
AMC 10

SET 4

PRACTICE EXAMINATION

TIME ALLOWED

75 : 00

25 QUESTIONS • 75 MINUTES • 150 POINTS

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____

Date _____

Start _____

Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2019) and appear in their original wording, rearranged into a new paper. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

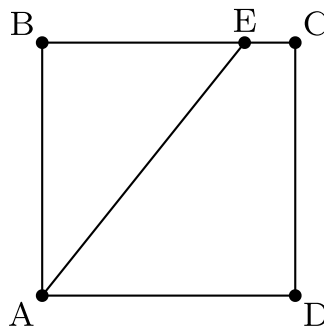
1. One can holds 12 ounces of soda. What is the minimum number of cans needed to provide a gallon (128 ounces) of soda?

(A) 7 (B) 8 (C) 9 (D) 10 (E) 11

2. What is the hundreds digit of $(20! - 15!)$?

(A) 0 (B) 1 (C) 2 (D) 4 (E) 5

3. Square $ABCD$ has side length 10. Point E is on $\overline{BC}$, and the area of $\triangle ABE$ is 40. What is BE ?



(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

4. A book that is to be recorded onto compact discs takes 412 minutes to read aloud. Each disc can hold up to 56 minutes of reading. Assume that the smallest possible number of discs is used and that each disc contains the same length of reading. How many minutes of reading will each disc contain?

(A) 50.2 (B) 51.5 (C) 52.4 (D) 53.8 (E) 55.2

5. How many subsets of $\{2, 3, 4, 5, 6, 7, 8, 9\}$ contain at least one prime number?

(A) 128 (B) 192 (C) 224 (D) 240 (E) 256

6. The average (mean) of 20 numbers is 30, and the average of 30 other numbers is 20. What is the average of all 50 numbers?

(A) 23 (B) 24 (C) 25 (D) 26 (E) 27

7. When the decimal point of a certain positive decimal number is moved four places to the right, the new number is four times the reciprocal of the original number. What is the original number?

(A) 0.0002 (B) 0.002 (C) 0.02 (D) 0.2 (E) 2

8. A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

(A) $\frac{b}{1080t}$ (B) $\frac{30t}{b}$ (C) $\frac{30b}{t}$ (D) $\frac{10t}{b}$ (E) $\frac{10b}{t}$

9. The function f is defined by

$$f(x) = \lfloor |x| \rfloor - \lfloor x \rfloor$$

for all real numbers x , where $\lfloor r \rfloor$ denotes the greatest integer less than or equal to the real number r . What is the range of f ?

(A) $\{-1, 0\}$ (B) The set of nonpositive integers
 (C) $\{-1, 0, 1\}$ (D) $\{0\}$
 (E) The set of nonnegative integers

10. Five positive consecutive integers starting with a have average b . What is the average of 5 consecutive integers that start with b ?

(A) $a + 3$ (B) $a + 4$ (C) $a + 5$ (D) $a + 6$ (E) $a + 7$

11. Which of the following describes the graph of the equation $(x + y)^2 = x^2 + y^2$?

(A) the empty set (B) one point (C) two lines
 (D) a circle (E) the entire plane

12. Three distinct integers are selected at random between 1 and 2016, inclusive. Which of the following is a correct statement about the probability p that the product of the three integers is odd?

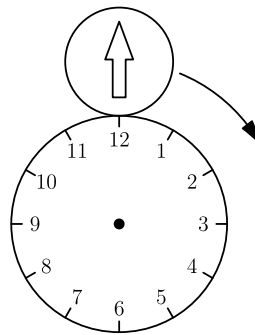
(A) $p < \frac{1}{8}$ (B) $p = \frac{1}{8}$ (C) $\frac{1}{8} < p < \frac{1}{3}$ (D) $p = \frac{1}{3}$ (E) $p > \frac{1}{3}$

13. What is the sum of all the solutions of

$$x = |2x - |60 - 2x||?$$

- (A) 32 (B) 60 (C) 92 (D) 120 (E) 124

14. The diagram below shows the circular face of a clock with radius 20 cm and a circular disk with radius 10 cm externally tangent to the clock face at 12 o'clock. The disk has an arrow painted on it, initially pointing in the upward vertical direction. Let the disk roll clockwise around the clock face. At what point on the clock face will the disk be tangent when the arrow is next pointing in the upward vertical direction?



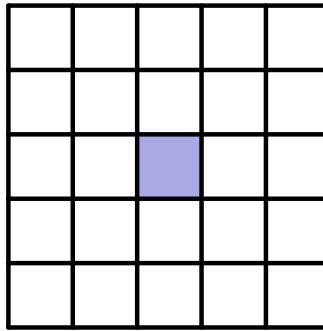
- (A) 2 o'clock (B) 3 o'clock (C) 4 o'clock (D) 6 o'clock (E) 8 o'clock

15. Two non-zero real numbers, a and b , satisfy $ab = a - b$. Find a possible value of

$$\frac{a}{b} + \frac{b}{a} - ab.$$

- (A) -2 (B) $-\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) 2

16. The 5×5 grid shown contains a collection of squares with sizes from 1×1 to 5×5 . How many of these squares contain the shaded center square?



- (A) 12 (B) 15 (C) 17 (D) 19 (E) 20
17. Jesse cuts a circular paper disk of radius 12 along two radii to form two sectors, the smaller having a central angle of 120 degrees. He makes two circular cones, using each sector to form the lateral surface of a cone. What is the ratio of the volume of the smaller cone to that of the larger?

- (A) $\frac{1}{8}$ (B) $\frac{1}{4}$ (C) $\frac{\sqrt{10}}{10}$ (D) $\frac{\sqrt{5}}{6}$ (E) $\frac{\sqrt{10}}{5}$

18. In how many ways can 345 be written as the sum of an increasing sequence of two or more consecutive positive integers?

- (A) 1 (B) 3 (C) 5 (D) 6 (E) 7

19. A number m is randomly selected from the set

$$\{11, 13, 15, 17, 19\},$$

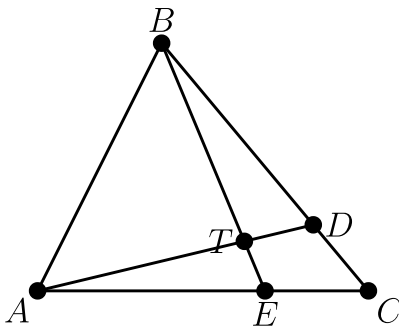
and a number n is randomly selected from

$$\{1999, 2000, 2001, \dots, 2018\}.$$

What is the probability that m^n has a units digit of 1?

- (A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{3}{10}$ (D) $\frac{7}{20}$ (E) $\frac{2}{5}$

20. In $\triangle ABC$ points D and E lie on $\overline{BC}$ and $\overline{AC}$, respectively. If $\overline{AD}$ and $\overline{BE}$ intersect at T so that $\frac{AT}{DT} = 3$ and $\frac{BT}{ET} = 4$, what is $\frac{CD}{BD}$?

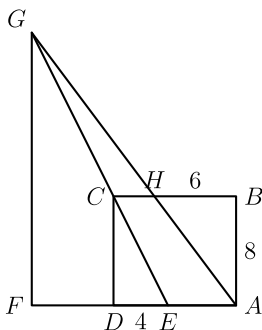


- (A) $\frac{1}{8}$ (B) $\frac{2}{9}$ (C) $\frac{3}{10}$ (D) $\frac{4}{11}$ (E) $\frac{5}{12}$

21. Let points $A = (0, 0, 0)$, $B = (1, 0, 0)$, $C = (0, 2, 0)$, and $D = (0, 0, 3)$. Points E , F , G , and H are midpoints of line segments $\overline{BD}$, $\overline{AB}$, $\overline{AC}$, and $\overline{DC}$ respectively. What is the area of $EFGH$?

- (A) $\sqrt{2}$ (B) $\frac{2\sqrt{5}}{3}$ (C) $\frac{3\sqrt{5}}{4}$ (D) $\sqrt{3}$ (E) $\frac{2\sqrt{7}}{3}$

22. In rectangle $ABCD$, we have $AB = 8$, $BC = 9$, H is on $\overline{BC}$ with $BH = 6$, E is on AD with $DE = 4$, line EC intersects line AH at G , and F is on line AD with $\overline{GF} \perp \overline{AF}$. Find the length $\overline{GF}$.



- (A) 16 (B) 20 (C) 24 (D) 28 (E) 30

- 23.** Rachel and Robert run on a circular track. Rachel runs counterclockwise and completes a lap every 90 seconds, and Robert runs clockwise and completes a lap every 80 seconds. Both start from the start line at the same time. At some random time between 10 minutes and 11 minutes after they begin to run, a photographer standing inside the track takes a picture that shows one-fourth of the track, centered on the starting line. What is the probability that both Rachel and Robert are in the picture?

(A) $\frac{1}{16}$ (B) $\frac{1}{8}$ (C) $\frac{3}{16}$ (D) $\frac{1}{4}$ (E) $\frac{5}{16}$

- 24.** For certain real numbers a , b , and c , the polynomial

$$g(x) = x^3 + ax^2 + x + 10$$

has three distinct roots, and each root of $g(x)$ is also a root of the polynomial

$$f(x) = x^4 + x^3 + bx^2 + 100x + c.$$

What is $f(1)$?

(A) -9009 (B) -8008 (C) -7007 (D) -6006 (E) -5005

- 25.** For each positive integer n , let $S(n)$ denote the sum of the digits of n . For how many values of n is $n + S(n) + S(S(n)) = 2007$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

END OF EXAMINATION

ANSWER SHEET

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Name _____

Date _____

- 1 (A) (B) (C) (D) (E)
2 (A) (B) (C) (D) (E)
3 (A) (B) (C) (D) (E)
4 (A) (B) (C) (D) (E)
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- 14 (A) (B) (C) (D) (E)
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22 (A) (B) (C) (D) (E)
23 (A) (B) (C) (D) (E)
24 (A) (B) (C) (D) (E)
25 (A) (B) (C) (D) (E)

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
×6	×1.5	×0	/ 150