AMC 10

SET 1

PRACTICE EXAMINATION

TIME ALLOWED

75:00

25 QUESTIONS • 75 MINUTES • 150 POINTS

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2019) and appear in their original wording, rearranged into a new paper. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

1. What is $(-1)^1 + (-1)^2 + \cdots + (-1)^{2006}$?

(A) -2006 **(B)** -1 **(C)** 0 **(D)** 1 **(E)** 2006

2. For what value of x does

$$10^x \cdot 100^{2x} = 1000^5?$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

3. The sum of 5 consecutive even integers is 4 less than the sum of the first 8 consecutive odd counting numbers. What is the smallest of the even integers?

(A) 6 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 14

4. Pablo, Sofia, and Mia got some candy eggs at a party. Pablo had three times as many eggs as Sofia, and Sofia had twice as many eggs as Mia. Pablo decides to give some of his eggs to Sofia and Mia so that all three will have the same number of eggs. What fraction of his eggs should Pablo give to Sofia?

(A) $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{1}{2}$

5. What is the greatest number of consecutive integers whose sum is 45?

(A) 9 **(B)** 25 **(C)** 45 **(D)** 90 **(E)** 120

6. A box contains 5 chips, numbered 1, 2, 3, 4, and 5. Chips are drawn randomly one at a time without replacement until the sum of the values drawn exceeds 4. What is the probability that 3 draws are required?

(A) $\frac{1}{15}$ **(B)** $\frac{1}{10}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{5}$ **(E)** $\frac{1}{4}$

7. A circle is inscribed in a square, then a square is inscribed in this circle, and finally, a circle is inscribed in this square. What is the ratio of the area of the smaller circle to the area of the larger square?

(A) $\frac{\pi}{16}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{3\pi}{16}$ **(D)** $\frac{\pi}{4}$ **(E)** $\frac{\pi}{2}$

8. At a gathering of 30 people, there are 20 people who all know each other and 10 people who know no one. People who know each other hug, and people who do not know each other shake hands. How many handshakes occur?

(A) 240 **(B)** 245 **(C)** 290 **(D)** 480 **(E)** 490

9. The two legs of a right triangle, which are altitudes, have lengths $2\sqrt{3}$ and 6. How long is the third altitude of the triangle?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

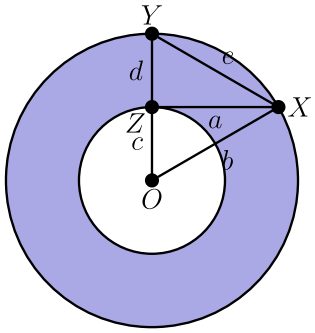
10. Suppose that a and b are nonzero real numbers, and that the equation $x^2+ax+b=0$ has solutions a and b. What is the pair (a,b)?

(A) $(-2,1)$ **(B)** $(-1,2)$ **(C)** $(1,-2)$ **(D)** $(2,-1)$ **(E)** $(4,4)$

11. The numbers from 1 to 8 are placed at the vertices of a cube in such a manner that the sum of the four numbers on each face is the same. What is this common sum?

(A) 14 **(B)** 16 **(C)** 18 **(D)** 20 **(E)** 24

12. An *annulus* is the region between two concentric circles. The concentric circles in the figure have radii b and c, with $b>c$. Let $\overline{OX}$ be a radius of the larger circle, let $\overline{XZ}$ be tangent to the smaller circle at Z, and let $\overline{OY}$ be the radius of the larger circle that contains Z. Let $a=XZ$, $d=YZ$, and $e=XY$. What is the area of the annulus?



(A) πa^2 **(B)** πb^2 **(C)** πc^2 **(D)** πd^2 **(E)** πe^2

13. How many even integers are there between 200 and 700 whose digits are all different and come from the set

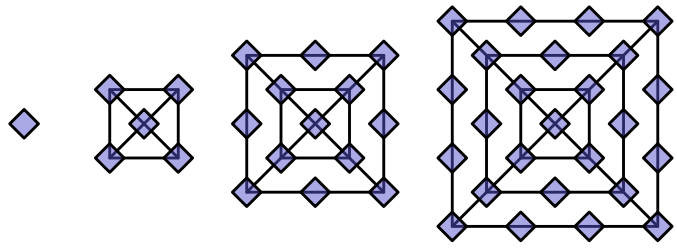
$$\{1, 2, 5, 7, 8, 9\}?$$

(A) 12 **(B)** 20 **(C)** 72 **(D)** 120 **(E)** 200

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?

(A) 26 **(B)** 27 **(C)** 36 **(D)** 37 **(E)** 41

15. The figures F_1, F_2, F_3, and F_4 shown are the first in a sequence of figures. For $n \geq 3$, F_n is constructed from F_{n-1} by surrounding it with a square and placing one more diamond on each side of the new square than F_{n-1} had on each side of its outside square. For example, figure F_3 has 13 diamonds. How many diamonds are there in figure F_{20}?



F_1 F_2 F_3 F_4

(A) 401 **(B)** 485 **(C)** 585 **(D)** 626 **(E)** 761

16. How many of the base-ten numerals for the positive integers less than or equal to 2017 contain the digit 0?

(A) 469 **(B)** 471 **(C)** 475 **(D)** 478 **(E)** 481

17. A solid cube has side length 3 inches. A 2-inch by 2-inch square hole is cut into the center of each face. The edges of each cut are parallel to the edges of the cube, and each hole goes all the way through the cube. What is the volume, in cubic inches, of the remaining solid?

(A) 7 **(B)** 8 **(C)** 10 **(D)** 12 **(E)** 15

18. Bricklayer Brenda would take 9 hours to build a chimney alone, and bricklayer Brandon would take 10 hours to build it alone. When they work together, they talk a lot, and their combined output is decreased by 10 bricks per hour. Working together, they build the chimney in 5 hours. How many bricks are in the chimney?

(A) 500 **(B)** 900 **(C)** 950 **(D)** 1000 **(E)** 1900

19. In base 10, the number 2013 ends in the digit 3. In base 9, on the other hand, the same number is written as $(2676)_9$ and ends in the digit 6. For how many positive integers b does the base-b-representation of 2013 end in the digit 3?

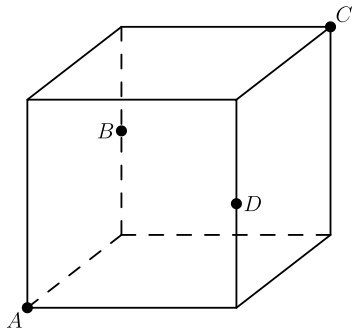
(A) 6 **(B)** 9 **(C)** 13 **(D)** 16 **(E)** 18

20. A 3×3 square is partitioned into 9 unit squares. Each unit square is painted either white or black with each color being equally likely, chosen independently and at random.

The square is then rotated 90° clockwise about its center, and every white square in a position formerly occupied by a black square is painted black. The colors of all other squares are left unchanged. What is the probability the grid is now entirely black?

(A) $\frac{49}{512}$ **(B)** $\frac{7}{64}$ **(C)** $\frac{121}{1024}$ **(D)** $\frac{81}{512}$ **(E)** $\frac{9}{32}$

21. A cube with side length 1 is sliced by a plane that passes through two diagonally opposite vertices A and C and the midpoints B and D of two opposite edges not containing A or C, as shown. What is the area of quadrilateral $ABCD$?

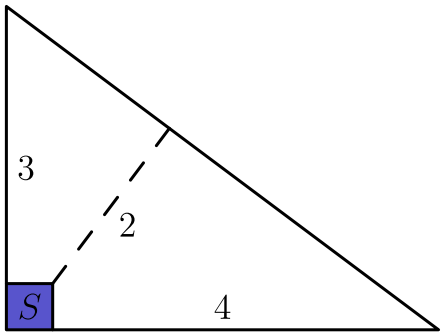


(A) $\frac{\sqrt{6}}{2}$ **(B)** $\frac{5}{4}$ **(C)** $\sqrt{2}$ **(D)** $\frac{3}{2}$ **(E)** $\sqrt{3}$

22. A player chooses one of the numbers 1 through 4. After the choice has been made, two regular four-sided (tetrahedral) dice are rolled, with the sides of the dice numbered 1 through 4. If the number chosen appears on the bottom of exactly one die after it is rolled, then the player wins \$1. If the number chosen appears on the bottom of both of the dice, then the player wins \$2. If the number chosen does not appear on the bottom of either of the dice, the player loses \$1. What is the expected return to the player, in dollars, for one roll of the dice?

(A) $-\frac{1}{8}$ **(B)** $-\frac{1}{16}$ **(C)** 0 **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{8}$

23. Farmer Pythagoras has a field in the shape of a right triangle. The right triangle's legs have lengths 3 and 4 units. In the corner where those sides meet at a right angle, he leaves a small unplanted square S so that from the air it looks like the right angle symbol. The rest of the field is planted. The shortest distance from S to the hypotenuse is 2 units. What fraction of the field is planted?



(A) $\frac{25}{27}$ **(B)** $\frac{26}{27}$ **(C)** $\frac{73}{75}$ **(D)** $\frac{145}{147}$ **(E)** $\frac{74}{75}$

24. A lattice point in an xy-coordinate system is any point (x, y) where both x and y are integers. The graph of $y = mx + 2$ passes through no lattice point with $0 < x \leq 100$ for all m such that $\frac{1}{2} < m < a$. What is the maximum possible value of a?

(A) $\frac{51}{101}$ **(B)** $\frac{50}{99}$ **(C)** $\frac{51}{100}$ **(D)** $\frac{52}{101}$ **(E)** $\frac{13}{25}$

25. Let $a > 0$, and let $P(x)$ be a polynomial with integer coefficients such that

$$P(1) = P(3) = P(5) = P(7) = a,$$

and

$$P(2) = P(4) = P(6) = P(8) = -a.$$

What is the smallest possible value of a?

(A) 105 **(B)** 315 **(C)** 945 **(D)** 7! **(E)** 8!

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**14**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**15**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**16**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**17**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**18**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**19**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**20**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**21**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**22**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**23**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**24**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**25**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ						

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 150