{0.02\ \text{m}} = 900\ \text{V}$$

This means that the work done by the electric field on a charge of q coulombs brought to a point 2 cm from Q would be $-900q$ joules.

Note that, like potential energy, electric potential is a *scalar*. In the preceding example, we didn't have to specify the direction of the vector from the position of Q to the point P, because it didn't matter. Imagine a sphere with a surface of 2 cm from Q; at any point on that sphere, the potential will be 900 V. These spheres around Q are called **equipotential surfaces**, and they're surfaces of constant potential. The equipotentials are always perpendicular to the electric field lines.

Remember
Equipotential surfaces are often imaginary. Whether we put a metal sphere in that location or whether we visualize a sphere that isn't really there doesn't change the electric potential in that region.

Example 7 How much work is done by the electric field as a charge moves along an equipotential surface?

Solution. If the charge always remains on a single equipotential, then, by definition, the potential, V, never changes. Therefore, $\Delta V = 0$, so $\Delta U_E = 0$. Since $W_E = -\Delta U_E$, the work done by the electric field is zero.

Addition of Electric Potential

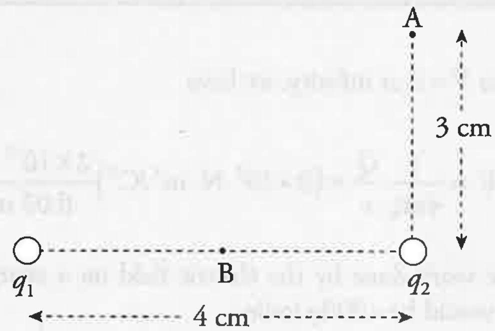
The formula $V = kq/r$ tells us how to find the potential due to a single point charge, q. Potential is scalar (we will not be concerned with direction, just the sign of charge). When we add up individual potentials, we're simply adding numbers; we're not adding vectors.

Equation Sheet

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

Just like Gravitational Potential Energy
The electric force is conservative. All we care about is the change in position when calculating the change in potential energy.

Example 8 How much work would it take to move a charge $q = +1 \cdot 10^{-2}$ C from Point A to Point B (the point midway between $q_1 = 4$ nC and $q_2 = -6$ nC)?



Solution. $\Delta U_E = q\Delta V$, so if we calculate the potential difference between Points A and B and multiply by q, we will have found the change in the electrical potential energy: $\Delta U_{A\to B} = q\Delta V_{A\to B}$. Then, since the work by the electric field is $-\Delta U$, the work required by an external agent is ΔU.

First, we need the potential at point A, V_A. Since there are two charges, q_1 and q_2, contributing to the potential at point A, we calculate the contribution of each using $V = kQ/r$ and sum the results. Remember that the potential is a scalar quantity, so no vector addition is required here. It is very important to keep track of the positive and negative signs, however.

$$V_A = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(9.0\times10^9 \ \tfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{4\times10^{-9}\text{ C}}{0.05\text{ m}} + \frac{-6\times10^{-9}\text{ C}}{0.03\text{ m}}\right)$$

$$V_A = -1080 \text{ V}$$

A similar calculation can be carried out for point B, although the distance from the charges will be different for point B than they were for point A.

$$V_B = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right) = \left(9.0\times10^9 \ \tfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{4\times10^{-9}\text{ C}}{0.02\text{ m}} + \frac{-6\times10^{-9}\text{ C}}{0.02\text{ m}}\right)$$

$$V_B = -900 \text{ V}$$

$\Delta V_{A \to B} = V_B - V_A = (-900 \text{ V}) - (-1080 \text{ V}) = +180 \text{ V}$. This means that the change in electrical potential energy as q moves from A to B is

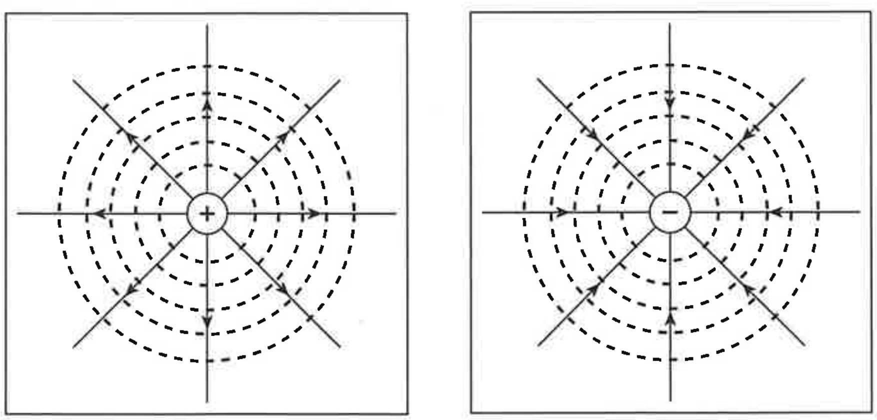
$$\Delta U_{A \to B} = q\Delta V_{A \to B} = (+1 \cdot 10^{-2} \text{ C})(+180 \text{ V}) = 1.8 \text{ J}$$

This is the work required by an external agent to move q from A to B.

EQUIPOTENTIAL CURVES AND EQUIPOTENTIAL MAPS

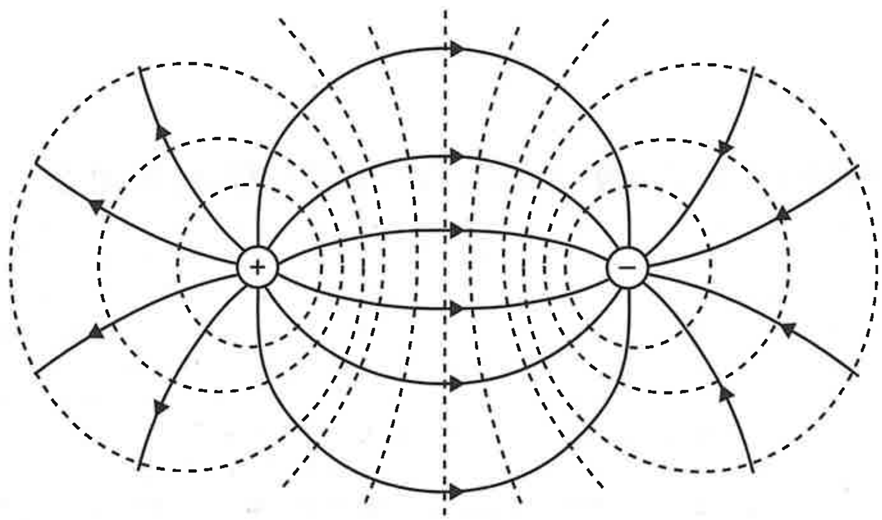
Electric field diagrams are a simple way to represent a sometimes very complex result of the influence of a charge distribution created by several charges. A similar image may be formed using isolines of electric potential instead of electric field vectors. In a downward gravitational field, if an object were to move from one position on a line at a constant height to another position on the line, no work would be done because the displacement would be perpendicular to the gravitational field. Electric equipotential curves are similar. A charge may be moved from one position on an equipotential curve to another without any work being required. A drawing of several equipotential curves at various values of the potential for a charge distribution (which may or may not be specified) is called an **equipotential map**. These images may look somewhat familiar as they are very similar to topographical maps, which plot lines of constant height in a uniformly downward directed gravitational field.

Here are the electric fields (which should look very familiar from Chapter 5) and some equipotential lines for isolated positive or negative point charges. The field lines are shown in solid and the equipotentials are shown as dashed lines.



The equipotential map for an isolated positive charge is exactly the same shape as that for an isolated negative charge. Both equipotential maps are made of concentric circles. Notice that everywhere the equipotential curves meet the field lines, the two types of curves meet at a right angle. This must be the case for an equipotential curve to require no work to move along. Imagine if a field component were directed opposite the motion along an equipotential curve. Then an external agent would have to push harder to maintain the same speed along the curve, and that would mean some outside work was being done. Despite the fields for positive and negative charges being identical, it is possible to determine whether the charge at the center is positive or negative whenever two or more of the equipotential lines are labeled with values. By definition, the potential will be higher (or more positive) close to a positive charge, and lower (or more negative) close to a negative charge.

Below is the equipotential map and some electric field lines for an electric dipole.



Example 9 Describe the equipotential lines in areas where the electric field is relatively strong compared to the areas where the electric field is relatively weak.

Solution. The electric field is strong where the field lines (solid in drawing above) are densely packed (center of image) and weak where they are spread out (left and right edges of drawing). Therefore, using the dipole field shown above as a reference, the field is strong between the two charges and weak off to the sides. Comparing the equipotential lines at those two locations, we can conclude that where the equipotential lines are close together, the field is strong, and as the distance between adjacent equipotential lines increases, the field strength decreases. This is consistent with the relationship $|\vec{E}| = \left|\frac{\Delta V}{r}\right|$, as the equipotential map shows lines at a constant electric potential difference ΔV, so the field strength, $|\vec{E}|$, is strong when the distance, r, between the lines is small.

THE ELECTRIC POTENTIAL OF A UNIFORM FIELD

Example 10 Consider a very large, flat plate that contains a uniform surface charge density σ. At points that are not too far from the plate, the electric field is uniform and given by the equation

$$E = \frac{\sigma}{2\varepsilon_0}$$

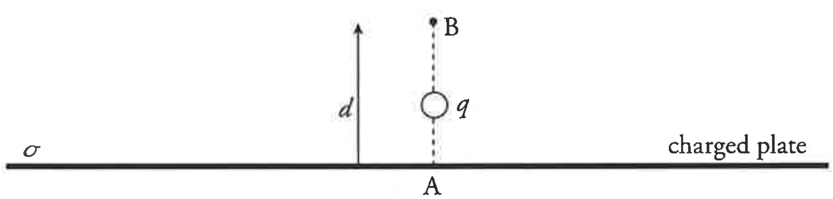
What is the potential at a point which is a distance *d* from the sheet (close to the plates) relative to the potential of the sheet itself?

Solution. Let A be a point on the plate and let B be a point a distance *d* from the sheet. Then

$$V_B - V_A = \frac{-W_{E,A\to B} \text{ on } q}{q}$$

Since the field is constant, the force that a charge q would feel is also constant, and is equal to

$$F_E = qE = q\frac{\sigma}{2\varepsilon_0}$$



Therefore,

$$\begin{aligned} W_{E,A\to B} &= F_E d \\ &= \frac{q\sigma}{2\varepsilon_0} d \end{aligned}$$

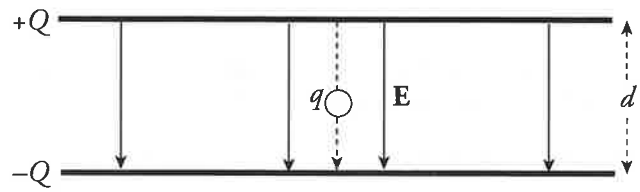
so applying the definition gives us

$$V_B - V_A = \frac{-W_{E,A\to B}}{q} = -\frac{\sigma}{2\varepsilon_0} d$$

This says that the potential decreases linearly as we move away from the plate.

Example 11 Two large flat plates—one carrying a charge of $+Q$, the other $-Q$—are separated by a distance d. The electric field between the plates, **E**, is uniform. Determine the potential difference between the plates.

Solution. Imagine a positive charge q moving from the positive plate to the negative plate:



Since the work done by the electric field is

$$W_{E,+\to-} = F_E d = qEd$$

the potential difference between the plates is

$$V_- - V_+ = \frac{-W_{E,+\to-}}{q} = \frac{-qEd}{q} = -Ed$$

This tells us that the potential of the positive plate is greater than the potential of the negative plate, by the amount Ed. This equation can also be written as

$$E = -\frac{V_- - V_+}{d}$$

Therefore, if the potential difference and the distance between the plates are known, then the magnitude of the electric field can be determined quickly. The magnitude is simply

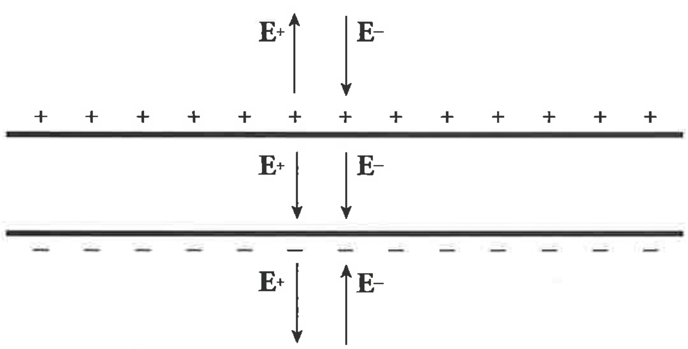
Equation Sheet

$$E = \left|\frac{\Delta V}{\Delta r}\right| = \left|\frac{\Delta V}{\Delta}\right|$$

CAPACITORS AND CAPACITANCE

Consider two conductors, separated by some distance, that carry equal but opposite charges, $+Q$ and $-Q$. Such a pair of conductors comprises a system called a **capacitor**. Work must be done to create this separation of charge, and, as a result, potential energy is stored. Capacitors are basically storage devices for electrical potential energy.

The conductors may have any shape, but the most common conductors are parallel metal plates or sheets. These types of capacitors are called **parallel-plate capacitors**. We'll assume that the distance d between the plates is small compared to the dimensions of the plates since, in this case, the electric field between the plates is uniform. When this is not the case, you must account for the **fringing fields**, which are discussed later in this chapter. The electric field due to *one* such plate, if its surface charge density is $\sigma = Q/A$, is given by the equation $E = \sigma/(2\varepsilon_0)$, with **E** pointing away from the sheet if σ is positive and toward the plate if σ is negative.



Therefore, with two plates, one with surface charge density $+\sigma$ and the other $-\sigma$, the electric fields combine to give a field that's zero outside the plates and that has the magnitude

$$E_{\text{total}} = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

in between.

$$E_{\text{total}} = \frac{Q}{\varepsilon_0 A}$$

In Example 11, we learned that the magnitude of the potential difference, ΔV, between the plates satisfies the relationship $\Delta V = Ed$, so combining this with the previous equation, we get

$$E = \frac{\sigma}{\varepsilon_0} \Rightarrow \frac{\Delta V}{d} = \frac{\sigma}{\varepsilon_0} \Rightarrow \frac{\Delta V}{d} = \frac{Q/A}{\varepsilon_0} \Rightarrow \frac{Q}{\Delta V} = \frac{\varepsilon_0 A}{d}$$

Q is the total charge stored on either plate of a capacitor, and ΔV is the potential difference between the plates. The ratio of Q to ΔV, for *any* capacitor, is defined as its **capacitance** (C).

Equation Sheet

$$C = Q/\Delta V$$

The capacitance measures the capacity for holding charge. The greater the capacitance, the more charge can be stored on the plates at a given potential difference. The capacitance of any capacitor depends only on the size, shape, and separation of the conductors, and the "dielectric constant," κ (the Greek letter kappa), which depends on what material is between the plates of the capacitor. For a parallel-plate capacitor, we get

Equation Sheet

$$C = \frac{\kappa\varepsilon_0 A}{d}$$

What Determines Capacitance?
Capacitance does NOT determine the charge on the capacitor, Q, or the potential difference, V, across it. It only shows a relationship between Q and V. The greater the potential difference applied to a capacitor, the greater the amount of charge the capacitor can hold. Think of capacitance as a property of a physical object. Physically, capacitance is determined by three things: the area of the plates, A; the plate separation, d; and the dielectric constant, κ.

An insulator (called a dielectric in this context) may fill the area between the plates, or the space between them may be in vacuum. When there is nothing in the gap between the plates except vacuum, then $\kappa = 1$. When an insulating material is in the gap between the plates, then $\kappa > 1$. From the definition, $C = Q/\Delta V$, the units of C are coulombs per volt. One coulomb per volt is renamed one **farad** (abbreviated F): 1 C/V = 1 F.

Example 12 A 10-nanofarad parallel-plate capacitor holds a charge of magnitude 50 ∝C on each plate.

(a) What is the potential difference between the plates?

(b) If the plates are separated by vacuum with a distance of 0.2 mm, what is the area of each plate?

Solution.

(a) From the definition, $C = Q/\Delta V$, we find that

$$\Delta V = \frac{Q}{C} = \frac{50\times10^{-6}\text{ C}}{10\times10^{-9}\text{ F}} = 5000\text{ V}$$

(b) Because $\kappa = 1$, we have the equation $C = \varepsilon_0\frac{A}{d}$, and can calculate the area, A, of each plate:

$$A = \frac{Cd}{\varepsilon_0} = \frac{(10\times10^{-9}\text{ F})(0.2\times10^{-3}\text{ m})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = 0.23\text{ m}^2$$

ELECTRIC FIELD AND CAPACITORS

For point charges, the electric field created by one or more point source charges varies, depending on the location. For example, as we move farther away from the source charge, the electric field gets weaker. Even if we stay at the same distance from, say, a single source charge, the direction of the field changes as we move around. Therefore, we could never obtain an electric field that was constant in both magnitude and direction throughout some region of space from point-source charges. However, the electric field that is created between the plates of a charged parallel-plate capacitor is constant in both magnitude and direction throughout the region between the plates; in other words, a charged parallel-plate capacitor can create a uniform electric field. The electric field, **E**, always points from the positive plate toward the negative plate, and its magnitude remains approximately the same at every point between the plates, whether we choose a point close to the positive plate, closer to the negative plate, or between them.

"Uniform electric field" is only approximately true. The electric field is only constant everywhere in space for an infinitely large sheet of charge, and such a thing is impossible to construct. However, whenever the observation location is sufficiently close to the plates, or the edges of the plates are sufficiently far away, the constant electric field approximation holds true.

Example 13 The charge on a parallel-plate capacitor is $4 \cdot 10^{-6}$ C. If the distance between the plates is 2 mm and the capacitance is 1 μF, what's the strength of the electric field between the plates?

Solution. Since $C = Q/\Delta V$, we have $\Delta V = Q/C = (4 \cdot 10^{-6}\text{ C})/(10^{-6}\text{ F}) = 4$ V. Now, using the equation $\Delta V = Ed$,

$$E = \Delta V/d = (4\text{ V})/(2 \cdot 10^{-3}\text{ m}) = 2000\text{ V/m}$$

Example 14 The plates of a parallel-plate capacitor are separated by a distance of 2 mm. The device's capacitance is 1 μF. How much charge needs to be transferred from one plate to the other in order to create a uniform electric field whose strength is 10^4 V/m?

Solution. Because $Q = C\Delta V$ and $\Delta V = Ed$, we find that

$$Q = CEd = (1 \cdot 10^{-6}\text{ F})(1 \cdot 10^{4}\text{ V/m})(2 \cdot 10^{-3}\text{ m}) = 2 \cdot 10^{-5}\text{ C} = 20\ \mu\text{C}$$

Fringing Fields

The mathematical analysis of capacitors up until now assumes that the plates of the capacitor are infinitely large. Because of this approximation, the analysis is very good in the region near the centers of the plates, but is not as precise near the edges of the plates. In the areas beyond the edges of the plates, we can approximate the field as the sum of the capacitor field and a dipole formed by the charges at the edges of the plates.

THE ENERGY STORED IN A CAPACITOR

To figure out the electrical potential energy stored in a capacitor, imagine taking a small amount of negative charge off the positive plate and transferring it to the negative plate. This requires that positive work be done by an external agent, and this is the reason that the capacitor stores energy. If the final charge on the capacitor is Q, then we transferred an amount of charge equal to Q, doing work to move the charge through a potential difference at each stage. If the final potential difference is ΔV, then the average potential difference during the charging process is $\frac{1}{2}\Delta V$; so, using the definition of potential difference, $\Delta V = \Delta U_E / Q$, we can write that for a capacitor $\Delta U_C = Q \cdot \frac{1}{2}\Delta V = \frac{1}{2}Q\Delta V$. At the beginning of the charging process, when there was no charge on the capacitor, we had $U_i = 0$, so $\Delta U_C = U_f - U_i = U_f - 0 = U_f$; therefore, we have

Equation Sheet

$$U_C = \frac{1}{2}Q\Delta V$$

This is the electrical potential energy stored in a capacitor. Because of the definition $C = Q/\Delta V$, the equation for the stored potential energy can be written as

$$U_C = \frac{1}{2}(C\Delta V) \cdot \Delta V = \frac{1}{2}C(\Delta V)^2$$

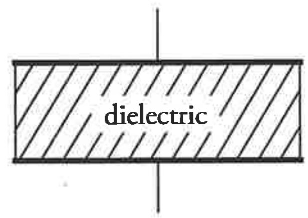
or

$$U_C = \frac{1}{2}Q \cdot \frac{Q}{C} = \frac{Q^2}{2C}$$

Interestingly, this work is typically done by a battery as the external agent. The battery supplies a total work of $Q\Delta V$, but the capacitor ends up storing only half of this energy. The other 50% of the energy supplied by the battery is dissipated as heat during this charging process.

CAPACITORS AND DIELECTRICS

One method of keeping the plates of a capacitor apart, which is necessary to maintain charge separation and store potential energy, is to insert an insulator (called a **dielectric**) between the plates.



Insulator?
Why not a conductor? A capacitor stores energy by holding charges apart from one another. If a conductor is placed between the plates, the charges are no longer separated and the device is no longer a capacitor.

A dielectric always increases the capacitance of a capacitor.

Let's see why this is true. Imagine charging a capacitor to a potential difference of ΔV with charge $+Q$ on one plate and $-Q$ on the other. Now disconnect the capacitor from the charging source and insert a dielectric. What happens? Although the dielectric is not a conductor, the electric field that exists between the plates causes the molecules within the dielectric material to polarize; there is more electron density on the side of the molecule near the positive plate.

The effect of this is to form a layer of negative charge along the top surface of the dielectric and a layer of positive charge along the bottom surface; this separation of charge induces its own electric field ($\mathbf{E}_i$), within the dielectric, which opposes the original electric field, **E**, within the capacitor.

So the overall electric field has been reduced from its previous value: $\mathbf{E}_{total} = \mathbf{E} + \mathbf{E}_i$, and $E_{total} = E - E_i$. Let's say that the electric field has been reduced by a factor of κ from its original value as follows:

$$E_C = \frac{Q}{\kappa \varepsilon_0 A}$$

Equation Sheet

Since $\Delta V = Ed$ for a parallel-plate capacitor, we see that ΔV must have decreased by a factor of κ. But $C = \frac{Q}{\Delta V}$, so if ΔV decreases by a factor of κ, then C increases by a factor of κ:

$$C_{\text{with dielectric}} = \kappa C_{\text{without dielectric}}$$

The value of κ, called the **dielectric constant**, varies from material to material, but it's always greater than 1. In general, the capacitance of parallel-plate capacitors is

Equation Sheet

$$C = \kappa \varepsilon_0 \frac{A}{d}$$

Chapter 5 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review

An experiment is conducted and data is gathered for the electric potential V at various positions r away from a uniformly charged sphere. All measurements are taken outside of the sphere. Which of the following graphs yields a straight line?

(A) V as a function of $\frac{1}{r^2}$

(B) V as a function of $\frac{1}{r}$

(C) V as a function of r

(D) V as a function of r^2

2 Mark for Review

Negative charges are accelerated by electric fields toward

(A) points of lower electric potential

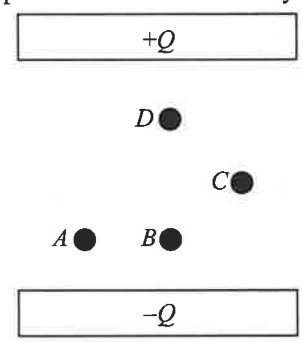
(B) points of higher electric potential

(C) points where the electric field is weaker

(D) points where the electric field is stronger

3 Mark for Review

A section near the center of a parallel-plate capacitor is shown below. There are 4 labeled positions between the plates shown as A, B, C, and D. Relative to A, which point has the largest potential difference and why?



(A) Point A because the potential difference is infinite when the position between points is 0 m

(B) Point B because it is the same distance from the $-Q$ plate as A

(C) Point C because it is farther in distance from Point A

(D) Point D because it is closest to the $+Q$ plate

4 Mark for Review

A charge q experiences a displacement within an electric field from Position A to Position B. The change in the electrical potential energy is ΔU_E, and the work done by the electric field during this displacement is W_E. Then

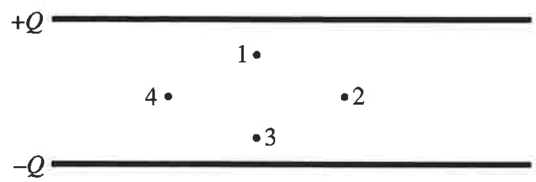
(A) $V_A - V_B = qW_E$

(B) $V_B - V_A = qW_E$

(C) $V_A - V_B = \Delta U_E/q$

(D) $V_B - V_A = \Delta U_E/q$

5 Mark for Review



Which points in this uniform electric field (between the plates of the capacitor) shown above lie on the same equipotential?

(A) 1 and 3 only

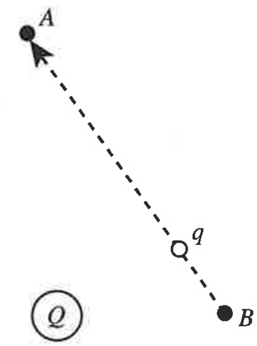
(B) 2 and 4 only

(C) None lie on the same equipotential.

(D) 1, 2, 3, and 4 all lie on the same equipotential since the electric field is uniform.

6 Mark for Review

A charge Q creates an electric field through which a second charge q moves, as shown below. q is initially at point B and is moved to point A, farther away. The potential from Q at position A is $V_A = 100\ V$ and at B is $V_B = 200\ V$. The charge on q is negative. What is the sign of Q and the sign of the work done by the electric field of Q as q is moved from B to A?



(A) Q is positive and the work done by the electric field is positive.

(B) Q is positive and the work done by the electric field is negative.

(C) Q is negative and the work done by the electric field is positive.

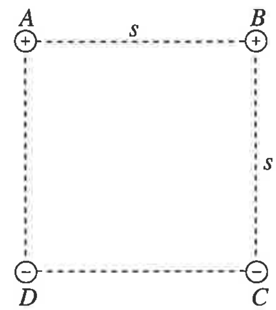
(D) Q is negative and the work done by the electric field is negative.

Section II: Free Response

1 Mark for Review

In Figure 1, four charges, each of magnitude Q, are situated at the corners of a square with side lengths s. The two charges on the top of the square are positively charged, while the two on the bottom of the square are negatively charged.

Figure 1



A. These charges were assembled in order by first bringing in charge A, then bringing in charge B, then bringing in C, and finally bringing in charge D. Rank the amount of energy in the charge distribution in the presence of only charge A, only charges A and B, only charges A, B, and C, and in the presence of all four charges. Negative numbers should be taken as smaller than positive numbers. Justify your answer.

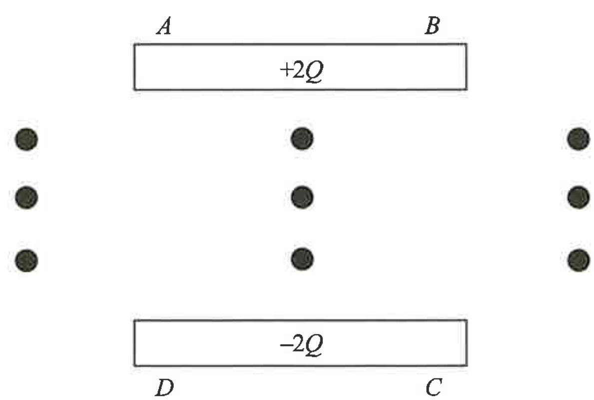
Greatest (most positive) ____ ____ ____ ____ Least (most negative)

B. Show that the potential at the exact center of the square is 0 V by calculating the potential from each charge at that location.

C. Sketch (on the diagram) the portion of the equipotential surface that lies in the plane of the figure and passes through the center of the square.

D. As shown in Figure 2, solid conducting bars are placed to connect points A to B and also to connect points C to D. The charge is allowed to distribute over these conductors. Sketch the electric field at each of the dots. Explain why the field is constant at the dots in the center but not at the dots on the edges.

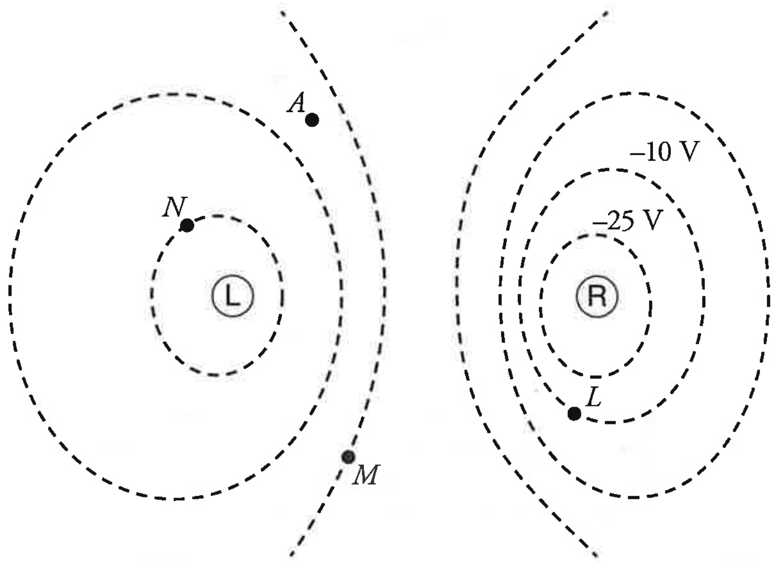
Figure 2



2 Mark for Review

Figure 1 shows the isolines of electric potential surrounding two charged spheres, labeled L and R. The spheres carry opposite charges and the potential difference between adjacent pairs of lines is $\Delta V = 15$ V. The isolines with electric potentials of –25 V and –10 V are indicated.

Figure 1



A. Sketch in the line of potential 0 V on the drawing.

B. Approximately what is the value of the potential at the point labeled A?

C.

i. Which sphere, L or R, carries a negative charge? Explain your answer.

ii. Which sphere, L or R, carries a greater magnitude of charge? Explain how you know.

D.

i. Draw arrows to indicate the electric fields at the points labeled L, M, and N.

ii. Rank the magnitude of the electric field strength at points L, M, and N. Explain your answer.

E. If the sphere labeled L were replaced with another sphere to have the same magnitude of charge but of the opposite sign, would the value of the potential at the point labeled N be larger, smaller, or stay the same? Justify your answer.

3 Mark for Review

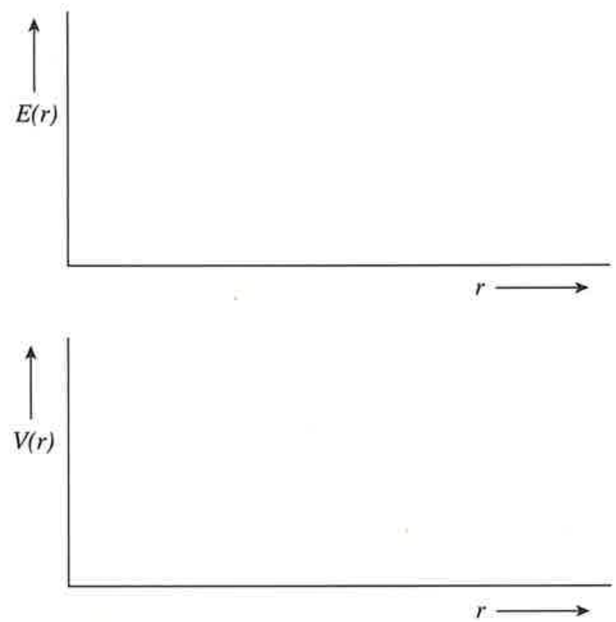
A solid conducting sphere of radius a carries an excess charge of Q.

A. Determine the electric field magnitude, $E(r)$, as a function of r, the distance from the sphere's center.

B. Determine the potential, $V(r)$, as a function of r. Take the zero of potential at $r = \infty$.

C. On Figure 1, sketch $E(r)$ and $V(r)$. (Cover at least the range $0 < r < 2a$.)

Figure 1



Chapter 5 Summary

- Electric potential energy is a type of energy arising from the interaction of a charge and an external electric field. As with all types of energy, it may be converted into other types of energy or may be added to the system or removed from the system through work.
- The electric potential difference (commonly referred to as the voltage) is defined as the change in electric potential energy per unit of charge: $\Delta V = \Delta U_E / q$.
- The electric potential energy is defined by $\Delta U_E = -W_E$ or $\Delta U_E = q\Delta V$. For a pair of interacting point charges q_1 and q_2, the electric potential energy in the system is $U_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r}$.
- The work done moving a charge q through a uniform electric field E for a distance d is given by $W = qEd$.
- Equipotential surfaces are surfaces along which the potential is constant. Moving a charge at a constant speed along an equipotential surface results in no change in energy of the charge.
- Equipotential surfaces are always perpendicular to the electric field at any point on the surface.
- The electric field is strong where the equipotential lines are close to one another and weaker where the lines are farther apart.
- Capacitors are devices that store electric potential energy in electric fields. Capacitance is given by $C = \frac{Q}{\Delta V}$. For parallel-plate capacitors, the capacitance is $C = \kappa\varepsilon_0 \frac{A}{d}$.
- A parallel-plate capacitor has a uniform electric field between the plates for regions close to the plates and far from the outer edges of the capacitor.
- The electrical energy stored in a capacitor is given by $U_C = \frac{1}{2}Q\Delta V = \frac{1}{2}C(\Delta V)^2$.
- When a capacitor is filled with a dielectric, its capacitance increases from the capacitance it had when it had a vacuum between its plates.