
AMC 10

SET 5

PRACTICE EXAMINATION

TIME ALLOWED

75 : 00

25 QUESTIONS • 75 MINUTES • 150 POINTS

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____

Date _____

Start _____

Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2019) and appear in their original wording, rearranged into a new paper. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

(A) 48 (B) 56 (C) 64 (D) 72 (E) 80

2. For the nonzero numbers a , b , and c , define

$$(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}.$$

Find $(2, 12, 9)$.

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

3. A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?

(A) 22 (B) 24 (C) 25 (D) 26 (E) 28

4. A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?

(A) 75 (B) 76 (C) 79 (D) 84 (E) 91

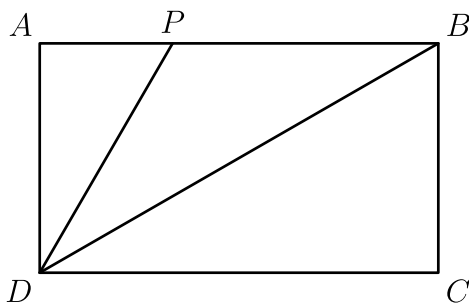
5. Triangle ABC lies in the first quadrant. Points A , B , and C are reflected across the line $y = x$ to points A' , B' , and C' , respectively. Assume that none of the vertices of the triangle lie on the line $y = x$. Which of the following statements is not always true?

(A) Triangle $A'B'C'$ lies in the first quadrant.
(B) Triangles ABC and $A'B'C'$ have the same area.
(C) The slope of line AA' is -1 .
(D) The slopes of lines AA' and CC' are the same.
(E) Lines AB and $A'B'$ are perpendicular to each other.

6. Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?

(A) 13 (B) 26 (C) 102 (D) 103 (E) 110

7. In rectangle $ABCD$, $AD = 1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



(A) $3 + \frac{\sqrt{3}}{3}$ (B) $2 + \frac{4\sqrt{3}}{3}$ (C) $2 + 2\sqrt{2}$ (D) $\frac{3 + 3\sqrt{5}}{2}$ (E) $2 + \frac{5\sqrt{3}}{3}$

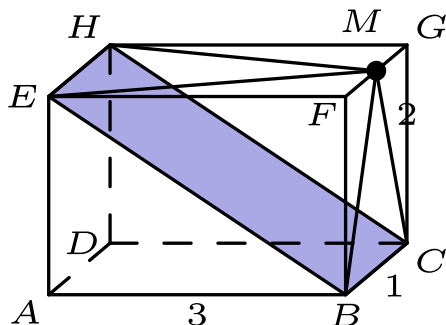
8. The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

9. The state income tax where Kristin lives is levied at the rate of $p\%$ of the first \$28000 of annual income plus $(p + 2)\%$ of any amount above \$28000. Kristin noticed that the state income tax she paid amounted to $(p + 0.25)\%$ of her annual income. What was her annual income?

(A) \$28000 (B) \$32000 (C) \$35000 (D) \$42000 (E) \$56000

10. In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of FG . What is the volume of the rectangular pyramid with base $BCHE$ and apex M ?



- (A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{2}$ (D) $\frac{5}{3}$ (E) 2
11. What is the tens digit in the sum $7! + 8! + 9! + \cdots + 2006!$?
- (A) 1 (B) 3 (C) 4 (D) 6 (E) 9
12. At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, $x\%$ of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x ?
- (A) 0 (B) 20 (C) 40 (D) 60 (E) 80
13. Jo and Blair take turns counting from 1 to one more than the last number said by the other person.
Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on.
What is the 53^{rd} number said?

- (A) 2 (B) 3 (C) 5 (D) 6 (E) 8

14. Older television screens have an aspect ratio of $4 : 3$. That is, the ratio of the width to the height is $4 : 3$. The aspect ratio of many movies is not $4 : 3$, so they are sometimes shown on a television screen by “letterboxing” — darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of $2 : 1$ and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?



- (A) 2 (B) 2.25 (C) 2.5 (D) 2.7 (E) 3
15. Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?
- (A) 0 (B) 1 (C) 2
(D) 3 (E) infinitely many
16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?
- (A) $\frac{1}{36}$ (B) $\frac{7}{72}$ (C) $\frac{1}{9}$ (D) $\frac{5}{36}$ (E) $\frac{1}{6}$
17. An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius? (Note: A cone with radius r and height h has volume $\frac{\pi r^2 h}{3}$, and a sphere with radius r has volume $\frac{4\pi r^3}{3}$.)
- (A) $2 : 1$ (B) $3 : 1$ (C) $4 : 1$ (D) $16 : 3$ (E) $6 : 1$

18. Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

(A) 32 (B) 40 (C) 48 (D) 56 (E) 64

19. Equiangular hexagon $ABCDEF$ has side lengths

$$AB = CD = EF = 1$$

and

$$BC = DE = FA = r.$$

The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r ?

(A) $\frac{4\sqrt{3}}{3}$ (B) $\frac{10}{3}$ (C) 4 (D) $\frac{17}{4}$ (E) 6

20. A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2, 2)$ to the circle of radius 3 centered at $A'(5, 6)$. What distance does the origin $O(0, 0)$, move under this transformation?

(A) 0 (B) 3 (C) $\sqrt{13}$ (D) 4 (E) 5

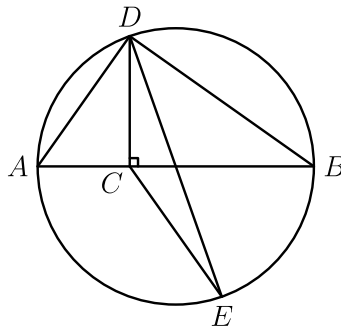
21. Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let p be the probability that all four slips bear the same number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value of $\frac{q}{p}$?

(A) 162 (B) 180 (C) 324 (D) 360 (E) 720

22. Two farmers agree that pigs are worth \$300 and that goats are worth \$210. When one farmer owes the other money, he pays the debt in pigs or goats, with “change” received in the form of goats or pigs as necessary. (For example, a \$390 debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?

(A) \$5 (B) \$10 (C) \$30 (D) \$90 (E) \$210

23. Let AB be a diameter of a circle and C be a point on AB with $2 \cdot AC = BC$. Let D and E be points on the circle such that $DC \perp AB$ and DE is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?



- (A) $\frac{1}{6}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$
24. A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and on the n th iteration, listing $n + 3$ and skipping n . The sequence begins

1, 2, 3, 4, 6, 7, 8, 9, 10, 13.

What is the 500,000th number in the sequence?

- (A) 996,506 (B) 996,507 (C) 996,508 (D) 996,509 (E) 996,510
25. All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?
- (A) 49 (B) 65 (C) 70 (D) 96 (E) 128

END OF EXAMINATION

ANSWER SHEET

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Name _____

Date _____

- 1 (A) (B) (C) (D) (E)
- 2 (A) (B) (C) (D) (E)
- 3 (A) (B) (C) (D) (E)
- 4 (A) (B) (C) (D) (E)
- 5 (A) (B) (C) (D) (E)
- 6 (A) (B) (C) (D) (E)
- 7 (A) (B) (C) (D) (E)
- 8 (A) (B) (C) (D) (E)
- 9 (A) (B) (C) (D) (E)
- 10 (A) (B) (C) (D) (E)
- 11 (A) (B) (C) (D) (E)
- 12 (A) (B) (C) (D) (E)
- 13 (A) (B) (C) (D) (E)

- 14 (A) (B) (C) (D) (E)
- 15 (A) (B) (C) (D) (E)
- 16 (A) (B) (C) (D) (E)
- 17 (A) (B) (C) (D) (E)
- 18 (A) (B) (C) (D) (E)
- 19 (A) (B) (C) (D) (E)
- 20 (A) (B) (C) (D) (E)
- 21 (A) (B) (C) (D) (E)
- 22 (A) (B) (C) (D) (E)
- 23 (A) (B) (C) (D) (E)
- 24 (A) (B) (C) (D) (E)
- 25 (A) (B) (C) (D) (E)

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
×6	×1.5	×0	/ 150