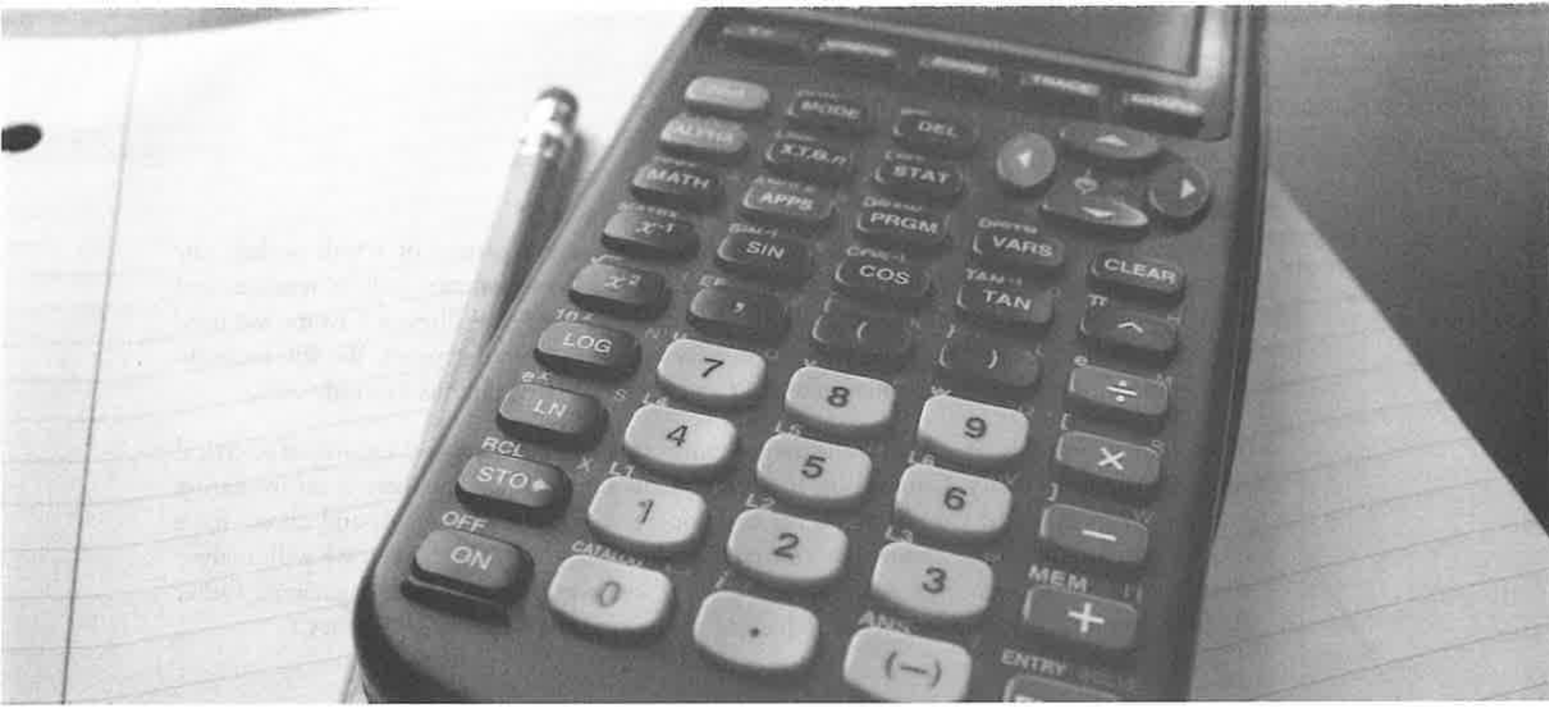


Chapter 6
Electric Circuits

INTRODUCTION

Electric circuits are comprised of an energy source (typically a battery or a wall outlet), one or more conducting materials (such as wires), and circuit components such as resistors and capacitors. The broad topic of electric circuits is vast, but for the AP Physics 2 Exam, we need to focus on only a small set of components in relatively simple arrangements. We will focus on direct current (DC) voltage supplies in resistor and capacitor (RC) circuits in steady state.

The most basic electric circuit is a collection of conductors, such as wire, and a source of electrical energy, such as a battery. Connecting the components together so that there is no insulating material (such as an air gap) between the components completes the circuit and allows for a continuous flow of charge, known as **current**, in the circuit. In this chapter, we will analyze circuits constructed of one or more batteries, wires, switches, resistors, and capacitors. Other circuit components, such as diodes or inductors, are beyond the scope of AP Physics 2.

BATTERIES AND VOLTAGE

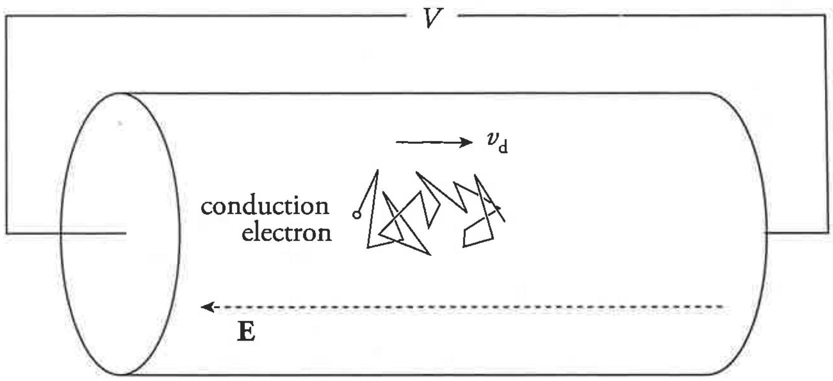
A battery is a device that maintains an electric potential difference between the two terminals. When a wire is connected between the two terminals of the battery, there is a potential difference between the ends of the wire. As we saw previously, a potential difference in a conducting material causes an electric field. Previously, when we were looking at electrostatics, the potential difference was always 0 V because we require, as a condition of electrostatics, that the electric field within a wire be 0 N/C. In circuits, when there is a battery present to *maintain* a potential difference, there will be a nonzero electric field within the conductor. As a result, there will be a flow of charge. We call this flow of charge electric current.

A Note on Convention for this Chapter
In the previous chapter, we referred to a change in potential as a voltage, ΔV. In this chapter, we will also use the standard convention, V. Make note of this as you move forward so you won't be confused.

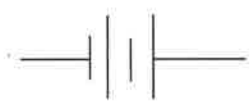
A common misconception is that the battery is the *supply* of the charges which create the current. As we have already seen, conducting materials such as metals contain vast numbers of electrons. The battery supplies the energy which is needed to arrange the electrons in the wire into a configuration where the current continuously flows. However, the electrons present within the wire constitute the current. The electrons present in the wire also create the electric field that is the source of the current.

Voltage Is like Electric Pressure
Voltage is a power source that allows for the movement of charges. It is like a region in the atmosphere of high pressure compared to a region of low pressure, which will result in a wind.

Any particular conduction electron will move through the circuit with a **drift velocity** of some millimeters per second, but the influence of the "electricity" moves through the circuit at the speed of light. As we will see, were the battery the source of the electric charge, not only would it take several hours for light bulbs to turn on, because of the slow drift velocity of electrons, but circuits with capacitors would not function at all.



In a circuit diagram, a battery will be represented by one of these two diagrams:



The diagram to the left shows a *single-cell* battery and the one on the right shows a battery with multiple cells. The longer line in the battery diagram represents the positive (higher potential) terminal, and the shorter line is the negative (lower potential) terminal.

The voltage source is a power source. Whether it is a wall plug, a battery, a capacitor, or an electric generator, it supplies energy to the circuit. When two points in a circuit have different voltages, charges will flow from one point to the other. As long as the voltage difference is maintained, the flow of charge will continue.

Voltage Has Many Names but One Meaning
When scientists were discovering the properties of electricity, it was not clear that all the concepts they were analyzing were the same phenomenon. In different contexts, voltage is referred to as potential difference, voltage drop, voltage, or electromotive force.

Voltage between two points in a conducting material causes charge to flow.

The voltage supplied by an ideal battery is often, for historical reasons, referred to as EMF (electromotive force, ε). Whether referred to as the potential difference, voltage drop, voltage, or EMF, the voltage is measured in units of **volts** (abbreviated V).

A battery never changes which pole is at high potential and which pole is at low potential. As a result, the battery always causes electricity to flow in the same direction through the circuit. This is called a **direct current** circuit.

RESISTORS AND RESISTANCE

Resistance vs. Resistivity
Resistance and resistivity are NOT the same. Resistance is the extent to which a resistor impedes the passage of electric current. Resistivity is a property of what molecules a material consists of and how the molecules that constitute a material are bound together. The units of resistivity are different as well.

Resistance is the impedance to the flow of electricity through a material. As a charge moves through a material, it eventually hits a non-moving nucleus in the material. These collisions can be thought of as the source of resistance.

The less distance on average that a charge moves between such collisions results in a larger resistance. A material with a long length through which the charge must move will have more collisions, so a longer material will have a greater resistance. A material with a wide cross-sectional surface area will have more room for the charge to move laterally to avoid collisions, so a greater area results in a smaller resistance. Finally, all materials have a property called **resistivity**, denoted by ρ, which can be thought of in this context as the density of the nuclei the electrons may strike. A greater density will cause more collisions. The resistance, R, of a material depends on its resistivity and its geometrical configuration, in particular, its length, ℓ, and its cross-sectional area, A.

Equation Sheet

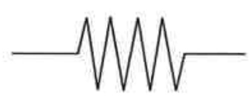
$$R = \frac{\rho \ell}{A}$$

This equation applies only to shapes with constant cross-sectional area, such as cylinders or rectangular prisms, and will not apply to those with a varying cross-sectional area, such as spheres or cones.

Resistors Are like Drinking Straws
A very narrow straw will be harder to drink through than a wide straw. A very long straw will be harder to drink through than a short one.

Resistance is measured in **ohms** (Ω, *omega*) and resistivity is measured in **ohm-meters** ($\Omega \cdot$m).

In a circuit diagram, a resistor will be represented by



A resistor is awkwardly named because all wires are technically "resistors." Materials with a very low resistivity are referred to as conductors. These include aluminum, silver, and copper, which all have resistivity on the order of 1×10^{-8} $\Omega \cdot$m. Electrical components called "resistors" are made of materials like carbon graphite, which have resistivity on the order of 10×10^{-5} $\Omega \cdot$m, approximately 10,000 times higher than the materials which comprise wire. Materials with a high resistivity, such as glass with resistivity of 10×10^{10} $\Omega \cdot$m, are referred to as insulators.

Example 1 The resistivity of copper is about 1.7×10^{-8} $\Omega \cdot$m. What is the resistance of a 5 m long 16-gauge light-duty extension cord whose diameter is 1.3 mm?

Solution.

$$R = \frac{\rho\ell}{A}$$

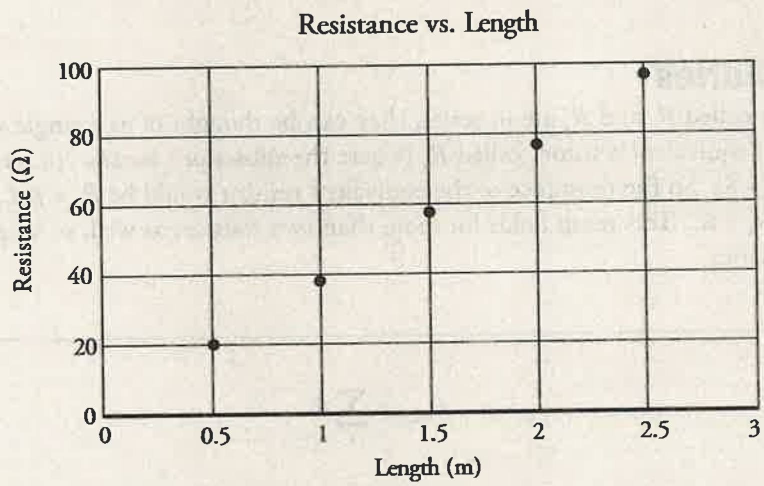
To find the cross-sectional area, we use $A = \pi r^2 = (3.14)(1.3 \times 10^{-3}\ \text{m}/2)^2$.

$$R = \frac{(1.7 \times 10^{-8}\ \Omega\cdot\text{m})(5\text{m})}{((3.14)(1.3 \times 10^{-3}\ \text{m}/2)^2)} = 0.064\ \Omega$$

Example 2 A wire with cross-sectional area 1.33 mm² has an unknown resistivity.

(a) To find the resistivity, you have a spool of this wire, a meter stick, wire cutters, and a device to measure resistance (such a device is called an Ohmmeter). Briefly describe what data you will collect and how you will determine the resistivity of the wire.

(b) Another student gathers data and makes the following plot. What is the resistivity of the wire in her experiment?



Solution.

(a) Cut a length of the wire with wire cutters. Measure the length of the wire, L, with the meterstick. Measure the resistance of the wire with the Ohmmeter. The resistivity can be found using $\rho = AR/L$.

(b) From the graph, the slope is $(98\ \Omega - 20\ \Omega)/(2.5\ \text{m} - 0.5\ \text{m}) = 39\ \Omega\cdot\text{m}$, which is R/L. Since we determined in (a) that $\rho = AR/L$, we can find the resistivity as $\rho = (1.33 \times 10^{-6}\ \text{m}^2)(39\ \Omega\cdot\text{m}) = 5.19 \times 10^{-5}\ \Omega\cdot\text{m}$.

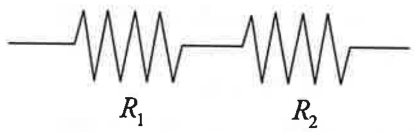
Multiple Resistors Are like Multiple Straws

If one very narrow straw will be hard to drink through, two very narrow straws side by side will act like a wider straw, decreasing the overall resistance.

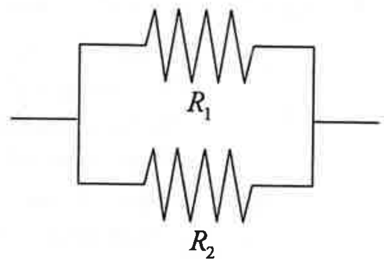
COMBINING RESISTORS AND EQUIVALENT RESISTANCE

To deal with arrangements that have multiple resistors, combinations of resistors can be considered. Resistors may be arranged in two ways: one after the other or side by side. In an arrangement in which the resistors are one after the other, we say the resistors are "in series." In an arrangement in which the resistors are side by side, we say the resistors are "in parallel." When two or more resistors are combined mathematically, the resulting resistance is called *equivalent* resistance.

Two resistors R_1 and R_2 arranged in series



Two resistors R_1 and R_2 arranged in parallel



Resistors in Series

When two resistors called R_1 and R_2 are in series, they can be thought of as a single wire of a greater length. The "equivalent resistor" called R_S (where the subscript s means "in series") has a length of $L_{eq} = L_1 + L_2$. So the resistance of the equivalent resistor would be $R_S = \rho(L_1 + L_2)/A = \rho L_1/A + \rho L_2/A = R_1 + R_2$. This result holds for more than two resistors as well, so long as they are all arranged in series.

Equation Sheet

$$R_{eq,s} = \sum_i R_i$$

Resistors arranged in series result in an overall resistance that is greater than the resistance of any of the individual resistors in the arrangement.

One Over R_p
The equation for equivalent resistance in parallel has R_p in the denominator. Don't forget to take the reciprocal as the final step in calculating the equivalent resistance.

Resistors in Parallel

When the resistors are in parallel, they behave like a wire with a greater cross-sectional area. It would be equivalent to replace those two resistors with a single "equivalent resistor" called R_{eq} of area $A_p = A_1 + A_2$. Then the resistance of the equivalent resistor would be

$$R_p = \rho L/(A_1 + A_2) \rightarrow \frac{1}{R_p} = \frac{(A_1 + A_2)}{\rho L} = \frac{A_1}{\rho L} + \frac{A_2}{\rho L} = \frac{1}{R_1} + \frac{1}{R_2}$$

This result holds for more than two resistors as well, so long as they are all arranged in parallel.

$$\frac{1}{R_{\text{eq},p}} = \sum_i \frac{1}{R_i}$$

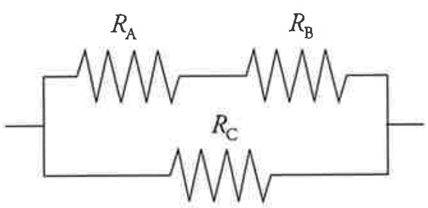
Equation Sheet

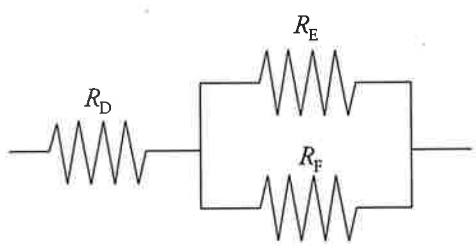
Resistors arranged in parallel result in an overall resistance that is less than the resistance of any of the resistors.

When dealing with arrangements of three or more resistors, it is possible for the arrangement to be a mixture of series and parallel.

(a) A pair of resistors R_A and R_B is in series. That combination is in parallel with a third resistor, R_C.

(b) A resistor R_D is in series with a pair of resistors R_E and R_F, which are in parallel.





When resistors appear in a mixture of parallel and series, the equivalent resistance is calculated in multiple steps. First, the equivalent resistance of the part of the circuit that is purely series or parallel must be found. Then that resistance is used for further calculations. This process may need to be repeated several times in circuits with many resistors.

Example 3 Calculate the total equivalent resistance of the two arrangements in the figure above.

A Shortcut for Pairs of Parallel Resistors
The equivalent resistance of a pair of resistors in parallel is the product of the resistances over their sum. This shortcut does **not** work for combinations of more than two resistors.

Solution.

(a) The figure labeled (a) shows two resistors in series, R_A and R_B, which are in parallel with the third resistor R_C. The equivalent resistance of the arrangement is found by first combining the series resistors into $R_{\text{eq,AB}}$ = $R_A + R_B$ and finally combining that with R_C as a parallel arrangement to get

$$\frac{1}{R_{\text{eq, total}}} = \frac{1}{R_A + R_B} + \frac{1}{R_C} \text{ so that } R_{\text{eq,total}} = \frac{R_C\left(R_A + R_B\right)}{R_A + R_B + R_C}$$

Wind as an Analogy to Current
Think of current as the wind. Blowing wind is comprised of many air molecules moving randomly and colliding with one another and with other things the air molecules encounter, but there is an overall flow of air in a particular direction. When talking about wind, the rate of motion of air molecules is considered. When talking about current, the rate of motion of electric charge is under consideration.

(b) The figure labeled (b) shows a resistor, R_D, in series with a pair of parallel resistors, R_E and R_F. The equivalent resistance of the arrangement is found by first combining the parallel resistors into $\frac{1}{R_{eq,EF}} = \frac{1}{R_E} + \frac{1}{R_F}$ so that $R_{eq,EF} = \frac{R_E R_F}{R_E + R_F}$. Then that is combined with R_D as a series arrangement to get $R_{eq,total} = R_D + \frac{R_E R_F}{R_E + R_F}$.

CURRENT

Establishing a potential difference across a wire creates an electric field in the wire, and as a result of that electric field, charges move. We can measure how much charge passes any particular point in the wire in a specified unit of time. When an amount of charge of magnitude ΔQ crosses an imaginary plane in a time interval Δt, then the **average current** is

Equation Sheet

$$I_{avg} = \frac{\Delta Q}{\Delta t}$$

Benjamin Franklin
Conventional direction of current is "wrong" because Benjamin Franklin did not have enough information to determine whether charge carriers were positive or negative charges (or both moving in opposite directions) when he was investigating electrostatics. He simply picked a direction based on the flow of positive charges.

Because current is charge per unit time, it's expressed in coulombs per second. One coulomb per second is an **ampere** (abbreviated **A**). Instead of calling the unit an ampere, it is often simply called an amp.

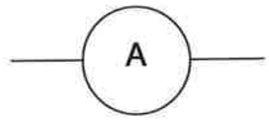
We know that the charge carriers that constitute the current within a metal are electrons. However, the direction of the current is taken to be the direction that positive charge carriers would move. This is referred to as *conventional current*, in contrast with *electron current*, which is the flow of electrons. On the AP Physics 2 Exam, assume all current flow is conventional current.

MEASURING CURRENT AND VOLTAGE IN A CIRCUIT

When a circuit is established, each electrical component in the circuit will have some electricity flowing through it and will have some energy being used up by it.

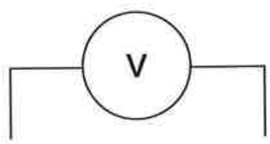
The current flowing past any point in a circuit is measured using an ammeter. An ammeter is a device with a very low resistance (essentially $R = 0\ \Omega$). To measure the current flowing through a resistor of resistance R, the ammeter is placed in series and the equivalent resistance of the resistor and ammeter is $R_S = R + 0 = R$. Because of the negligible resistance of the ammeter, the resistance of the pair is just R. The fact that this arrangement does not change the equivalent resistance means the ammeter reading is the same as the reading through the resistor when the ammeter is not present. Because current flows through the resistor, the reading will be the same no matter on which side of the resistor the meter is placed.

In a circuit diagram, an ammeter will be represented by



Current Is Not "Used Up"
Current flows through the devices in a circuit. The same amount of current flows into, and then back out of, each resistor.

The electrical energy is not measured directly, but instead the electric potential difference—commonly referred to as voltage or voltage drop—is measured using a voltmeter. A voltmeter has an extremely high resistance (essentially $R = \infty$). One lead is attached to each of the two points where the voltage difference is desired. To measure the voltage across a resistor of resistance R, the voltmeter must be in parallel with the resistor. Because they are in parallel, the equivalent resistance of the pair is found from $\frac{1}{R_P} = \frac{1}{R} + \frac{1}{\infty}$, so $R_P = R$. Because of the divergent resistance of the voltmeter, the resistance of the pair is just R. The fact that this arrangement does not change the equivalent resistance means the voltmeter reading is the same as the reading across the resistor when the meter is not present. In a circuit diagram, a voltmeter will be represented by



Example 4 A non-ideal ammeter with a resistance of 2 Ω used in series with a resistor with resistance of 25 Ω. By what percentage would adding the ammeter change the equivalent resistance of the circuit?

Solution. Since the two elements are in series, the equivalent resistance is found using

$$R_{eq} = R_1 + R_2 = 25\ \Omega + 2\ \Omega = 27\ \Omega$$

The percentage change is found by looking at the difference between the two things and dividing by the original.

$$\%_{change} = \frac{27\ \Omega - 25\ \Omega}{25\ \Omega} \times 100 = 8\%$$

So adding this non-ideal ammeter increases the equivalent resistance of the circuit by 8%.

OHM'S LAW

For many resistors, a plot of the current flowing through the resistor for various voltage drops results in a graph showing the relationship between ΔV and I is directly proportional. Moreover, the slope of such a graph is equal to the resistance R of the resistor. Whenever this is the case, the device is said to be an "Ohmic resistor." Ohm's Law states that the voltage drop across an Ohmic resistor is directly proportional to the current flowing through the resistor and that the proportionality constant is the resistance. Because the resistance of a component is determined by the geometric properties of its construction and the voltage supplied is determined by the power source, Ohm's Law tells the amount of current that will flow through the device and is written as

Equation Sheet

$$I = \frac{\Delta V}{R}$$

Not every device obeys Ohm's Law. However, unless a question explicitly states a device is non-Ohmic or asks to explain whether a device is Ohmic or not, every resistor you encounter in AP Physics 2 will be Ohmic. You can determine whether a resistive element in a circuit obeys Ohm's Law or not by making a plot of the voltage versus the current. A nonlinear plot indicates that the resistor is non-Ohmic, while a directly proportional graph will indicate that the resistor is Ohmic.

Ohm's Law can be applied to each individual resistor in a circuit as well as to the overall circuit. Applying Ohm's Law to an individual resistor yields the voltage drop across that resistor and the current flowing through the resistor. Applied to the entire circuit, Ohm's Law tells the voltage drop across the battery and the current which flows through the battery.

POWER DISSIPATION

The power dissipated by a circuit component is given by the product of the current through the component and the voltage drop across it.

Equation Sheet

$$P = I\Delta V$$

Heat and Power
Power in a circuit is very closely associated with heat given off. If we were to touch an incandescent light bulb, we would notice it becomes hot. The brighter the lightbulb is, the greater the heat given off.

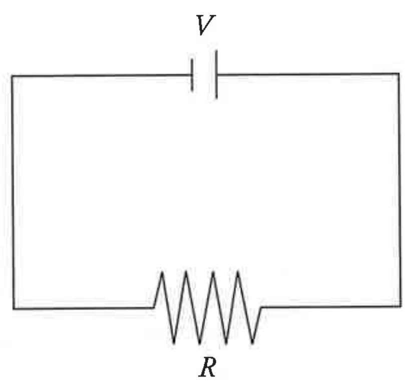
The power dissipated by each individual component must sum to equal the power supplied by the battery (or other power source). For Ohmic resistors, the power equation may be combined with Ohm's Law to yield

$$P = I^2R \text{ and } P = V^2/R$$

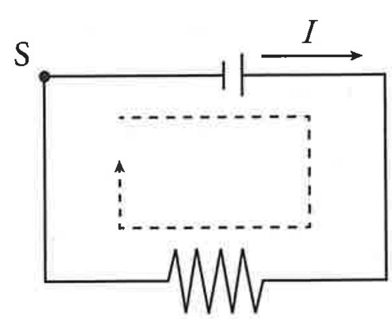
KIRCHHOFF'S RULES

In addition to Ohm's Law, Kirchhoff's rules are used when analyzing circuits. These two laws are statements of conservation laws applied to circuit analysis. The **Loop Rule** states that the voltage drop across *any complete loop* in a circuit is 0 V. This statement follows from Conservation of Energy when applied to circuits. A corollary to the Loop Rule is that any pair of resistors in parallel will have identical voltage drops across them. The **Junction Rule** states that the sum of all current flowing into any junction is equal to the current flowing out of the junction. This statement follows from conservation of charge. A corollary of the Junction Rule is that for two resistors in series, the current must be the same through each because there is no junction.

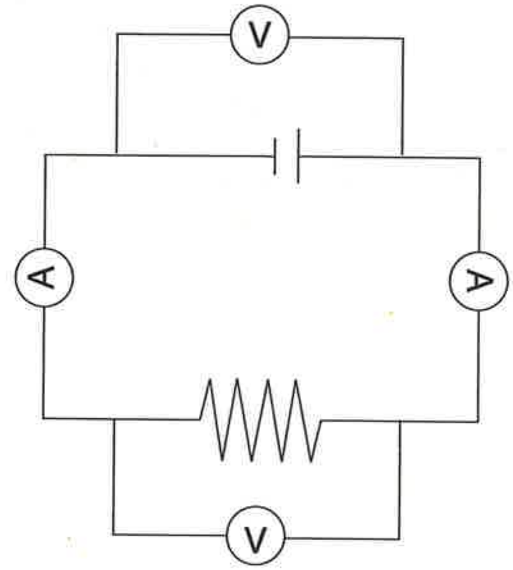
A very simple circuit contains one battery with voltage V and one resistor of resistance R connected by wires.



Let's consider how to apply Kirchhoff's Loop Rule to this circuit. First, assign the direction of current flow for all segments of the circuit. There are no junctions in this circuit, so one current flows throughout, and a current I flows from the positive terminal of the battery to the negative terminal, clockwise around the circuit. Then, choose a loop. In this simple circuit, there is only a single loop. Next, choose a starting point and a direction to traverse the loop. It is generally advisable to choose the direction of current, so start at point S and go clockwise.



As we traverse the loop clockwise from point S, the first change in electric potential we experience is the increase in electric potential $+V$ from the battery. The next change in potential in the loop is a voltage drop across the resistor, $-IR$, after which we're back at starting point S. The loop must have an overall change in voltage of zero, so $V + (-IR) = 0$, and $V = IR$. (Note that if the loop went opposite the direction of current, the battery would correspond to a voltage drop and the resistor a voltage gain—this is why it's important to assign the direction of current first.) Add a pair of voltmeters and a pair of ammeters to the circuit in their proper positions to measure the current and voltage of the battery and the resistor.

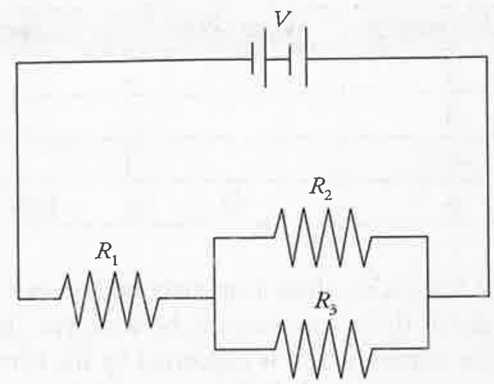


The reading in the top voltmeter is the voltage drop across the battery. The reading in the lower voltmeter is the voltage across the resistor. As we discovered from the Loop Rule, the readings in these voltmeters must be the same; the voltage supplied by the battery is equal to the voltage drop across the resistor. As there are no junctions in the circuit (no current flows through the voltmeters due to their nearly infinite resistance), each ammeter will also read an equal value for the current. In this simple circuit, all the current that flows out of the battery flows into the resistor, and all the current that flows out of the resistor flows back into the battery.

ANALYSIS OF CIRCUITS WITH RESISTORS

Circuits with several resistors can be computationally intensive. Problems frequently involve iteratively solving for the current and voltage of several devices in several steps. The typical approach to quantitative analysis is to determine the overall equivalent resistance of the circuit. Then given the battery voltage, the current supported by the battery can be calculated as well as the power supplied by the battery. From the Junction Rule, Ohm's Law is then applied to any resistors in series with the battery, as the current supported by the battery flows through each of those resistors. The iterative process then requires using the Loop Rule to determine the voltage drop across the paths that contain parallel branches.

Example 5 The circuit below shows a 12 V battery connected to three resistors. The resistances include $R_1 = 4\ \Omega$, $R_2 = 3\ \Omega$, and $R_3 = 6\ \Omega$. Determine the current supported by the battery and the voltage drop across each resistor.



Solution. To begin, it is helpful to make a chart of the resistance, voltage, current, and power for each device. We need a row for each resistor and an additional row for the entire circuit. Any time that any two of the columns are known in a row, Ohm's Law and the power equation can be used to solve for the other two columns. We can immediately fill out any known resistance, current, voltage, or power values. Here, we have three resistances and one voltage as given.

	Resistance (Ω)	**Voltage (V)**	**Current (A)**	**Power (W)**
R_1	4			
R_2	3			
R_3	6			
Entire Circuit		12		

Next, we can use the concept of equivalent resistance to get resistance of the entire circuit, which will give us two values in the bottom row.

To begin, we combine R_2 and R_3 to get an equivalent resistor $R_{2,3}$. Since R_2 and R_3 are in parallel,

$$\frac{1}{R_{2,3}} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{3} + \frac{1}{6} = \frac{2}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$$R_{2,3} = 2\ \Omega$$

We now have R_1 in series with $R_{2,3}$. The equivalent resistance $R_{1,2,3} = R_1 + R_{2,3} = 4 + 2 = 6\ \Omega$. The total resistance of the circuit is $R_{total} = 6\ \Omega$. We add that to the bottom row of the table. Then, using Ohm's Law and the power equation, we can complete the bottom row of the table.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	4			
R_2	3			
R_3	6			
Entire Circuit	**6**	12	12/6 = 2	$12^2/6 = 24$

According to the image of the circuit, there is no junction between V and R_1. The corollary to the Junction Rule states that if there is no junction between two devices, the current through each must be equal. All the current which is supported by the battery must flow through R_1. This allows us to fill in the current through R_1. Then, using Ohm's Law and the power equation, we can complete the R_1 row of the table.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	4	$2 * 4 = 8$	**2**	$2^2 * 4 = 16$
R_2	3			
R_3	6			
Entire Circuit	6	12	2	24

After the Junction Rule is used to complete a row, the Loop Rule is typically used to complete the next step. We know that the total voltage drop through any complete loop must be equal to the voltage supplied by the battery. This circuit has two completed loops: $V = V_1 + V_2$ and $V = V_1 + V_3$. We know V is 12 V and we know the voltage drop across R_1 is $V_1 = 8$ V, so the voltage drop across either R_2 or R_3 must be 4 V. This conclusion is consistent with the corollary to the Loop Rule that the voltage drops across resistors which are in parallel are equal. We can now complete the rest of the table.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	4	8	2	16
R_2	3	4	4/3 = 1.33	$4^2/3 = 5.33$
R_3	6	4	4/6 = 0.67	$4^2/6 = 2.67$
Entire Circuit	6	12	2	24

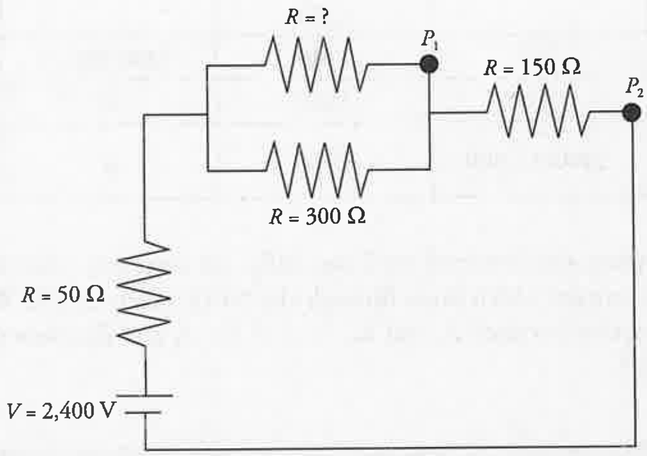
This example did not ask for the power dissipated by the circuit or by any component, but it is useful to have that column. As you will note, the resistances do not sum to the entire resistance. The voltages do not sum to the entire voltage. The currents do not sum to the entire current. However, the power will always sum to the total power. Here, $24 = 16 + 5.33 + 2.67$ is true. This simple check allows you to verify the likelihood that your answer is correct and that you didn't make any algebra mistakes in calculating the voltages or currents.

Example 6 In this circuit, the voltage of the battery and the resistance of three of the resistors is known. An ammeter placed at P_2 reads 6 A.

(a) What is the value of the unknown resistor?

(b) What will a voltmeter read that is placed between P_1 and P_2?

(c) What percentage of the power dissipated by the circuit is dissipated by the 150 Ω resistor?



Solution. For the chart, we will need to label each resistor. Going in the direction of current flow from the battery, the 50 Ω resistor is R_1, the unknown resistor is R_2, the 300 Ω resistor is R_3, and the 150 Ω resistor is R_4. Fill out the given three resistances and one voltage.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	50			
R_2				
R_3	300			
R_4	150			
Entire Circuit		2400		

Additionally, we are given the current at point P_2 is 6 A. Whenever there is no junction between two devices, the current through each must be equal, so any current through an ammeter at P_2 must flow through the battery, R_1, and R_4.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	50	50 * 6 = 300	6	6^2 * 50 = 1800
R_2				
R_3	300			
R_4	150	150 * 6 = 900	6	6^2 * 150 = 5400
Entire Circuit	2400/6 = 400	2400	6	2400 * 6 = 14,400

Since the last step involved the Junction Rule, the next step will involve the Loop Rule. The voltage across R_2 (which we know is equal to the voltage across R_3) is $V - V_1 - V_4 = 2{,}400 - 900 - 300 = 1{,}200$ V.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	50	300	6	1800
R_2		**1200**		
R_3	300	**1200**	1200/300 = 4	$1200^2/300 = 4800$
R_4	150	900	6	5400
Entire Circuit	2400/6 = 400	2400	6	14,400

Now, since the previous step involved the Loop Rule, the next step will involve the Junction Rule. We know the current which flows through the 50 Ω resistor is 6 A. We know this must all flow into the junction between R_2 and R_3. So $I_1 = I_2 + I_3$ and therefore $6 = I_2 + 4$. We can conclude that I_2 is 2 A.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	50	300	6	1800
R_2	1200/2 = 600	1200	**2**	2 * 1200 = 2400
R_3	300	1200	1200/300 = 4	$1200^2/300 = 4800$
R_4	150	900	6	5400
Entire Circuit	2400/6 = 400	2400	6	14,400

Finally, we can check to see if the sum of the power of each resistor correctly adds to the entire power: 1800 + 2400 + 4800 + 5400 = 14,400 is correct.

With the analysis complete, we can address the questions that were asked in the problem.

(a) The unknown resistance (R_2) is 600 Ω.

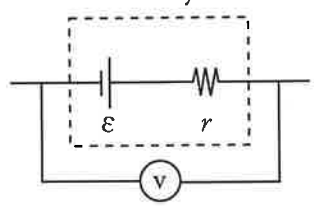
(b) A voltmeter placed between P_1 and P_2 will be in parallel with R_3 and will therefore have a reading of 1200 V.

(c) The 150 Ω resistor dissipates 5400 W of the entire 14,400 W, which is 37.5%.

EMF and Internal Resistance

The voltage supplied by an ideal battery is often, for historical reasons, referred to as EMF (electromotive force, ε). This is simply another name for voltage and electric potential difference, but it is almost always encountered in the context of "voltage supplied by the battery" (or in the context of Faraday's Law in the magnetism chapter).

A battery in a circuit can be modeled as an ideal EMF in series with a resistor, r. In this model, r is called the "internal resistance" of the battery and the voltage measured across the EMF source, and r is the "terminal voltage." This would be the measurement reading from a voltmeter placed across the terminals of the battery.



Example 7 If the 50 Ω resistor in the previous example were an internal resistor, what would the terminal voltage of the battery in that example equal?

Solution. The terminal voltage would be the voltage measured from one side of the battery across the internal resistor. Here, that would just be $V_{\text{terminal}} = V - V_1 = 2400 - 300 = 2100$ V.

CAPACITORS IN CIRCUITS

In the previous chapter, we looked at the electric field generated by parallel-plate capacitors. The electric field between the plates of a capacitor allows capacitors to be used in circuits as devices to store energy. They also have interesting properties related to the amount of time they take to charge, but those non-steady-state conditions are beyond the scope of the AP Physics 2 Exam, as their analysis requires calculus.

A capacitor stores energy in an electric field. A parallel-plate capacitor is comprised of two plates of area A and separation d. An insulator (called a dielectric in this context) may fill the area between the plates, or the space between them may be in vacuum.

The capacitance, which determines the amount of charge that can be stored on the plates for a given voltage, depends on the plate area, separation, and the dielectric constant, κ, of whatever material is between the plates (for vacuum, $\kappa = 1$).

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

Equation Sheet

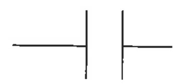
As current flows in a circuit with a capacitor, charges move onto one plate and off of the other plate. As a charge excess builds up on one plate, the electric field between the plates builds. The ratio of the charge on either plate to the voltage across the plates is the capacitance.

Equation Sheet

$$\Delta V = Q/C$$

Capacitance is measured in a unit called **Farad** (named after Michael Faraday). Rearranging the equation above to $C = Q/\Delta V$, you can see that one Farad is one Coulomb per Volt, 1F = 1C/V.

In a circuit diagram, a capacitor will be represented by this symbol:



Combinations of Capacitors and Equivalent Capacitance

Multiple capacitors may be in a circuit in which the equivalent capacitance may be calculated. Two capacitors which are arranged in parallel are equivalent to a single capacitor of area $A_p = A_1 + A_2$. Following a similar analysis to a pair of resistors, we see that

$$C_{eq,p} = \kappa\varepsilon_0 \frac{A_1 + A_2}{d} = \kappa\varepsilon_0 \frac{A_1}{d} + \kappa\varepsilon_0 \frac{A_2}{d} = C_1 + C_2$$

This result holds for more than two capacitors as well, so long as they are all arranged in parallel.

Equation Sheet

$$C_{eq,p} = \sum_i C_i$$

Similarly, the capacitors may be arranged in series. That arrangement is equivalent to a single capacitor of separation: $d_s = d_1 + d_2$. Following a similar analysis to a pair of resistors in parallel, we see that for capacitors in series

$$C_{eq,s} = \kappa\varepsilon_0 \frac{A}{d_1 + d_2} \rightarrow \frac{1}{C_{eq,s}} = \frac{(d_1 + d_2)}{\kappa\varepsilon_0 A} = \frac{d_1}{\kappa\varepsilon_0 A} + \frac{d_2}{\kappa\varepsilon_0 A} = \frac{1}{C_1} + \frac{1}{C_2}$$

This result holds for more than two capacitors as well, so long as they are all arranged in series.

Equation Sheet

$$\frac{1}{C_{eq,s}} = \sum_i \frac{1}{C_i}$$

Example 8 Calculate the equivalent capacitance of a pair of capacitors that are in series when that combination is in parallel with a third capacitor. All the capacitors have capacitance C.

Solution. In the same way we combined part of the resistor circuits, we first combine only the portion of this mixed series and parallel capacitor arrangement in which the capacitors are in series: $\frac{1}{C_{1,2}} = \frac{1}{C_1} + \frac{1}{C_2}$ so $C_{1,2} = \frac{C_1 C_2}{C_1 + C_2}$. Because all the capacitors have capacitance C, $C_{1,2} = \frac{C^2}{2C} = \frac{C}{2}$. Then combining that equivalent capacitor with the third capacitor in parallel, $C_{1,2,3} = C_{1,2} + C_3 = \frac{C}{2} + C = \frac{3C}{2}$.

Charging and Discharging Capacitors

An uncharged capacitor may become charged by connecting it to a battery. The instant that connection is made, the uncharged capacitor has a potential difference of 0 V across it and current flows into the capacitor just like current would flow through a wire. For some amount of time, the capacitor is partially charged and the potential difference across the plates grows as more and more charge builds up on the plate. Eventually, the capacitor reaches a state at which the potential difference across the plates is equal to the voltage of the battery and the current flow onto the capacitor plates ceases. In this state, the capacitor acts like an open switch and no current flows onto or off of the plates.

A charged capacitor may be discharged by connecting the two plates to a resistor. The voltage between the two plates causes current to flow across the resistor. As the current flows, the amount of charge on the plates, Q, decreases and the voltage ΔV also decreases. Eventually, when the capacitor is fully discharged so that $Q = 0$ C, the voltage across the resistor is $\Delta V = 0$ V, from $\Delta V = Q/C$. Ohm's Law, $V = IR$, indicates that no more charge flows across the resistor.

Calculating the amount of time required for a capacitor to charge and discharge requires calculus. It is not part of the AP Physics 2 Exam. Instead, the exam focuses on capacitors in "steady state," in which enough time has passed for the capacitor to begin each problem in a state which is either completely discharged or completely charged.

Altering the Capacitance of a Capacitor

The capacitance is given by

Equation Sheet

$$C = \kappa\varepsilon_0 \frac{A}{d}$$

Three things may be done to alter the capacitance of a capacitor: a dielectric may be inserted (altering κ), the area of the plates may change (changing *A*), or the spacing between the plates may change (which changes *d*).

There are two scenarios for making these changes to a charged capacitor in a circuit:

1. The battery may remain connected, causing the potential difference to remain constant.
2. The battery may be disconnected, causing the charge on each plate to remain constant.

Which of these scenarios unfolds will influence an analysis of the capacitor in terms of its charge on each plate, potential difference, stored electric field, and stored energy.

Let's look at what happens when the capacitance increases as a result of adding a dielectric (κ increases).

Capacitance is increased with a battery connected. *V* is constant and *Q* changes.	
V	Constant
$Q = CV$	Increases
$E = V/d$	Constant
$U_C = \frac{1}{2}QV$	Increases

Capacitance is increased with a battery disconnected. *Q* is constant and *V* changes.	
$V = Q/C$	Decreases
Q	Constant
$E = V/d$	Decreases
$U_C = \frac{1}{2}QV$	Decreases

If the capacitance were decreased instead of increased, say by removing a dielectric from a charged capacitor (κ decreases), the same analysis would apply. The results would be as follows.

Capacitance is decreased with a battery connected. *V* is constant and *Q* changes.	
V	Constant
$Q = CV$	Decreases
$E = V/d$	Constant
$U_C = \frac{1}{2}QV$	Decreases

Capacitance is decreased with a battery disconnected. *Q* is constant and *V* changes.	
$V = Q/C$	Increases
Q	Constant
$E = V/d$	Increases
$U_C = \frac{1}{2}QV$	Increases

RC CIRCUITS WITH CAPACITORS IN STEADY STATE

The two possible steady state conditions for a capacitor in a circuit occur when the capacitor is discharged or when it is fully charged.

When a capacitor is discharged, the voltage between the plates is 0 V. As a result, the discharged capacitor *acts like a wire.* This causes all of the current from any junction where one parallel path has the discharged capacitor and the other has a resistor to flow into the capacitor and none to flow into the resistor. Essentially, the discharged capacitor short circuits any resistors arranged in parallel with the capacitor.

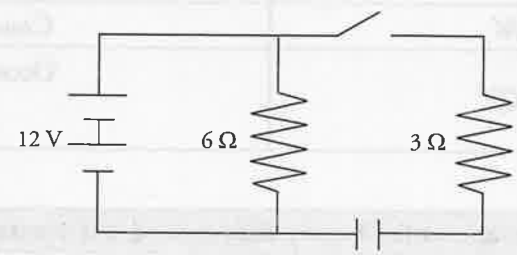
The situation is reversed when a capacitor is fully charged. The voltage between the plates is the maximum amount that can be supplied by the battery. As a result, the fully charged capacitor *acts like a broken wire.* This causes all of the current from any junction where one parallel path has the fully charged capacitor and the other has a resistor to flow into the resistor and none to flow into the capacitor. Essentially, the fully charged capacitor short circuits any resistors arranged in series with the capacitor.

When we analyze circuits with capacitors, we still use a chart of the resistance, voltage, current, and power. However, because our analysis is limited to one of two particular instants in time (the discharged capacitor or the fully charged capacitor), both the "current" and the "power" columns are not applicable to the capacitor, as both of those quantities explicitly depend on Δt.

Example 9 The image shows an RC circuit with an uncharged capacitor. Determine the following:

(a) the current through the capacitor when the switch has just been closed

(b) the voltage across the capacitor after the switch has been closed for a long time



Solution.

(a) Immediately after the switch is closed, the uncharged capacitor acts like a wire.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6			
R_2	3			
C	**0**	**0**	N/A	N/A
Entire Circuit		12		

We can find the resistance of the entire circuit by noting the 6 Ω and 3 Ω resistors are in parallel, so the equivalent resistance is $\frac{1}{R}=\frac{1}{3}+\frac{1}{6}$ so that $R = 2\ \Omega$.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6			
R_2	3			
C	0	0	N/A	N/A
Entire Circuit	**2**	12	12/2 = 6	$12^2/2 = 72$

Now that we have used the current, we look to the Loop Rule. Since the 6 Ω and 3 Ω resistors are both in parallel with the battery, they all have equal voltages.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6	**12**	12/6 = 2	$12^2/6 = 24$
R_2	3	**12**	12/3 = 4	$12^2/3 = 48$
C	0	0	N/A	N/A
Entire Circuit	2	12	6	72

Finally, we return to the Junction Rule and see that the capacitor is in series with the 3 Ω resistor, so the current flowing to the capacitor must be 4 A.

Note that as with wires, we cannot apply Ohm's Law to the capacitor because we would be dividing by 0 when calculating the current.

(b) After the switch is closed for a long time, the capacitor has filled. The capacitor acts like a broken wire. Now, R_2 is in series with the fully charged capacitor, so it has been shorted out of the circuit. Therefore, when we complete the table, we place a value of 0 Ω in for R_2 instead of 3 Ω. The resistance of the resistor is still 3 Ω, but because it is shorted out of the circuit, its effect is 0 Ω.

	Resistance (Ω)	**Voltage (V)**	**Current (A)**	**Power (W)**
R_1	6			
R_2	**0**			
C	∞		N/A	N/A
Entire Circuit		12		

The equivalent resistance of the circuit is just 6 Ω.

	Resistance (Ω)	**Voltage (V)**	**Current (A)**	**Power (W)**
R_1	6			
R_2	0			
C	∞		N/A	N/A
Entire Circuit	**6**	12	12/6 = 2	$12^2/6 = 24$

All of the current supported by the battery flows through R_1 and none through R_2. Because no current flows across R_2, there must be 0 V voltage between the two ends of the resistor and it must be dissipating 0 W of power.

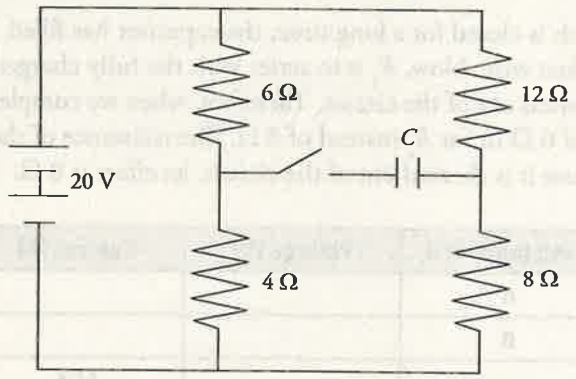
	Resistance (Ω)	**Voltage (V)**	**Current (A)**	**Power (W)**
R_1	6	6 * 2 = 12	2	$12^2/6 = 24$
R_2	0	**0**	**0**	**0**
C	∞		N/A	N/A
Entire Circuit	**6**	12	2	24

We know that each parallel branch has to have the same voltage. Since the current, voltage, and resistance of R_2 are all 0, then the voltage across the filled capacitor must be 12 V.

Example 10 Determine the current through and the voltage across each resistor and the battery in the following circuit when

(a) the switch has just been closed

(b) the switch has been closed for a long period of time



Solution.

(a) If we imagine the capacitor as a wire, the 6 Ω and 12 Ω resistors are in parallel, with an equivalent resistance of 4 Ω. The 4 Ω and 8 Ω resistors are in parallel, with an equivalent resistance of $\frac{8}{3}$ Ω. The equivalent 4 Ω resistor and $\frac{8}{3}$ Ω resistor are in series. The total resistance of the circuit is $\frac{20}{3}$ Ω.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6			
R_2	12			
R_3	4			
R_4	8			
C	**0**	**0**	N/A	N/A
Entire Circuit	**20/3**	20	20/6.67 = 3	20^2/6.67 = 60

Now, we again have a mixture of parallel and series. We could continue our methodical approach and find the equivalent resistance combination of R_1 and R_2 since we know the current through that combination is 3 A. That would allow us to calculate the voltage across each of those resistors. However, since we know the 3 A of total current divides between the 6 Ω and 12 Ω resistors, and since those are in parallel, they must have the same voltage, so $I_1 * 6 = I_2 * 12$, and we can conclude that the current in R_1 is 2 A while the current in R_2 is 1 A. Similarly, the 3 A of total current divides between the 4 Ω and 8 Ω resistors. Since those are in parallel, they must have the same voltage, so $I_3 * 4 = I_4 * 8$, and we can conclude that the current in R_3 is 2 A while the current in R_4 is 1 A.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6	$6 * 2 = 12$	**2**	$2^2 * 6 = 24$
R_2	12	$12 * 1 = 12$	**1**	$1^2 * 12 = 12$
R_3	4	$4 * 2 = 8$	**2**	$2^2 * 4 = 16$
R_4	8	$8 * 1 = 8$	**1**	$1^2 * 8 = 8$
C	**0**	**0**	N/A	N/A
Entire Circuit	20/3	20	3	60

(b) When the switch has been closed for a long time, it acts like a broken wire. This makes the 6 Ω and 4 Ω resistors in series, acting as a single 10 Ω resistor, and that combination is in parallel with the combination of the 12 Ω and 8 Ω resistors in series, acting as a single 20 Ω resistor. The equivalent resistance of the entire circuit is $\frac{1}{R} = \frac{1}{10} + \frac{1}{20}$, so $R = 6.67$.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6			
R_2	12			
R_3	4			
R_4	8			
C	∞		N/A	N/A
Entire Circuit	**6.67**	20	20/6.67 = 3	$20^2/6.67 = 60$

Now, we again have a mixture of parallel and series. We could continue our methodical approach and find the equivalent resistance combination of R_1 and R_3 since we know the voltage across that combination is 20 V. That would allow us to calculate the current through each of those resistors. However, since we know the 6 A of total current divides between the 10 Ω and 20 Ω paths, we know that 2 A goes through R_1 and R_3 while 1 A goes through R_2 and R_4.

	Resistance (Ω)	Voltage (V)	Current (A)	Power (W)
R_1	6	$6 * 2 = 12$	**2**	$12^2/6 = 24$
R_2	12	$12 * 1 = 12$	**1**	$12^2/2 = 12$
R_3	4	$4 * 2 = 8$	**2**	$8^2/4 = 16$
R_4	8	$8 * 1 = 8$	**1**	$8^2/8 = 8$
C	∞		N/A	N/A
Entire Circuit	6.67	20	20/6.67 = 3	$20^2/6.67 = 60$

Chapter 6 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review

A wire made of brass and a wire made of silver have the same length, but the diameter of the brass wire is 4 times the diameter of the silver wire. The resistivity of brass is 5 times greater than the resistivity of silver. If R_B denotes the resistance of the brass wire and R_S denotes the resistance of the silver wire, which of the following is true?

(A) $R_B = \frac{5}{16}R_S$

(B) $R_B = \frac{4}{5}R_S$

(C) $R_B = \frac{5}{4}R_S$

(D) $R_B = \frac{5}{2}R_S$

2 Mark for Review

For an Ohmic conductor, doubling the voltage without changing the resistance will cause the current to

(A) decrease by a factor of 4

(B) decrease by a factor of 2

(C) increase by a factor of 2

(D) increase by a factor of 4

3 Mark for Review

A battery, an uncharged capacitor, and resistor are all placed in parallel. Which answer best describes the current flowing in this circuit?

(A) The capacitor and resistor both experience the same current.

(B) Initially, all the current flows through the path with the resistor, then, after a long time, all the current flows through the path with the capacitor.

(C) Initially, all the current flows through the path with the capacitor, then, after a long time, all the current flows through the path with the resistor.

(D) No current will flow in this circuit.

4 Mark for Review

A student wants to determine the resistivity of copper. She has a voltmeter reading for a copper wire of known length. What other information will she need?

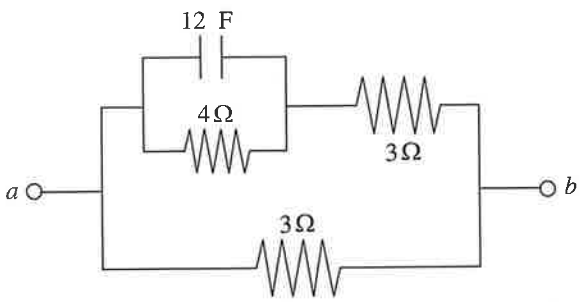
(A) An ammeter reading

(B) The diameter of the wire

(C) Both an ammeter reading and the diameter of the wire

(D) Neither an ammeter reading nor the diameter of the wire

5 Mark for Review



The circuit shown above has a constant voltage between points a and b. The voltage has been applied for a long time. What is the resistance of the circuit?

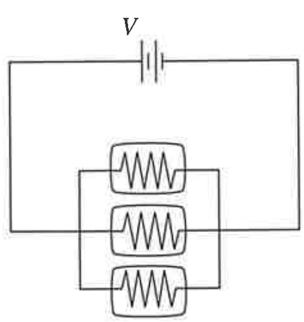
(A) 0.47 Ω

(B) 1.5 Ω

(C) 2.1 Ω

(D) 10 Ω

6 Mark for Review



Three identical light bulbs are connected to a source of emf, as shown in the diagram above. What will happen if the middle bulb burns out?

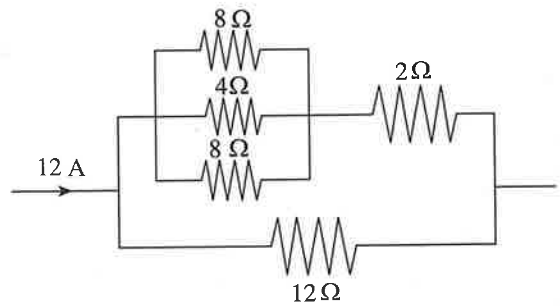
(A) The light intensity of the other two bulbs will decrease (but they won't go out).

(B) The light intensity of the other two bulbs will increase.

(C) The light intensity of the other two bulbs will remain the same.

(D) More current will be supported by the source of emf.

7 Mark for Review



What is the voltage drop across the 12 Ω resistor in the portion of the circuit shown above?

(A) 24 V

(B) 36 V

(C) 48 V

(D) 72 V

8 Mark for Review

A simple DC circuit with a single ohmic resistor is constructed. A graph plots the voltage drop across the resistor versus the current flowing through the resistor. For an ideal battery, the graph shows a directly proportional relationship. If the battery has some internal resistance, how, if at all, would such a graph change?

(A) The graph would continue to show a directly proportional relationship.

(B) The graph would be linear, but would have a negative y-intercept.

(C) The graph would be linear, but would have a positive y-intercept.

(D) The graph would be nonlinear.

9 Mark for Review

How much energy is dissipated as heat in 20 s by a 100 Ω resistor that carries a current of 0.5 A?

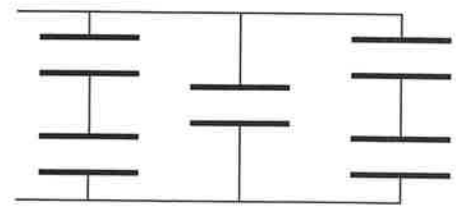
(A) 50 J

(B) 100 J

(C) 250 J

(D) 500 J

10 Mark for Review



If each of the capacitors in the array shown above is C, what is the capacitance of the entire combination?

(A) $C/2$

(B) $2C/3$

(C) $5C/6$

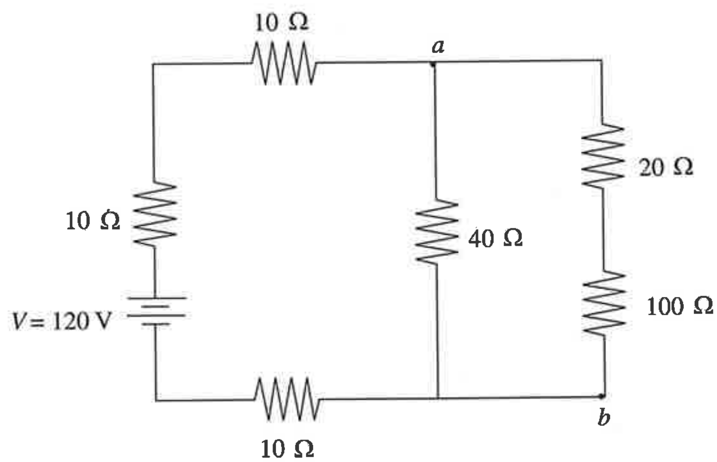
(D) $2C$

Section II: Free Response

1 Mark for Review

Consider the following circuit in Figure 1:

Figure 1



A. At what rate does the battery deliver energy to the circuit?

B. Find the current through the 40 Ω resistor.

C.

i. Determine the potential difference between points *a* and *b*.

ii. At which of these two points is the potential higher?

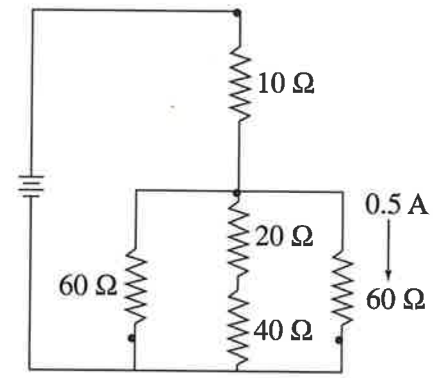
D. Given that the 100 Ω resistor is a solid cylinder that's 4 cm long, composed of a material whose resistivity is 0.45 Ω • m, determine the radius of its cross-section.

E. Which resistor or resistors, if any, could be replaced with a capacitor so that when the capacitor is fully charged, no current flow is supported by the battery? Explain your answer.

2 Mark for Review

Consider the following circuit in Figure 1:

Figure 1



A. What is the current through each resistor?

B. What is the potential difference across each resistor?

C. What is the equivalent resistance of the circuit?

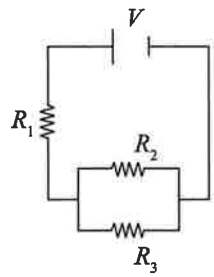
D. The leftmost 60 Ω resistor is replaced with a capacitor.

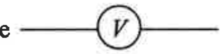
  i. When the capacitor is uncharged and just beginning to charge, will the current in the rightmost 60 Ω resistor be greater than, equal to, or less than the 0.5 A it was before the capacitor was inserted into the circuit?

  ii. When the capacitor is fully charged, will the current in the rightmost 60 Ω resistor be greater than, equal to, or less than the 0.5 A it was before the capacitor was inserted into the circuit?

3 Mark for Review

Figure 1



The circuit in Figure 1 contains a battery of voltage V and three resistors with resistances R_1, R_2, and R_3, respectively. As part of an experiment, a student has been given two measuring devices: a voltmeter and an ammeter. The first can be used to measure the changes in voltage of a circuit. The second can be used to measure the current flowing through a particular segment of wire. For answering the questions below, a voltmeter and ammeter look like 

and 

, respectively, when drawn in a circuit diagram.

A. In terms of the known variables, what is the voltage drop across the first resistor?

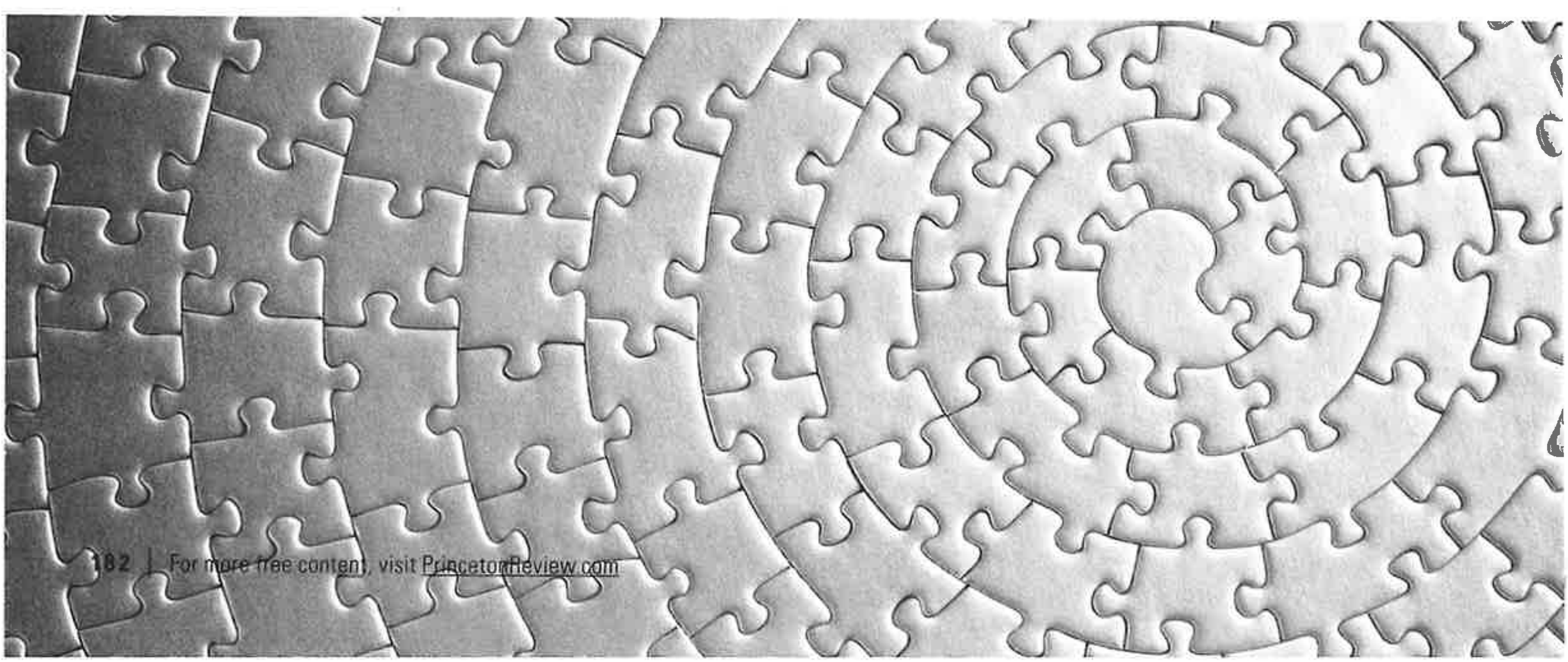
B. Draw a diagram showing how you would integrate the voltmeter to measure the voltage drop across resistor labeled R_1. Explain the reasoning behind your decision.

C. Draw a diagram showing how you would integrate the ammeter to measure the current passing through the resistor labeled R_1. Explain the reasoning behind your decision.

D. What would be the ideal resistances for each device to have? Explain why each would be ideal for that device.

Chapter 6 Summary

- The resistance of an object can be determined by $R = \frac{\rho \ell}{A}$, where ρ is the resistivity (a property of the material), ℓ is the length, and A is the cross-sectional area.
- The current is the rate at which charge is transferred and given by $I_{avg} = \frac{\Delta Q}{\Delta t}$.
- Many objects obey Ohm's Law, which is given by $V = IR$.
- The electrical power in a circuit is given by $P = IV$ or $P = I^2R$ or $P = \frac{V^2}{R}$.
  This is the same power we've encountered in our discussion of energy $P = \frac{W}{t}$.
In a series circuit	In a parallel circuit
$V_{Bat} = V_1 + V_2 + \ldots$	$V_{Bat} = V_1 = V_2 = \ldots$
$I_{Bat} = I_1 = I_2 = \ldots$	$I_{Bat} = I_1 + I_2 + \ldots$
$R_{eq,s} = R_1 + R_2 + \ldots$	$\frac{1}{R_{eq,p}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
$R_{eq,s} = \sum_i R_i$	$\frac{1}{R_{eq,p}} = \sum_i \frac{1}{R_i}$
- Kirchhoff's Loop Rule tells us that the sum of the potential differences in any closed loop in a circuit must be zero.
- Kirchhoff's Junction Rule (Node Rule) tells us the total current that enters a junction must equal the total current that leaves the junction.



- To find the combined capacitance of capacitors in parallel (side by side), simply add their capacitances:

$$C_{eq,p} = C_1 + C_2 + \ldots \text{ or } C_{eq,p} = \sum_i C_i$$

- To find the capacitance of capacitors in series (one after another), add their inverses:

$$\frac{1}{C_{eq,s}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots \text{ or } \frac{1}{C_{eq,s}} = \sum_i \frac{1}{C_i}$$

- When a capacitor is in a circuit and is completely discharged, it will act like a closed switch (equivalent to a resistor with $R = 0\ \Omega$). When the capacitor is fully charged, it acts like an open switch (equivalent to an infinite resistor).