Chapter 4
Electric Forces and Fields

INTRODUCTION

The existence of mass in an object results in that object having the ability to experience the gravitational force, and thereby interact with other objects that also have mass. All objects in the universe that have been discovered thus far have mass (although the particle known as the neutrino has such a small mass that it is practically massless). Another fundamental property of objects is charge. A charged object will interact with another charged object through the electromagnetic force, similar to massive objects. The property of charge is as fundamental to physics as the property of mass.

ELECTRIC CHARGE

The basic components of atoms are protons, neutrons, and electrons. Protons and neutrons form the nucleus (and are referred to collectively as *nucleons*), while the electrons keep their distance, swarming around the nucleus. Most of an atom consists of empty space. In fact, if a nucleus were the size of the period at the end of this sentence, then the electrons would be 5 meters away. So what holds such an apparently tenuous structure together? One of the most powerful forces in nature: the *electromagnetic force.* Protons and electrons have a quality called **electric charge** that gives them an attractive force. Electric charge comes in two varieties: positive and negative. A positive particle always attracts a negative particle, and particles of the same charge always repel each other. Protons are positively charged, and electrons are negatively charged.

Protons and electrons are intrinsically charged, but bulk matter is not. This is because the amount of charge on a proton exactly balances the charge on an electron, which is quite remarkable in light of the fact that protons and electrons are very different particles. Since most atoms contain an equal number of protons and electrons, their overall electric charge is 0, because the negative charges cancel out the positive charges. Therefore, in order for matter to be **charged**, an imbalance between the numbers of protons and electrons must exist. This can be accomplished by either the removal or addition of electrons (that is, by the **ionization** of some of the object's atoms). If you remove electrons, then the object becomes positively charged, while if you add electrons, then it becomes negatively charged. Furthermore, charge is **conserved**. For example, if you rub a glass rod with a piece of silk, then the silk will acquire a negative charge and the glass will be left with an *equal* positive charge. *Net charge cannot be created or destroyed.* (*Charge* can be created or destroyed—it happens all the time—but *net* charge cannot.)

The magnitude of charge on an electron (and therefore on a proton) is denoted e. This stands for **elementary charge** because it's the basic unit of electric charge. The charge of an ionized atom must be a whole number times e because charge can be added or subtracted only in lumps of size e. For this reason, we say that charge is **quantized**. To remind us of the quantized nature of electric charge, the charge of a particle (or object) is denoted by the letter q. In the SI system of units, charge is expressed in **coulombs** (abbreviated **C**). One coulomb is a tremendous amount of charge, about 10^{18} electrons. The value of e is about 1.6×10^{-19} C.

COULOMB'S LAW

The electric force between two charged particles obeys a law that is very similar to that describing the gravitational force between two masses: they are both inverse-square laws. The **electric force** between two particles with charges of q_1 and q_2, separated by a distance r, has a magnitude given by the equation

$$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2} = k\frac{|q_1q_2|}{r^2}$$

Equation Sheet

This is **Coulomb's Law**. If we were to leave off the absolute value bars, we would interpret a negative F_E as an attraction between the charges and a positive F_E as a repulsion. The value of the proportionality constant, k, depends on the material between the charged particles. In empty space (vacuum)—or air, for all practical purposes—it is called **Coulomb's constant** and has the approximate value $k_0 = 9 \times 10^9$ N·m²/C². The expression k_0 is sometimes written in terms of a fundamental constant known as the **permittivity of free space**, denoted ε_0:

$$k_0 = \frac{1}{4\pi\varepsilon_0}$$

You should notice a similarity between the form of Coulomb's Law and Newton's Law of Universal Gravitation you learned in AP Physics 1. Both of these force laws depend on the product of the material property that generates the force (charge or mass), and both are inversely proportional to the square of the separation distance. These two laws are therefore known as **inverse square laws**.

> **Example 1** Consider the proton and electron in hydrogen. The proton has a mass of 1.6×10^{-27} kg and a charge of $+e$. The electron has a mass of 9.1×10^{-31} kg and a charge of $-e$. In hydrogen, they are separated by a Bohr radius, about 0.5×10^{-10} m. What is the electric force between the proton and electron in hydrogen? What about the gravitational force between the proton and electron?

Solution. The electric force between the proton and the electron is given by Coulomb's Law:

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} = (9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.6\times10^{-19}\ \text{C})(-1.6\times10^{-19}\ \text{C})}{(0.5\times10^{-10}\ \text{m})^2} = -9.24\times10^{-8}\ \text{N}$$

The fact that F_E is negative means that the force is one of *attraction*, which we naturally expect, since one charge is positive and the other is negative. The force between the proton and electron is along the line that joins the charges, as we've illustrated below. The two forces shown form an action/reaction pair.

Now, the gravitational force between the two charges is given by Newton's Law of Gravitation:

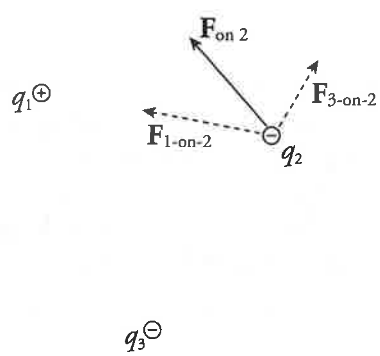
$$\mathbf{F}_G = G \times \frac{(m_1 m_2)}{r^2} = (6.67 \times 10^{-11} \frac{\text{kg} \cdot \text{m}^2}{\text{kg}^2}) \times \frac{(1.67 \times 10^{-27}\ \text{kg})(9.11 \times 10^{-31}\ \text{kg})}{(0.5 \times 10^{-10}\ \text{m})^2} = 4.06 \times 10^{-47}\ \text{N}$$

Now, compare the orders of magnitude of the electric force to the gravitational force. The electric force has an order of magnitude of 10^{-8} N, but the gravitational force has an order of magnitude of 10^{-47} N! This means that the electric force is something like 10^{39} times larger! This is the reason that in problems in which we calculate electrostatic forces, we often neglect gravitational forces.

Addition of Electric Forces

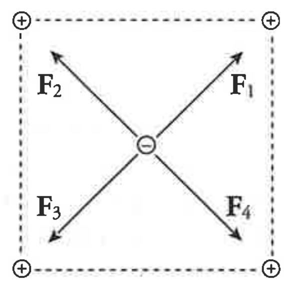
Consider three point charges: q_1, q_2, and q_3. The total electric force acting on, say, q_2 is simply the sum of $\mathbf{F}_{1\text{-on-}2}$, the electric force on q_2 due to q_1, and $\mathbf{F}_{3\text{-on-}2}$, the electric force on q_2 due to q_3:

$$\mathbf{F}_{\text{on }2} = \mathbf{F}_{1\text{-on-}2} + \mathbf{F}_{3\text{-on-}2}$$



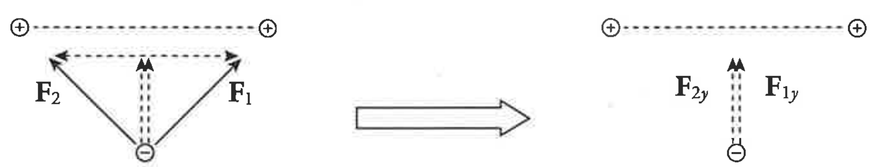
Example 2 Consider four equal, positive point charges that are situated at the vertices of a square. Find the net electric force on a negative point charge placed at the square's center.

Solution. Refer to the diagram below. The attractive forces due to the two charges on each diagonal cancel out: $F_1 + F_3 = 0$, and $F_2 + F_4 = 0$, because the distances between the negative charge and the positive charges are all the same and the positive charges are all equivalent. Therefore, by symmetry, the net force on the center charge is zero.



Example 3 If the two positive charges on the bottom side of the square in the previous example were removed, what would be the net electric force on the negative charge? Assume that each side of the square is 4.0 cm, each positive charge is 1.5 ∝C, and the negative charge is –6.2 nC.

Solution. If we break down $\mathbf{F}_1$ and $\mathbf{F}_2$ into horizontal and vertical components, then by symmetry the two horizontal components will cancel each other out, and the two vertical components will add:



Since the diagram on the left shows the components of $\mathbf{F}_1$ and $\mathbf{F}_2$ making right triangles with legs each of length 2 cm, it must be that $F_{1y} = F_1 \sin 45°$ and $F_{2y} = F_2 \sin 45°$. Also, the magnitude of $\mathbf{F}_1$ equals that of $\mathbf{F}_2$. So the net electric force on the negative charge is $F_{1y} + F_{2y} = 2F \sin 45°$, where F is the strength of the force between the negative charge and each of the positive charges.

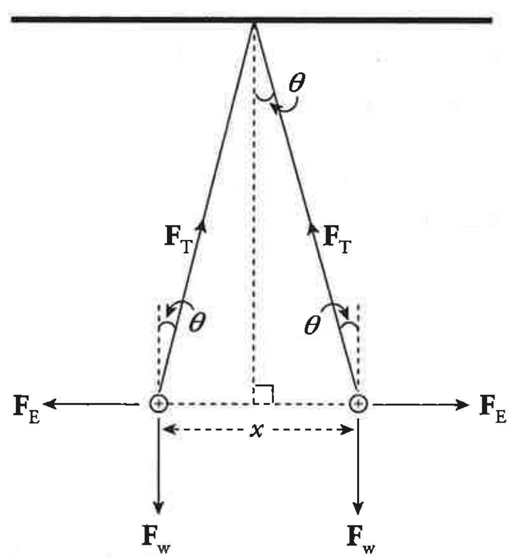
If s is the length of each side of the square, then the distance r between each positive charge and the negative charge is $r = \frac{1}{2}s\sqrt{2}$ and

$$F_E = 2F\sin 45° = 2\frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\sin 45°$$
$$= 2(9\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(1.5\times10^{-6}\ \text{C})(6.2\times10^{-9}\ \text{C})}{(\frac{1}{2}\cdot 4.0\times10^{-2}\cdot\sqrt{2}\ \text{m})^2}\sin 45°$$
$$= 0.15\ \text{N}$$

The direction of the net force is straight upward, toward the center of the line that joins the two positive charges.

Example 4 Two pith balls of mass m are each given a charge of $+q$. They are hung side-by-side from two threads each of length L, and move apart as a result of their electrical repulsion. Find the equilibrium separation distance x in terms of m, q, and L. (Use the fact that if θ is small, then $\tan\theta \approx \sin\theta$.)

Solution. Three forces act on each ball: weight, tension, and electrical repulsion:



When the balls are in equilibrium, the net force each feels is zero. Therefore, the vertical component of $\mathbf{F}_T$ must cancel out $\mathbf{F}_w$ and the horizontal component of $\mathbf{F}_T$ must cancel out $\mathbf{F}_E$:

$$F_T\cos\theta = F_w \quad \text{and} \quad F_T\sin\theta = F_E$$

Dividing the second equation by the first, we get $\tan\theta = F_E/F_w$. Therefore,

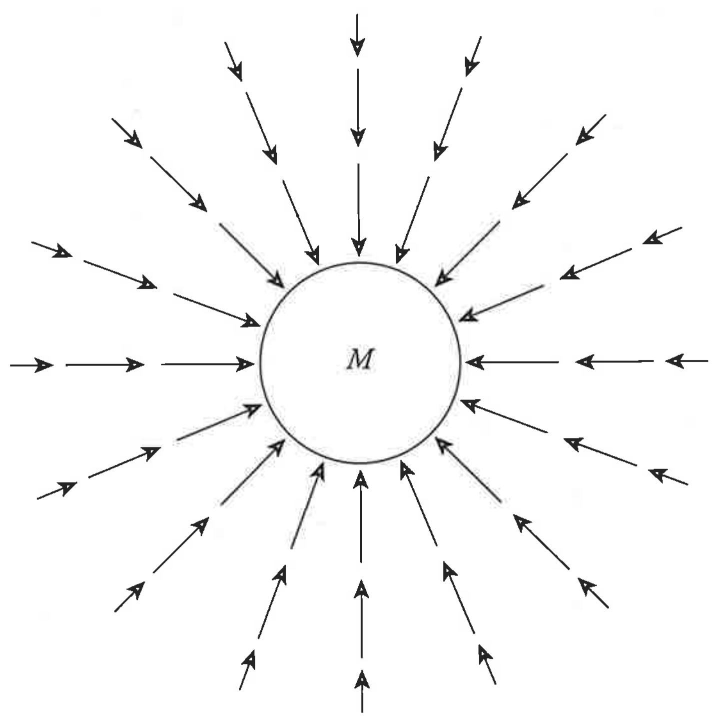
$$\tan\theta = \frac{k\frac{q^2}{x^2}}{mg} = \frac{kq^2}{mgx^2}$$

Now, to approximate: if θ is small, $\tan\theta \approx \sin\theta$, and from the diagram, $\sin\theta = \frac{1}{2}x/L$. Therefore, the equation above becomes

$$\frac{\frac{1}{2}x}{L} = \frac{kq^2}{mgx^2} \Rightarrow \frac{1}{2}mgx^3 = kq^2L \Rightarrow x = \sqrt[3]{\frac{2kq^2L}{mg}}$$

THE ELECTRIC FIELD

The presence of a massive body such as the Earth causes objects to experience a gravitational force directed toward the Earth's center. For objects located outside the Earth, this force varies inversely with the square of the distance and directly with the mass of the gravitational source. A vector diagram of the gravitational field surrounding the Earth looks like this:



We can think of the space surrounding the Earth as permeated by a **gravitational field** created by the Earth. Any mass placed in this field then experiences a gravitational force due to an interaction with this field.

In general, a field is a physical quantity that has a value at each point in space-time. The values can be scalars (for example, the temperature at each location in a house) or vectors (like the gravitational field in space due to the mass of the Earth). All non-contact forces in physics—such as the gravitational force and the electromagnetic force—are described as the interaction between one object and the field generated by another object.

This is how we describe the electric force. Rather than having two charges reach out across empty space to each other to produce a force, we will instead interpret the interaction in the following way: the presence of a charge creates an **electric field** in the space that surrounds it. Another charge placed in the field created by the first will experience a force due to the field.

Consider a point charge Q in a fixed position and assume that it's positive. Now imagine moving a tiny positive test charge q around to various locations near Q. At each location, measure the force that the test charge experiences, and call it $\mathbf{F}_{\text{on } q}$. Divide this force by the test charge q; the resulting vector is the **electric field vector, E**, at that location:

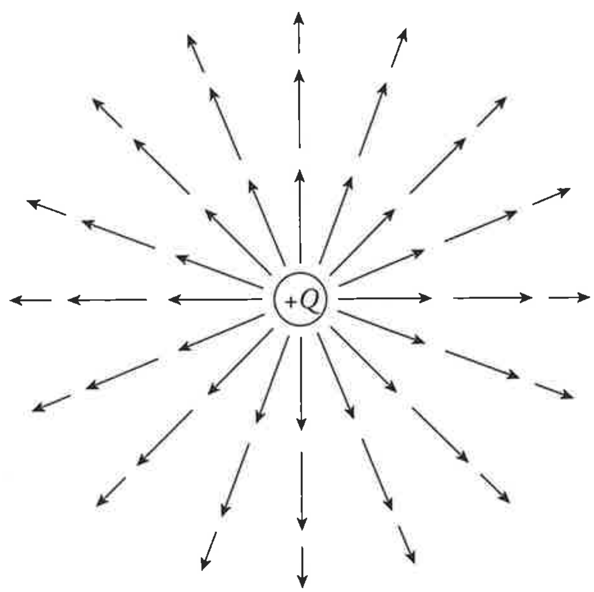
Equation Sheet

$$\vec{E} = \frac{\vec{F}_E}{q}$$

The reason for dividing by the test charge is simple. If we were to use a different test charge with, say, twice the charge of the first one, then each of the forces we'd measure would be twice as much as before. But when we divided this new, stronger force by the new, greater test charge, the factors of 2 would cancel, leaving the same ratio as before. So this ratio tells us the intrinsic strength of the field due to the source charge, independent of whatever test charge we may use to measure it.

There are three types of electric fields that you should expect to encounter on the AP Physics 2 Exam. The first type of field is a radial field. This occurs from a point charge (or from a charged sphere). The field points along lines that radiate outward from (for positive charges) or point inward toward (for negative charges) the center of the circle. These fields are inversely proportional to the square of the distance from the point charge (or center of the sphere), just like the gravitational field. The second type of field is that generated by a collection of point charges with specified locations and charges. This distribution of charge results in an electric field that is determined by superposition; the field at a particular observation location is determined from each charge separately, and then the vector sum of the individual fields is taken to determine the net field. This is identical to finding the vector sum of the forces of the charge distribution before dividing by the test charge. Lastly, an infinite sheet of charge will result in a field that is constant in both magnitude and direction. Because infinite sheets of charge are impossible to construct in the real world, the "infinite sheet of charge" is a useful approximation when the distance between the observation location and the sheet of charge is small compared to the distance between the observation location and the edges of the sheet.

First, let's examine the point charge. Since the test charge used to measure the field is positive, every electric field vector would point radially away from the source charge. *If the source charge is positive, the electric field vectors point away from it; if the source charge is negative, then the field vectors point toward it.* And, since the force decreases as we get farther away from the charge (as $1/r^2$), so does the electric field. This is why the electric field vectors farther from the source charge are shorter than those that are closer.



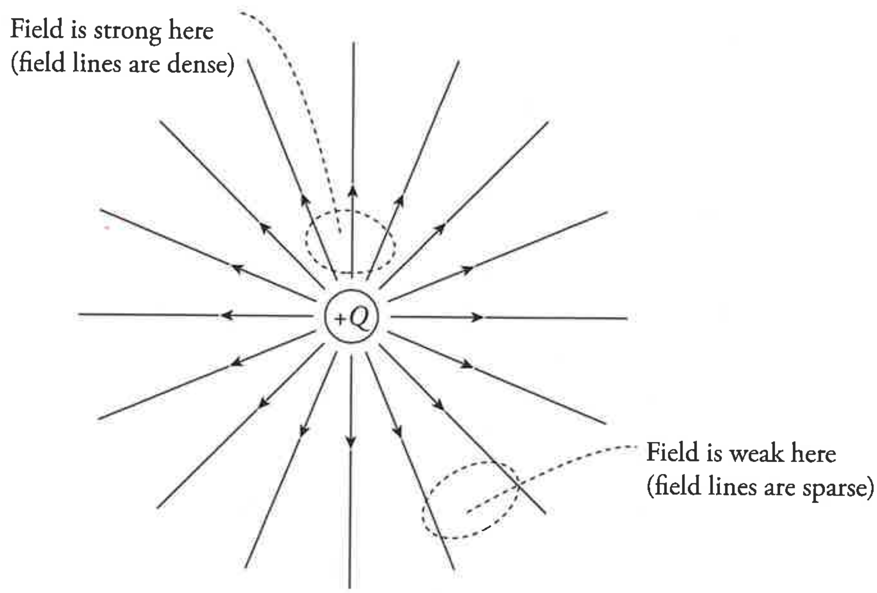
Since the force on any test charge q, due to some source Q, has a strength of $qQ/4\pi\varepsilon_0 r^2$, when we divide this by q, we get the expression for the strength of the electric field created by a point-charge source of magnitude Q:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

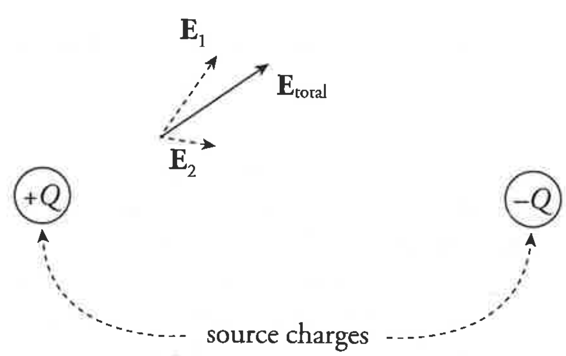
This is the electric field in the space surrounding a **point charge**.

It is of utmost importance to note that this equation resulted from considering the electric field of a single point charge. Any other distribution of charge, such as placing charge on a sheet to generate a **parallel-plate capacitor** (described later), will have a different electric field and this equation will be invalid. Adding to the confusion, there is one way to distribute a large amount of charge and get the same exact equation for the electric field as having just a single point charge. When the charges were spread out over a sphere (for example, a charged metal ball) instead of a single point charge, at any location outside of the sphere the electric field would be the same as the point charge field.

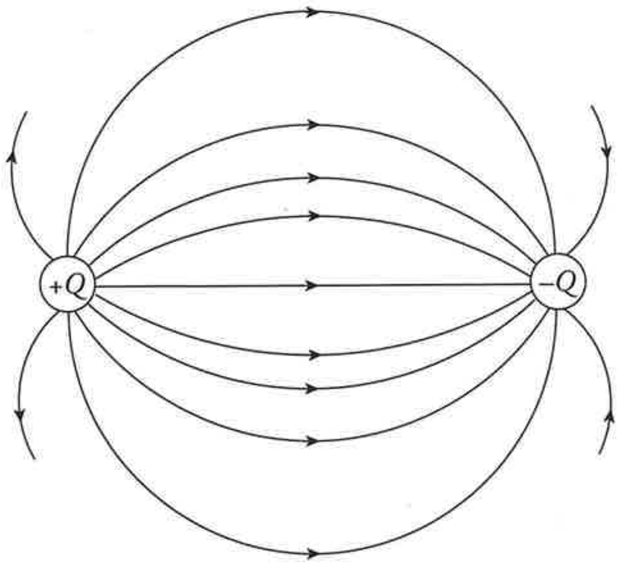
Sometimes electric fields are drawn, as shown above, with arrows at various locations. The arrows point in the field direction, and their lengths indicate the field strength. However, sometimes they are sketched simply as continuous lines, as shown below, from the source such that the electric field vector is always tangent to the line everywhere it's drawn (a single arrowhead on the line indicates the direction to draw the field line along) and the strength is determined by how close the lines are together.



Electric fields obey the same addition properties as the electric force. If we had two source charges, their fields would overlap and effectively add; a third charge would feel the net influence of both charges. At each position in space (referred to as *observation locations*), add the electric field vector due to one of the charges to the electric field vector due to the other charge: $\mathbf{E}_{total} = \mathbf{E}_1 + \mathbf{E}_1$. This extends to any number of source charges. In the diagram below, $\mathbf{E}_1$ is the electric field vector at a particular location due to the charge $+Q$, and $\mathbf{E}_2$ is the electric field vector at that same location due to the other charge, $-Q$. Adding these vectors gives the overall field vector $\mathbf{E}_{total}$ at that location.

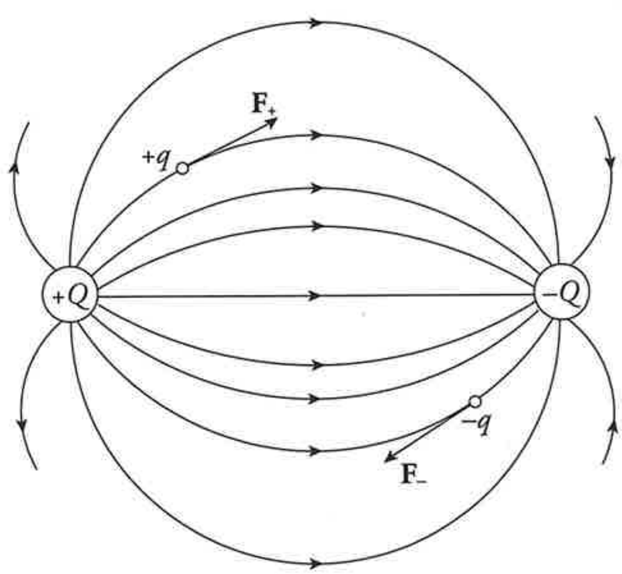


If this is done at enough locations, the electric field lines can be sketched.



Note that, like electric field vectors, electric field lines always point away from positive source charges and toward negative ones. Two equal but opposite charges, like the ones shown in the diagram above, form a pair called an **electric dipole**.

If a positive charge $+q$ were placed in the electric field above, it would experience a force that is tangent to, and in the same direction as, the field line passing through $+q$'s location. After all, electric field lines indicate the magnitude and direction of the force a positive test charge would experience. On the other hand, if a negative charge $-q$ were placed in the electric field, it would experience a force that is tangent to, but in the direction opposite from, the field line passing through $-q$'s location.



Finally, notice that electric field lines never cross.

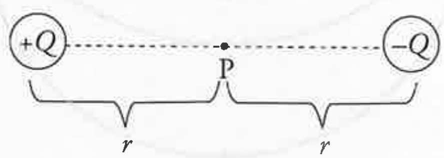
Example 5 A charge $q = +3.0$ nC is placed at a location at which the electric field strength is 400 N/C. Find the force felt by the charge q.

Solution. From the definition of the electric field, we have the following equation:

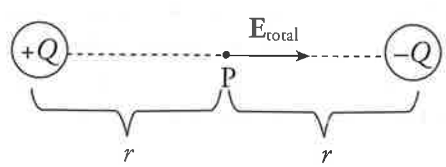
$$\mathbf{F}_{\text{on } q} = q\mathbf{E}$$

Therefore, in this case, $F_{\text{on } q} = qE = (3 \cdot 10^{-9}\ \text{C})(400\ \text{N/C}) = 1.2 \cdot 10^{-6}\ \text{N}$.

Example 6 A dipole is formed by two point charges, each of magnitude 4.0 nC, separated by a distance of 6.0 cm. What is the strength of the electric field at the point midway between them?



Solution. Let the two source charges be denoted $+Q$ and $-Q$. At Point P, the electric field vector due to $+Q$ would point directly away from $+Q$, and the electric field vector due to $-Q$ would point directly toward $-Q$. Therefore, these two vectors point in the same direction (from $+Q$ to $-Q$), so their magnitudes would add.



Using the equation for the electric field strength due to a single point charge, we find that

$$E_{\text{total}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} + \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 2\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

$$= 2(9\times10^{-9}\text{N}\cdot\text{m}^2/\text{C}^2)\frac{4.0\times10^{-9}\ \text{C}}{\left(\frac{1}{2}(6.0\times10^{-2}\ \text{m})\right)^2}$$

$$= 8.0\times10^{4}\ \text{N/C}$$

Example 7 If a charge $q = -5.0$ pC were placed at the midway point described in the previous example, describe the force it would feel. ("p" is the abbreviation for "pico-," which means 10^{-12}.)

Solution. Since the field E at this location is known, the force felt by q is easy to calculate:

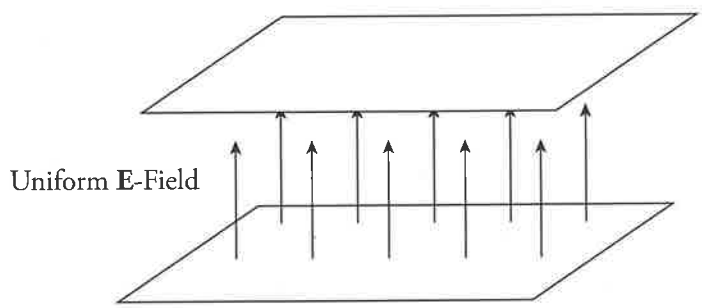
$$\mathbf{F}_{\text{on } q} = q\mathbf{E} = (-5.0 \cdot 10^{-12}\ \text{C})(8.0 \cdot 10^{4}\ \text{N/C to the right}) = 4.0 \cdot 10^{-7}\ \text{N to the } \textit{left}$$

Example 8 What can you say about the electric force that a charge would feel if it were placed at a location at which the electric field was zero?

Solution. Remember that $\mathbf{F}_{\text{on } q} = q\mathbf{E}$. So if $\mathbf{E} = 0$, then $\mathbf{F}_{\text{on } q} = 0$. (Zero field means zero force.)

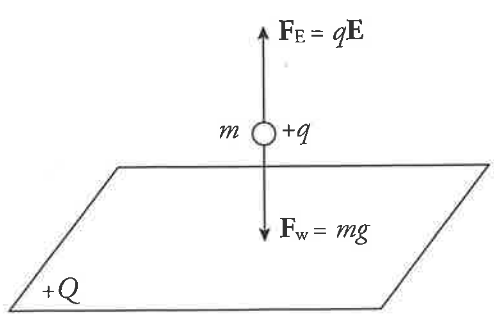
The Uniform Electric Field

An important subset of problems deals with uniform electric fields. One method of creating this uniform field is to have two large conducting sheets, each storing some charge Q, some distance d apart. Near the edges of each sheet, the field may not be uniform, but near the middle, for all practical purposes, the field is uniform. Having a uniform field means having a constant force (and therefore a constant acceleration), so you can use kinematic equations just as if you had a uniform gravitational field (as you do near the surface of the Earth).



Example 9 Positive charge is distributed uniformly over a large, horizontal plate, which then acts as the source of a vertical electric field. An object of mass 5 g is placed at a distance of 2 cm above the plate. If the strength of the electric field at this location is 10^6 N/C, how much charge would the object need to have in order for the electrical repulsion to balance the gravitational pull?

Solution. Clearly, since the plate is positively charged, the object would also have to carry a positive charge so that the electric force would be repulsive.



Let q be the charge on the object. Then, in order for F_E to balance mg, we must have

$$qE = mg \quad \Rightarrow \quad q = \frac{mg}{E} = \frac{(5\times10^{-3}\text{ kg})(10\text{ N/kg})}{10^6\text{ N/C}} = 5\times10^{-8}\text{ C} = 50\text{ nC}$$

> **Example 10** A proton, neutron, and electron are in a uniform electric field of 20 N/C that is caused by two large charged plates that are 30 cm apart. The particles are far enough apart that they don't interact with each other. They are released from rest equidistant from each plate.
>
> (a) What is the magnitude of the net force acting on each particle?
> (b) What is the magnitude of the acceleration of each particle?
> (c) How much work will be done on the particle as it moves to coincide with one of the charged plates?
> (d) What is the speed of each particle when it strikes the plate?
> (e) How long does it take to reach the plate?

Solution.

(a) Since $F = qE$, plugging in the values, we get

proton: $F = (1.6\cdot10^{-19}\text{ C})(20\text{ N/C}) = 3.2\cdot10^{-18}\text{ N}$

electron: $F = (1.6\cdot10^{-19}\text{ C})(20\text{ N/C}) = 3.2\cdot10^{-18}\text{ N}$

neutron: $F = (0\text{ C})(20\text{ N/C}) = 0\text{ N}$

Note: Because the proton and electron have the same magnitude, they will experience the same force. If you're asked for the direction, the proton travels in the same direction as the electric field and the electron travels in the opposite direction as the electric field.

(b) Since $F = ma$, $a = \frac{F}{m}$. Plugging in the values, we get

proton: $a = \frac{3.2\times10^{-18}\text{ N}}{1.67\times10^{-27}\text{ kg}} = 1.9\times10^{9}\text{ m/s}^2$

electron: $a = \frac{3.2\times10^{-18}\text{ N}}{9.11\times10^{-31}\text{ kg}} = 3.5\times10^{12}\text{ m/s}^2$

neutron: $a = \frac{0\text{ N}}{1.67\times10^{-27}\text{ kg}} = 0\text{ m/s}^2$

Notice that although the charges have the same magnitude of force, the electron experiences an acceleration almost 2000 times greater due to its mass being almost 2000 times smaller than the proton's mass.

(c) Since $W = Fd$, we get $W = qEd$. Plugging in the values, recalling that the charges start midway between the plates, we get

proton: $W = (1.6 \cdot 10^{-19}\text{ C})(20\text{ N/C})\ (0.15\text{ m}) = 4.8 \cdot 10^{-19}\text{ J}$

electron: $W = (1.6 \cdot 10^{-19}\text{ C})(20\text{ N/C})\ (0.15\text{ m}) = 4.8 \cdot 10^{-19}\text{ J}$

neutron: $W = (0\text{ C})\ (20\text{ N/C})(0.15\text{ m}) = 0\text{ J}$

(d) From the Work–Energy Theorem,

$W = \Delta K = 1/2mv^2$, so $v = \sqrt{2W/m}$

Using the answer from part (c) for the work, we get

proton: $v_f = \sqrt{2(4.8 \times 10^{-19}\text{ J})/(1.67 \times 10^{-27}\text{ kg})} \rightarrow v_f = 24{,}000\text{ m/s}$

electron: $v_f = \sqrt{2(4.8 \times 10^{-19}\text{ J})/(9.11 \times 10^{-31}\text{ kg})} \rightarrow v_f = 1.0 \times 10^6\text{ m/s}$

neutron: The neutron never hits the plate, so the question about what speed it hits the plate with is not well posed.

Notice that, even though the force is the same and the same work is done on both charges, there is a significant difference in final velocities due to the large mass difference. An alternative solution to this would be using kinematics. You would have obtained the same answers.

(e) Recall the definition of the final velocity of a uniformly accelerated object:

$$v_f = v_i + at \rightarrow t = \frac{v_{fi} - v}{a} \rightarrow t = \frac{v_f}{a}$$

proton: $t = \dfrac{24{,}000\text{ m/s}}{1.9 \times 10^9\text{ m/s}^2} \rightarrow 1.3 \cdot 10^{-5}\text{ s}$

electron: $t = \dfrac{1 \times 10^6\text{ m/s}}{3.5 \times 10^{12}\text{ m/s}^2} \rightarrow 2.9 \cdot 10^{-7}\text{ s}$

neutron: The neutron never accelerates, so it will never hit the plate.

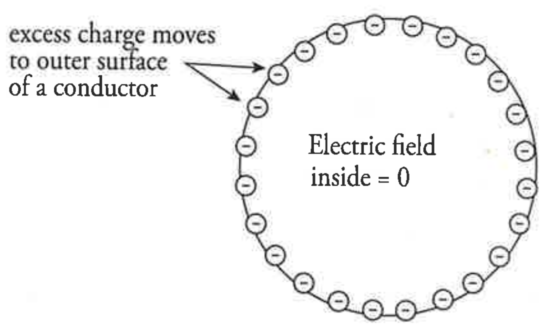
CONDUCTORS AND INSULATORS

Materials can be classified into broad categories based on their ability to permit the flow of charge. If electrons were placed on a metal sphere, they would quickly spread out and cover the outside of the sphere uniformly. These electrons would be free to flow through the metal and redistribute themselves, moving to get as far away from each other as they could. Materials that permit the flow of excess charge are called **conductors**; they conduct electricity. Metals are the best examples of conductors. Aqueous solutions that contain dissolved electrolytes (such as salt water) are also conductors. Metals conduct electricity because they bind all but their outermost electrons very tightly. That outermost electron is free to move about the metal. This creates a sort of sea of mobile (or conduction) electrons.

Insulators, on the other hand, closely guard their electrons—and even extra ones that might be added. Electrons are not free to roam throughout the atomic lattice. Examples of insulators are glass, wood, rubber, and plastic. If excess charge is placed on an insulator, it stays put. This process, called **charging by friction**, involves simply rubbing the insulator against another material, thereby stripping electrons off one material and depositing them on the other material.

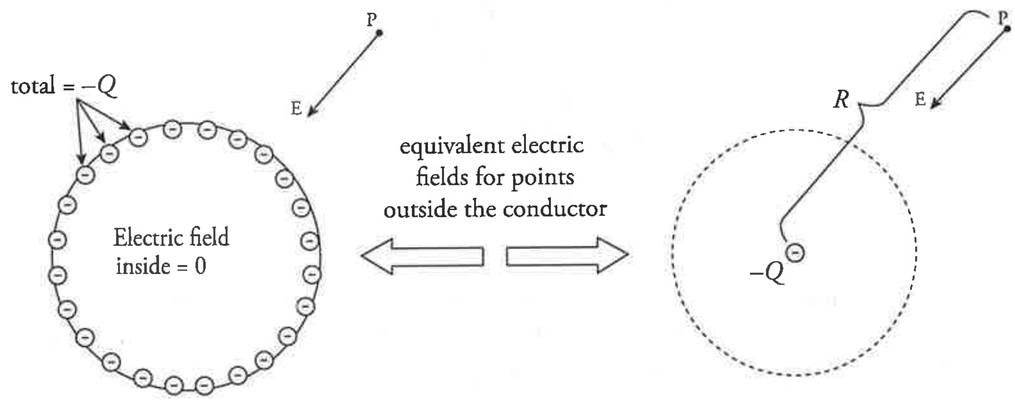
Example 11 A solid sphere of copper is given a negative charge. Discuss the electric field inside and outside the sphere.

Solution. In order for the electrons to eventually come to rest, the net force on any electron *that is free* to move must be 0 N. Copper is a conducting material. Therefore, since any excess charges that reside within the body of the sphere must have a net force of 0 N, we conclude that the electric field inside a conductor is zero. Otherwise, charges within the sphere would experience a force and would move, violating the "static" part of electrostatic equilibrium.

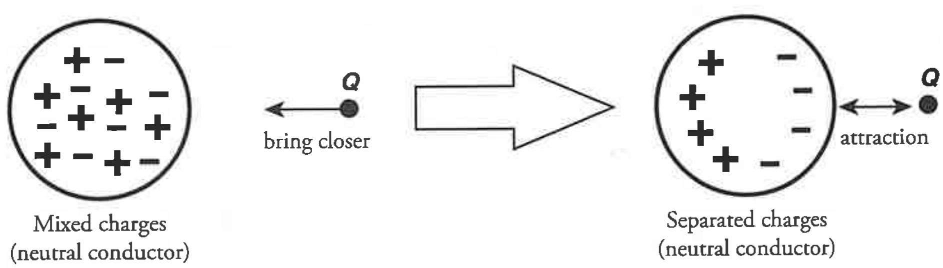


All the excess electrons that are deposited on the sphere arrange themselves on the outer surface, where they are constrained in their motion by the insulating air surrounding the sphere. The electric field within the sphere is 0 N and outside the sphere the field is found to be $E = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$, which is exactly the same as the field from a single point charge Q located at the center of the conducting sphere.

In fact, you can shield yourself from electric fields simply by surrounding yourself with metal. Charges may move around on the outer surface of your cage, but within the cage, the electric field will be zero. For points outside the sphere, it can be shown that the sphere behaves as if all its excess charge were concentrated at its center. (Remember that this is just like the gravitational field due to a uniform spherical mass.) Also, *the electric field is always perpendicular to the surface, no matter what shape the surface may be.* See the diagram below.

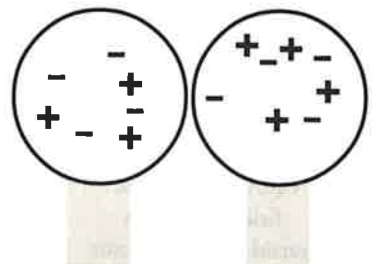


Now, let's take our previous sphere and put it in a different situation. Start with a neutral metal sphere and bring a positive charge Q nearby without touching the original metal sphere. What will happen? The positive charge will attract free electrons in the metal, leaving the far side of the sphere positively charged. Since the negative charge is closer to Q than the positive charge, there will be a net attraction between Q and the sphere. So, even though the sphere as a whole is electrically neutral, the separation of charge induced by the presence of Q will create a force of electrical attraction between them. This process for rearranging charges within a conductor is called **charge conduction**.

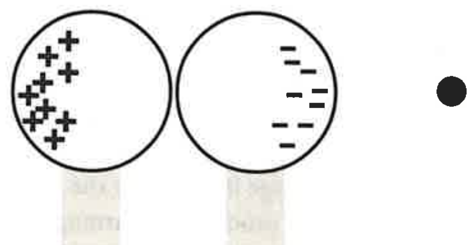


Charging by Induction

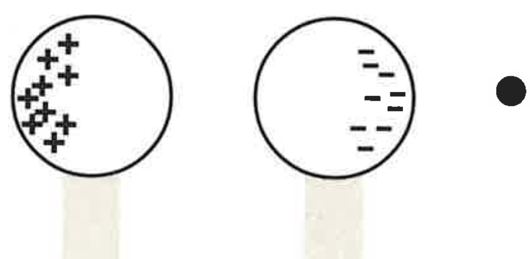
The process of charging by induction may be used to redistribute charges among a pair of neutrally charged spheres, so that in the end both spheres are charged. Imagine two neutrally charged spheres that are each set on an insulating stand. The spheres are arranged so that they are in contact with one another.



A positive charge is brought near the side of one of the spheres, as shown in the figure, resulting in the same attraction as that which occurred with the single sphere. However, because we have two conducting spheres, the excess negative charges are located on the right sphere and the positive excess charges are on the left sphere.

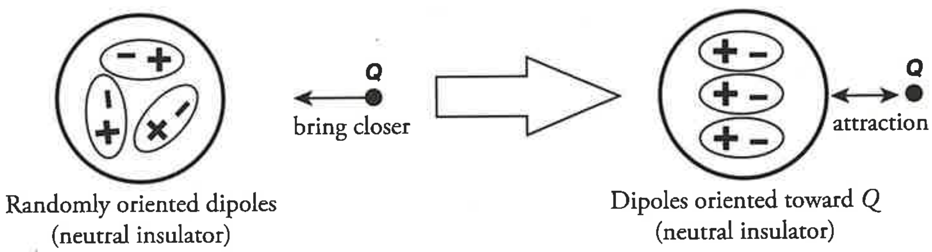


With the external charge still nearby, the two conducting spheres are separated. Because the conductors are no longer in contact, the negative charge has been trapped on the right sphere and the positive charge has been trapped on the left sphere.



Note that the net charge is unchanged between the two spheres—the distribution of the charges has simply been changed.

Now, what if the sphere was made of glass (an insulator)? Although there aren't free electrons that can move to the near side of the sphere, the atoms that make up the sphere will become polarized. That is, their electrons will feel a tug toward Q, and so will spend more time on the side of the atom closer to Q than on the side opposite Q, causing the atoms to develop a distribution of charge that is more negative on the side nearby Q (and a partial positive charge on the other side from Q). The effect isn't as dramatic as the mass movement of free electrons in the case of a metal sphere, but the polarization is still enough to cause an electrical attraction between the sphere and Q. For example, if you comb your hair, the comb will pick up extra electrons, making it negatively charged. If you place this electric field source near little bits of paper, the paper will become polarized and will then be attracted to the comb.



The same phenomenon, in which the presence of a charge tends to cause polarization in a nearby collection of charges, is responsible for a kind of intermolecular force. Dipole-induced forces are caused by a shifting of the electron cloud of a neutral molecule toward positively charged ions or away from negatively charged ions; in either case, the resulting force between the ion and the atom is attractive.

Chapter 4 Review Questions

Answers and explanations can be found in Chapter 11.

Section I: Multiple Choice

1 Mark for Review

An experiment is conducted measuring the electrostatic force, F, on a test object at various distances, r, from a point source. In order to create a plot with a straight line, what should be graphed?

(A) F versus r^2

(B) F versus r

(C) F versus r^{-1}

(D) F versus r^{-2}

2 Mark for Review

Two 1 kg spheres each carry a charge of magnitude 1 C. How does F_E, the strength of the electric force between the spheres, compare to F_G, the strength of their gravitational attraction?

(A) $F_E < F_G$

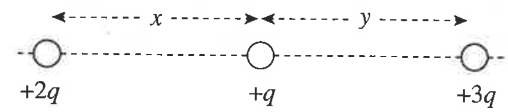
(B) $F_E = F_G$

(C) $F_E > F_G$

(D) If the charges on the spheres are of the same sign, then $F_E > F_G$; but if the charges on the spheres are of the opposite sign, then $F_E < F_G$.

3 Mark for Review

The figure below shows three point charges, all positive. If the net electric force on the center charge is zero, what is the value of y/x?



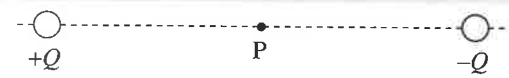
(A) $\frac{4}{9}$

(B) $\sqrt{\frac{2}{3}}$

(C) $\sqrt{\frac{3}{2}}$

(D) $\frac{3}{2}$

4 Mark for Review



The figure above shows two point charges, $+Q$ and $-Q$. If the negative charge were absent, the electric field at Point P due to $+Q$ would have strength E. With $-Q$ in place, what is the strength of the total electric field at P, which lies at the midpoint of the line segment joining the charges?

(A) 0

(B) $\frac{E}{2}$

(C) E

(D) $2E$

5 Mark for Review

A sphere of charge $+Q$ is fixed in position. A smaller sphere of charge $+q$ is placed near the larger sphere and released from rest. The small sphere will move away from the large sphere with

(A) decreasing velocity and decreasing acceleration

(B) decreasing velocity and increasing acceleration

(C) increasing velocity and decreasing acceleration

(D) increasing velocity and increasing acceleration

6 Mark for Review

An object of charge $+q$ feels an electric force $\mathbf{F}_E$ when placed at a particular location in an electric field, $\mathbf{E}$. Therefore, if an object of charge $-2q$ were placed at the same location where the first charge was, it would feel an electric force of

(A) $\frac{-\mathbf{F}_E}{2}$

(B) $-2\mathbf{F}_E$

(C) $-2q\mathbf{F}_E$

(D) $\frac{-2\mathbf{F}_E}{q}$

7 Mark for Review

A charge of $-3Q$ is transferred to a solid metal sphere of radius r. How will this excess charge be distributed?

(A) $-Q$ at the center, and $-2Q$ on the outer surface

(B) $-3Q$ at the center

(C) $-3Q$ on the outer surface

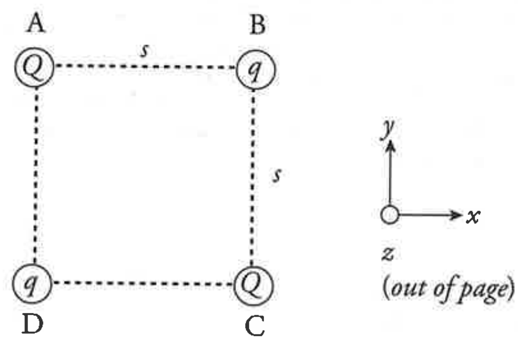
(D) $-Q$ at the center, $-Q$ in a ring of radius $\frac{1}{2}r$, and $-Q$ on the outer surface

Section II: Free Response

1 Mark for Review

In Figure 1, four charges are situated at the corners of a square of side length s. The charges on opposite corners are equal to one another and are labeled Q for corners A and C and labeled q for corners B and D. The charge on Q is positive for all experiments.

Figure 1



A. In a first experiment, it is found that the force on the charge at position C is 0 N.

   i. Justify the assertion that the charges q cannot have different magnitudes in this experiment.

   ii. Derive an expression for the magnitude and sign of charge on q in this experiment.

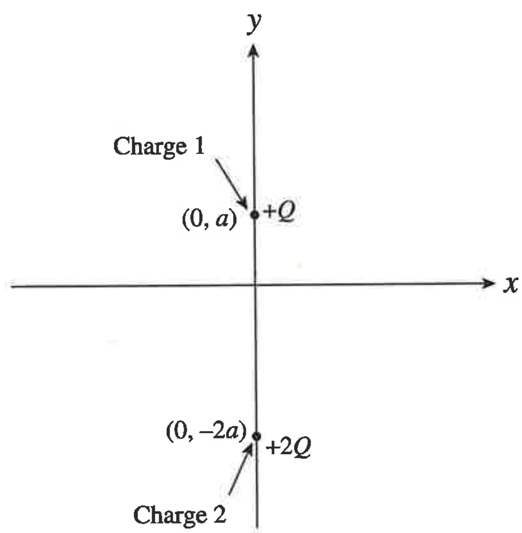
B. In a second experiment, the charge at point C is removed. If the charges q are positive, is there anywhere within the boundary square where the electric field could be 0 N/C?

C. The charges are reassembled into their original positions. In a clear, coherent paragraph-length response, explain why the electric field at the center of the square must be 0 N/C regardless of the magnitudes of the charges q or Q and regardless of their signs.

2 Mark for Review

Two charges, $+Q$ and $+2Q$, are fixed in place along the y-axis of an xy-coordinate system as shown in Figure 1. Charge 1 is at the point $(0, a)$, and Charge 2 is at the point $(0, -2a)$.

Figure 1



A. Find the electric force (magnitude and direction) felt by Charge 1 due to Charge 2.

B. Find the electric field (magnitude and direction) at the origin created by both Charges 1 and 2.

C. Is there a point on the x-axis where the total electric field is zero? If so, where? If not, explain briefly.

D. Is there a point on the y-axis where the total electric field is zero? If so, where? If not, explain briefly.

Chapter 4 Summary

- Coulomb's Law describes magnitude of the force acting on two point charges and is given by

$$\left|\vec{F}_E\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2} = k\frac{|q_1 q_2|}{r^2}$$

  where $\frac{1}{4\pi\varepsilon_0} = k$ is a constant equal to $9.0 \times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}$.

- The electric field is given by $\vec{E} = \frac{\vec{F}_E}{q}$.

- The electric field magnitude a distance r away from a point charge is $\left|\vec{E}\right| = \frac{1}{4\pi\varepsilon_0}\frac{|q|}{r^2} = k\frac{|q|}{r^2}$.

- Both the electric force and field are vector quantities, and therefore all the rules for vector addition apply.

- Electric field lines are continuous lines pointing in the direction of the electric field. They always point away from positive source charges and towards negative ones.

- Conductors allow for the free motion of charge. Insulators keep any added charge in place. The electric field inside any conductor is always zero.