AMC 10

PRACTICE EXAMINATION

SET 5

SOLUTIONS

Answer key • Sources • Step-by-step solutions

All 25 problems are taken from past AMC 10 contests (2000–2019) and appear in their original wording, rearranged into a new paper. Problems © Mathematical Association of America.

Answer Key and Sources

No.	Ans.	Source	Topic
1	**C**	2012 AMC 10B #1	Algebra
2	**C**	2002 AMC 10A #2	Algebra
3	**B**	2007 AMC 10B #3	Algebra
4	**B**	2019 AMC 10A #4	Counting & Probability
5	**E**	2019 AMC 10B #5	Geometry
6	**D**	2016 AMC 10A #6	Number Theory
7	**B**	2000 AMC 10 #7	Geometry
8	**D**	2012 AMC 10A #8	Algebra
9	**B**	2001 AMC 10 #9	Algebra
10	**E**	2018 AMC 10B #10	Geometry
11	**C**	2006 AMC 10B #11	Number Theory
12	**D**	2010 AMC 10B #12	Algebra
13	**E**	2013 AMC 10B #13	Algebra
14	**D**	2008 AMC 10A #14	Geometry
15	**B**	2015 AMC 10A #15	Number Theory
16	**B**	2014 AMC 10B #16	Counting & Probability
17	**B**	2003 AMC 10B #17	Geometry
18	**D**	2015 AMC 10B #18	Counting & Probability
19	**E**	2010 AMC 10A #19	Geometry
20	**C**	2016 AMC 10B #20	Geometry
21	**A**	2005 AMC 10B #21	Counting & Probability
22	**C**	2006 AMC 10A #22	Number Theory
23	**C**	2005 AMC 10A #23	Geometry
24	**A**	2014 AMC 10A #24	Algebra
25	**A**	2013 AMC 10A #25	Counting & Probability

1 Algebra

Source: 2012 AMC 10B #1

Each third-grade classroom at Pearl Creek Elementary has 18 students and 2 pet rabbits. How many more students than rabbits are there in all 4 of the third-grade classrooms?

(A) 48 **(B)** 56 **(C)** 64 **(D)** 72 **(E)** 80

Key idea. Find the difference in one classroom first, then multiply by the number of classrooms.

In one classroom there are $18 - 2 = 16$ more students than rabbits.

All 4 classrooms are identical, so the total difference is $4 \times 16 = 64$.

You can also count everything first: there are $4 \times 18 = 72$ students and $4 \times 2 = 8$ rabbits, and $72 - 8 = 64$.

Watch out. Stopping at the number of students, $4 \times 18 = 72$, gives choice (D). The question asks how many *more* students than rabbits.

Answer: (C) 64

2 Algebra

Source: 2002 AMC 10A #2

For the nonzero numbers a, b, and c, define

$$(a, b, c) = \frac{a}{b} + \frac{b}{c} + \frac{c}{a}.$$

Find $(2, 12, 9)$.

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Key idea. Substitute $a = 2$, $b = 12$, $c = 9$ into the definition, simplify each fraction, then add over a common denominator.

With $a = 2$, $b = 12$, $c = 9$:

$$(2, 12, 9) = \frac{2}{12} + \frac{12}{9} + \frac{9}{2} = \frac{1}{6} + \frac{4}{3} + \frac{9}{2}.$$

The common denominator is 6:

$$\frac{1}{6} + \frac{8}{6} + \frac{27}{6} = \frac{36}{6} = 6.$$

Watch out. The order matters: each letter is divided by the *next* one, and the last fraction is $\frac{c}{a} = \frac{9}{2}$, not $\frac{a}{c}$.

Answer: (C) 6

3 Algebra

Source: 2007 AMC 10B #3

> A college student drove his compact car 120 miles home for the weekend and averaged 30 miles per gallon. On the return trip the student drove his parents' SUV and averaged only 20 miles per gallon. What was the average gas mileage, in miles per gallon, for the round trip?
>
> **(A)** 22 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 28

Key idea. Average mileage is *total miles divided by total gallons*. Find the gallons used on each leg.

Going home: 120 miles at 30 miles per gallon uses $\dfrac{120}{30} = 4$ gallons.

Coming back: 120 miles at 20 miles per gallon uses $\dfrac{120}{20} = 6$ gallons.

The round trip is 240 miles on $4 + 6 = 10$ gallons, so the average is

$$\frac{240}{10} = 24 \text{ miles per gallon.}$$

Watch out. Averaging the two rates, $\frac{30+20}{2} = 25$, gives choice (C). That is wrong because the student spends more gallons on the low-mileage (SUV) leg, so the 20 mpg leg counts more heavily.

Answer: (B) 24

4 Counting & Probability

Source: 2019 AMC 10A #4

> A box contains 28 red balls, 20 green balls, 19 yellow balls, 13 blue balls, 11 white balls, and 9 black balls. What is the minimum number of balls that must be drawn from the box without replacement to guarantee that at least 15 balls of a single color will be drawn?
>
> **(A)** 75 **(B)** 76 **(C)** 79 **(D)** 84 **(E)** 91

Key idea. Imagine the unluckiest possible draw: take as many balls as you can *without* reaching 15 of any color. One more ball then forces it.

Step 1. To avoid 15 of one color, you may take at most 14 of each color. Blue, white, and black have fewer than 15 balls, so you can take all of them: $13 + 11 + 9 = 33$ balls.

Step 2. Red, green, and yellow each have at least 15 balls, so take only 14 of each: $3 \times 14 = 42$ balls.

Step 3. That is $33 + 42 = 75$ balls with no color reaching 15. The only balls left are red, green, or yellow, so the next ball makes 15 of that color. The answer is $75 + 1 = 76$.

Watch out. Answering 75, choice (A), counts the worst case but forgets the one extra ball needed to *guarantee* the 15th.

Answer: (B) 76

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5 Geometry

Source: 2019 AMC 10B #5

> Triangle ABC lies in the first quadrant. Points A, B, and C are reflected across the line $y = x$ to points A', B', and C', respectively. Assume that none of the vertices of the triangle lie on the line $y = x$. Which of the following statements is *not* always true?
>
> **(A)** Triangle $A'B'C'$ lies in the first quadrant.
> **(B)** Triangles ABC and $A'B'C'$ have the same area.
> **(C)** The slope of line AA' is -1.
> **(D)** The slopes of lines AA' and CC' are the same.
> **(E)** Lines AB and $A'B'$ are perpendicular to each other.

Key idea. Reflecting across $y = x$ just swaps coordinates: $(x, y) \mapsto (y, x)$. Test each statement with that rule, and look for one simple counterexample.

(A) and (B). If $x > 0$ and $y > 0$, then the swapped point (y, x) also has both coordinates positive, so $A'B'C'$ stays in the first quadrant. A reflection does not change lengths, so the area is unchanged. Both are always true.

(C) and (D). For $A = (x, y)$ with $x \neq y$, the image is $A' = (y, x)$, and the slope of AA' is

$$\frac{x - y}{y - x} = -1.$$

The same holds for every vertex, so AA' and CC' both have slope -1. Both are always true.

(E). Take $A = (2, 1)$, $B = (3, 2)$, $C = (4, 1)$: all in the first quadrant, none on $y = x$. Then $A' = (1, 2)$ and $B' = (2, 3)$. Line AB has slope $\frac{2-1}{3-2} = 1$ and line $A'B'$ has slope $\frac{3-2}{2-1} = 1$. The lines are parallel, not perpendicular, so (E) fails.

In fact, reflecting in $y = x$ turns a line of slope $m \neq 0$ into a line of slope $\frac{1}{m}$. The product of the two slopes is 1, not -1, so these lines are never perpendicular. Only a horizontal AB (whose image $A'B'$ is vertical) gives perpendicular lines — so (E) is sometimes true, but not always.

Answer: (E) Lines AB and $A'B'$ are perpendicular to each other

6 Number Theory

Source: 2016 AMC 10A #6

Ximena lists the whole numbers 1 through 30 once. Emilio copies Ximena's numbers, replacing each occurrence of the digit 2 by the digit 1. Ximena adds her numbers and Emilio adds his numbers. How much larger is Ximena's sum than Emilio's?

(A) 13 **(B)** 26 **(C)** 102 **(D)** 103 **(E)** 110

Key idea. Do not add the lists. Track only how much each change of a digit 2 to a digit 1 lowers a number — and that depends on which place the 2 is in.

Units digits. Changing a units digit 2 to 1 lowers the number by 1. The numbers from 1 to 30 with units digit 2 are 2, 12, 22: a loss of 3.

Tens digits. Changing a tens digit 2 to 1 lowers the number by 10. The numbers with tens digit 2 are $20, 21, \ldots, 29$ — ten of them — for a loss of $10 \times 10 = 100$.

Altogether Emilio's sum is $3+100 = 103$ less than Ximena's. (Check: Ximena's sum is 465 and Emilio's is 362.)

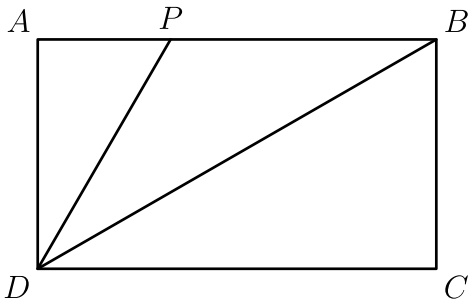
Watch out. The number 22 has *two* 2s and loses 11, not 10. Counting it only once gives 102, choice (C). Counting every changed digit as a loss of 1 gives 13, choice (A).

Answer: (D) 103

7 Geometry

Source: 2000 AMC 10 #7

In rectangle $ABCD$, $AD = 1$, P is on $\overline{AB}$, and $\overline{DB}$ and $\overline{DP}$ trisect $\angle ADC$. What is the perimeter of $\triangle BDP$?



(A) $3+\frac{\sqrt{3}}{3}$ **(B)** $2+\frac{4\sqrt{3}}{3}$ **(C)** $2+2\sqrt{2}$ **(D)** $\frac{3+3\sqrt{5}}{2}$ **(E)** $2+\frac{5\sqrt{3}}{3}$

Key idea. Trisecting a right angle gives three 30° angles, so the triangles ADP and ADB are 30–60–90 triangles with the known side $AD = 1$.

Step 1. $\angle ADC = 90^\circ$ splits into three 30° angles, so $\angle ADP = 30^\circ$ and $\angle ADB = 60^\circ$.

Step 2. In right triangle ADP (right angle at A), the angle at D is 30°, so

$$AP = \tan 30^\circ = \frac{\sqrt{3}}{3}, \qquad DP = \frac{1}{\cos 30^\circ} = \frac{2\sqrt{3}}{3}.$$

Step 3. In right triangle ADB, the angle at D is 60°, so

$$AB = \tan 60^\circ = \sqrt{3}, \qquad DB = \frac{1}{\cos 60^\circ} = 2.$$

Step 4. Then $PB = AB - AP = \sqrt{3} - \frac{\sqrt{3}}{3} = \frac{2\sqrt{3}}{3}$, and the perimeter is

$$DP + PB + DB = \frac{2\sqrt{3}}{3} + \frac{2\sqrt{3}}{3} + 2 = 2 + \frac{4\sqrt{3}}{3}.$$

Watch out. Using all of $AB = \sqrt{3}$ instead of PB gives $2 + \frac{2\sqrt{3}}{3} + \sqrt{3} = 2 + \frac{5\sqrt{3}}{3}$, choice (E). The side of $\triangle BDP$ on top is only the piece PB.

Answer: (B) $2 + \frac{4\sqrt{3}}{3}$

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8 Algebra

Source: 2012 AMC 10A #8

> The sums of three whole numbers taken in pairs are 12, 17, and 19. What is the middle number?
>
> **(A)** 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 8

Key idea. Adding all three pair sums counts each number twice, which gives the total of the three numbers at once.

Call the numbers $a < b < c$. (They are all different, because the three pair sums are different.) The smallest sum uses the two smallest numbers and the largest sum uses the two largest:

$$a + b = 12, \qquad a + c = 17, \qquad b + c = 19.$$

Adding all three equations gives $2(a + b + c) = 48$, so $a + b + c = 24$.

The middle number is the one missing from $a + c$: $b = 24 - 17 = 7$.

Check: $a = 24 - 19 = 5$ and $c = 24 - 12 = 12$, and indeed $5 + 7 = 12$, $5 + 12 = 17$, $7 + 12 = 19$.

Answer: (D) 7

9 Algebra

Source: 2001 AMC 10 #9

The state income tax where Kristin lives is levied at the rate of $p\%$ of the first \$28000 of annual income plus $(p+2)\%$ of any amount above \$28000. Kristin noticed that the state income tax she paid amounted to $(p+0.25)\%$ of her annual income. What was her annual income?

(A) \$28000 **(B)** \$32000 **(C)** \$35000 **(D)** \$42000 **(E)** \$56000

Key idea. Write the tax two ways — by the rule and by what Kristin noticed — and set them equal. The unknown rate p will cancel.

Let her income be x dollars. Her average rate $(p+0.25)\%$ is above $p\%$, so she earned more than \$28000 and $x > 28000$.

By the rule, her tax is $\frac{p}{100} \cdot 28000 + \frac{p+2}{100}(x - 28000)$. By her observation, it is $\frac{p+0.25}{100}\, x$. Multiply both by 100:

$$28000p + (p+2)(x-28000) = (p+0.25)x.$$

Expanding the left side: $28000p + px - 28000p + 2x - 56000 = px + 2x - 56000$. The right side is $px + 0.25x$. Cancel px:

$$2x - 56000 = 0.25x \quad \Longrightarrow \quad 1.75x = 56000 \quad \Longrightarrow \quad x = 32000.$$

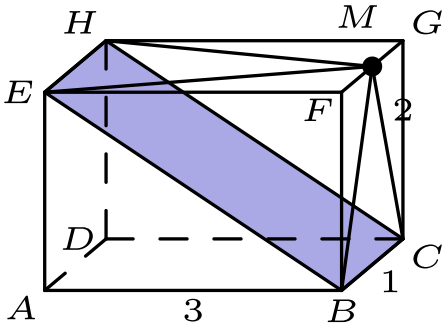
Check with any p, say $p = 5$: tax $= 1400 + 7\% \cdot 4000 = 1680$, and $5.25\% \cdot 32000 = 1680$.

Answer: (B) \$32000

10 Geometry

Source: 2018 AMC 10B #10

In the rectangular parallelepiped shown, $AB = 3$, $BC = 1$, and $CG = 2$. Point M is the midpoint of FG. What is the volume of the rectangular pyramid with base $BCHE$ and apex M?



(A) 1 **(B)** $\frac{4}{3}$ **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{3}$ **(E)** 2

Key idea. Volume $= \frac{1}{3} \cdot \text{base} \cdot \text{height}$. The base $BCHE$ is a rectangle, and because $\overline{FG}$ runs parallel to it, the height is just the distance from F to line EB in the front face.

Step 1: the base. $BCHE$ is a rectangle with sides $BC = 1$ and BE. In the front face, $\triangle EFB$ has a right angle at F with $EF = AB = 3$ and $FB = CG = 2$, so $BE = \sqrt{3^2 + 2^2} = \sqrt{13}$. The base has area $1 \cdot \sqrt{13} = \sqrt{13}$.

Step 2: the height. Segment $\overline{FG}$ is parallel to $\overline{BC}$, which lies in the base plane, so every point of $\overline{FG}$ — including M — is the same distance from that plane as F is. That distance is the altitude from F to the hypotenuse EB of right triangle EFB. Computing the area of $\triangle EFB$ two ways, $\frac{1}{2} \cdot 3 \cdot 2 = \frac{1}{2} \cdot \sqrt{13} \cdot h$, so $h = \dfrac{6}{\sqrt{13}}$.

Step 3: the volume.

$$V = \frac{1}{3} \cdot \sqrt{13} \cdot \frac{6}{\sqrt{13}} = \frac{6}{3} = 2.$$

Another way. Cut the box along plane $BCHE$. The half containing F and G is a triangular prism with volume $\frac{1}{2}(3 \cdot 1 \cdot 2) = 3$. From M (which sits on the faces $EFGH$ and $BFGC$) this prism splits into three pyramids, over the faces $BCHE$, $\triangle EFB$ and $\triangle HGC$. Each triangle has area $\frac{1}{2} \cdot 3 \cdot 2 = 3$, and M is $\frac{1}{2}$ away from each, so each of those two pyramids has volume $\frac{1}{3} \cdot 3 \cdot \frac{1}{2} = \frac{1}{2}$. The pyramid we want is $3 - \frac{1}{2} - \frac{1}{2} = 2$.

Answer: (E) 2

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11 Number Theory

Source: 2006 AMC 10B #11

What is the tens digit in the sum $7! + 8! + 9! + \cdots + 2006!$?

(A) 1 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 9

Key idea. Only the last two digits matter. Once a factorial contains both a 5 and a 10 as factors, it ends in 00 and stops affecting the tens digit — so almost all of the 2000 terms can be thrown away.

Step 1. For $n \geq 10$, the product $n! = 1 \cdot 2 \cdots n$ contains the three different factors 2, 5 and 10, whose product is 100, so it is a multiple of 100 and ends in 00. Adding such a number does not change the last two digits of the sum.

Step 2. So the last two digits of the whole sum are those of

$$7! + 8! + 9! = 5040 + 40320 + 362880 = 408240.$$

Using only the last two digits of each term: $40 + 20 + 80 = 140$, which ends in 40.

The tens digit is 4.

Watch out. $9! = 362880$ does *not* end in 00 — it has only one factor of 5. Every term from 7! to 9! must be kept.

Answer: (C) 4

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12 Algebra

Source: 2010 AMC 10B #12

At the beginning of the school year, 50% of all students in Mr. Wells' math class answered "Yes" to the question "Do you love math", and 50% answered "No." At the end of the school year, 70% answered "Yes" and 30% answered "No." Altogether, x% of the students gave a different answer at the beginning and end of the school year. What is the difference between the maximum and the minimum possible values of x?

(A) 0 **(B)** 20 **(C)** 40 **(D)** 60 **(E)** 80

Key idea. Split the class into four groups by (start answer, end answer). The given percentages fix the row and column totals, and x is the sum of the two "switch" groups — so make that as small and as large as the totals allow.

Step 1: set up. Let a% say Yes→Yes, b% Yes→No, c% No→Yes, d% No→No. Then

$$a + b = 50, \quad c + d = 50, \quad a + c = 70, \quad b + d = 30,$$

and $x = b + c$. From the first and third equations, $c = 70 - a$ and $b = 50 - a$, so $x = 120 - 2a$.

Step 2: how big can a be? a is at most 50 (only 50% started with Yes). With $a = 50$: $b = 0$, $c = 20$, $d = 30$ — all allowed. So the minimum is $x = 120 - 100 = 20$: just the 20% needed to raise Yes from 50 to 70 switch.

Step 3: how small can a be? $c = 70 - a$ cannot exceed 50 (only 50% started with No), so $a \geq 20$. With $a = 20$: $b = 30$, $c = 50$, $d = 0$ — all allowed. So the maximum is $x = 120 - 40 = 80$: every No switches to Yes, and $b = 30$ students switch from Yes to No — that is 30% of the class, which is 60% of the students who started with Yes.

The difference is $80 - 20 = 60$.

Watch out. The question asks for the *difference*. Stopping at the maximum 80 gives choice (E), and stopping at the minimum 20 gives choice (B).

Answer: (D) 60

13 Algebra

Source: 2013 AMC 10B #13

Jo and Blair take turns counting from 1 to one more than the last number said by the other person.

Jo starts by saying "1", so Blair follows by saying "1, 2". Jo then says "1, 2, 3", and so on.

What is the 53^{rd} number said?

(A) 2 **(B)** 3 **(C)** 5 **(D)** 6 **(E)** 8

Key idea. Each turn is one number longer than the one before, so the turn lengths are $1, 2, 3, \ldots$ The running totals are triangular numbers — find the last turn that ends before the 53rd number.

Step 1: how the counting goes. The turns are 1; then $1, 2$; then $1, 2, 3$; and so on. The kth turn is the list $1, 2, \ldots, k$, which has k numbers. Who is speaking does not matter — only the order of the numbers.

Step 2: count numbers said. After n turns the total number of numbers said is

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

For $n = 9$ this is $\frac{9 \cdot 10}{2} = 45$; for $n = 10$ it is 55. Since $45 < 53 \leq 55$, the 53rd number falls in the 10th turn.

Step 3: locate it. The 10th turn is $1, 2, \ldots, 10$, and its first number is the 46th number said overall. The 53rd number is therefore entry number $53 - 45 = 8$ of that turn, which is 8.

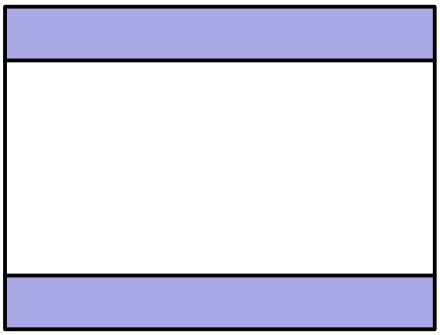
Check from the other end: the 10th turn ends with "10" as the 55th number, so the 54th is 9 and the 53rd is 8.

Answer: (E) 8

14 Geometry

Source: 2008 AMC 10A #14

Older television screens have an aspect ratio of $4:3$. That is, the ratio of the width to the height is $4:3$. The aspect ratio of many movies is not $4:3$, so they are sometimes shown on a television screen by "letterboxing" — darkening strips of equal height at the top and bottom of the screen, as shown. Suppose a movie has an aspect ratio of $2:1$ and is shown on an older television screen with a 27-inch diagonal. What is the height, in inches, of each darkened strip?



(A) 2 **(B)** 2.25 **(C)** 2.5 **(D)** 2.7 **(E)** 3

Key idea. A $4:3$ rectangle has width, height and diagonal in the ratio $4:3:5$ (a 3–4–5 right triangle), so the 27-inch diagonal gives the screen's width and height at once. The movie uses the full width.

Step 1: the screen. Width : height : diagonal $= 4:3:5$, and the diagonal is 27, so one "part" is $\frac{27}{5} = 5.4$ inches. The width is $4 \times 5.4 = 21.6$ inches and the height is $3 \times 5.4 = 16.2$ inches.

Step 2: the movie. The movie is wider than the screen's shape, so it fills the full width of 21.6 inches. With aspect ratio $2:1$, its height is $\frac{21.6}{2} = 10.8$ inches.

Step 3: the strips. The unused height is $16.2 - 10.8 = 5.4$ inches, shared equally by the top and bottom strips. Each strip is $\frac{5.4}{2} = 2.7$ inches high.

Watch out. 5.4 inches is the *total* dark height. The problem asks for each strip, so it must be halved.

Answer: (D) 2.7

15 Number Theory

Source: 2015 AMC 10A #15

Consider the set of all fractions $\frac{x}{y}$, where x and y are relatively prime positive integers. How many of these fractions have the property that if both numerator and denominator are increased by 1, the value of the fraction is increased by 10%?

(A) 0 **(B)** 1 **(C)** 2

(D) 3 **(E)** infinitely many

Key idea. Translate the condition into an equation in x and y. Clearing fractions leaves an equation with an xy term plus x and y terms — the signal to factor it as a product of two brackets equal to a constant, which leaves only finitely many cases.

Step 1: the equation. "Increased by 10%" means multiplied by $\frac{11}{10}$:

$$\frac{x+1}{y+1} = \frac{11}{10} \cdot \frac{x}{y}.$$

Cross-multiplying: $10y(x+1) = 11x(y+1)$, so $10xy + 10y = 11xy + 11x$, which rearranges to

$$xy + 11x - 10y = 0.$$

Step 2: factor. Since $(x-10)(y+11) = xy + 11x - 10y - 110$, the equation becomes

$$(x-10)(y+11) = -110.$$

Step 3: the cases. $y+11$ is a positive integer greater than 11, so $x-10$ must be negative: $1 \le x \le 9$. Then $y+11$ is a divisor of 110 greater than 11, namely 22, 55 or 110:

- $y+11 = 22$: $x-10 = -5$, giving $\frac{5}{11}$;
- $y+11 = 55$: $x-10 = -2$, giving $\frac{8}{44}$;
- $y+11 = 110$: $x-10 = -1$, giving $\frac{9}{99}$.

Step 4: relatively prime. $\frac{8}{44}$ and $\frac{9}{99}$ are not in lowest terms (gcd = 4 and 9). Only $\frac{5}{11}$ counts. Check: $\frac{6}{12} = \frac{1}{2}$ and $\frac{11}{10} \cdot \frac{5}{11} = \frac{1}{2}$.

Watch out. Forgetting that x and y must be relatively prime counts all three pairs and gives 3, choice (D).

Answer: (B) 1

16 Counting & Probability

Source: 2014 AMC 10B #16

Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

(A) $\frac{1}{36}$ **(B)** $\frac{7}{72}$ **(C)** $\frac{1}{9}$ **(D)** $\frac{5}{36}$ **(E)** $\frac{1}{6}$

Key idea. "At least three" splits cleanly into two cases that cannot overlap: *exactly* three dice match, or *all four* match. Count ordered outcomes in each case and divide by 6^4.

Step 1. The dice are distinguishable (think of them as red, blue, green, white), so there are $6^4 = 1296$ equally likely ordered outcomes.

Step 2: exactly three match. Build such an outcome in three choices: the repeated value (6 ways), the different value (5 ways, since it must differ from the repeated one), and which of the four dice shows the different value (4 ways). That is $6 \cdot 5 \cdot 4 = 120$ outcomes.

Step 3: all four match. The common value can be any of 6, so there are 6 outcomes.

Step 4. The two cases are disjoint, so

$$P = \frac{120 + 6}{1296} = \frac{126}{1296} = \frac{7}{72}.$$

Watch out. Forgetting to choose *which* die is the odd one out counts only $6 \cdot 5 = 30$ outcomes in Step 2; then $\frac{30+6}{1296} = \frac{1}{36}$, choice (A). Going the other way — "pick three dice to match (4 ways) and let the fourth be anything (6 ways)" — gives $6 \cdot 4 \cdot 6 = 144$, which counts each all-four-equal roll four times; $\frac{144}{1296} = \frac{1}{9}$ is choice (C).

Answer: (B) $\frac{7}{72}$

17 Geometry

Source: 2003 AMC 10B #17

An ice cream cone consists of a sphere of vanilla ice cream and a right circular cone that has the same diameter as the sphere. If the ice cream melts, it will exactly fill the cone. Assume that the melted ice cream occupies 75% of the volume of the frozen ice cream. What is the ratio of the cone's height to its radius? (Note: A cone with radius r and height h has volume $\frac{\pi r^2 h}{3}$, and a sphere with radius r has volume $\frac{4\pi r^3}{3}$.)

(A) 2 : 1 **(B)** 3 : 1 **(C)** 4 : 1 **(D)** 16 : 3 **(E)** 6 : 1

Key idea. "Exactly fill the cone" is an equation between two volumes. Because the cone and the sphere share a diameter, they share a radius r, so the equation has only one other unknown, h, and solving for it gives the ratio $h : r$ directly.

Step 1. Let r be the common radius. The frozen sphere has volume $\frac{4}{3}\pi r^3$. Melting shrinks it to $75\% = \frac{3}{4}$ of that:

$$\frac{3}{4} \cdot \frac{4}{3}\pi r^3 = \pi r^3.$$

Step 2. This melted volume exactly fills the cone of radius r and height h:

$$\pi r^3 = \frac{1}{3}\pi r^2 h.$$

Step 3. Divide both sides by πr^2 (which is not zero): $r = \frac{1}{3}h$, so $h = 3r$. The ratio of height to radius is $3 : 1$.

Watch out. Ignoring the 75% and setting $\frac{4}{3}\pi r^3 = \frac{1}{3}\pi r^2 h$ gives $h = 4r$, choice (C). Applying the 75% backwards — dividing by $\frac{3}{4}$ instead of multiplying — gives $\frac{16}{9}\pi r^3 = \frac{1}{3}\pi r^2 h$, so $h = \frac{16}{3}r$, choice (D).

Answer: (B) $3 : 1$

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18 Counting & Probability

Source: 2015 AMC 10B #18

Johann has 64 fair coins. He flips all the coins. Any coin that lands on tails is tossed again. Coins that land on tails on the second toss are tossed a third time. What is the expected number of coins that are now heads?

(A) 32 **(B)** 40 **(C)** 48 **(D)** 56 **(E)** 64

Key idea. Every coin is treated by the same rule, independently of the others, so find the probability that *one* coin ends heads and multiply by 64 (linearity of expectation). For one coin the complement is simplest: it ends tails only if *every* toss it gets is tails.

Step 1. Follow a single coin. It is tossed again only after a tail, and it stops at most after the third toss. So it ends showing tails exactly when its first, second and third tosses are all tails, which has probability

$$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{8}.$$

In every other case it has landed heads at some point and stopped there, so it ends heads with probability $1 - \frac{1}{8} = \frac{7}{8}$.

Step 2. The expected number of heads is the sum, over the 64 coins, of each coin's probability of ending heads:

$$64 \cdot \frac{7}{8} = 56.$$

Watch out. Stopping after the second toss — so that a coin ends tails with probability $\frac{1}{4}$ — gives $64 \cdot \frac{3}{4} = 48$, choice (C). Read carefully: there are *three* chances to get heads.

Another way. Track the expected counts round by round. On average, half of the 64 coins, 32, land heads on the first toss and 32 are tossed again. Half of those, 16, land heads and 16 are tossed a third time; half of those, 8, land heads. The expected total is $32 + 16 + 8 = 56$.

Answer: (D) 56

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19 Geometry

Source: 2010 AMC 10A #19

Equiangular hexagon $ABCDEF$ has side lengths

$$AB = CD = EF = 1$$

and

$$BC = DE = FA = r.$$

The area of $\triangle ACE$ is 70% of the area of the hexagon. What is the sum of all possible values of r?

(A) $\frac{4\sqrt{3}}{3}$ **(B)** $\frac{10}{3}$ **(C)** 4 **(D)** $\frac{17}{4}$ **(E)** 6

Key idea. Cutting the hexagon along $\overline{AC}$, $\overline{CE}$, $\overline{EA}$ leaves $\triangle ACE$ plus three corner triangles, and each corner triangle has sides 1 and r around a 120° angle. So both areas can be written in terms of r alone. The phrase "sum of all possible values" warns that the 70% condition will be a quadratic in r — and then Vieta gives the sum without solving.

Step 1: the corner triangles. An equiangular hexagon has every interior angle equal to $\frac{(6-2)\cdot 180^\circ}{6} = 120^\circ$. Triangles ABC, CDE, EFA each have two sides 1 and r with a 120° angle between them, so they are congruent (SAS). Each has area

$$\frac{1}{2} \cdot 1 \cdot r \cdot \sin 120^\circ = \frac{\sqrt{3}}{4} r.$$

Step 2: $\triangle ACE$. Since those three triangles are congruent, $AC = CE = EA$, so $\triangle ACE$ is equilateral. By the Law of Cosines in $\triangle ABC$, using $\cos 120^\circ = -\frac{1}{2}$,

$$AC^2 = 1^2 + r^2 - 2 \cdot 1 \cdot r \cdot \left(-\tfrac{1}{2}\right) = r^2 + r + 1,$$

so $[ACE] = \frac{\sqrt{3}}{4} AC^2 = \frac{\sqrt{3}}{4}(r^2 + r + 1)$.

Step 3: the hexagon. Adding the three corner triangles,

$$[ABCDEF] = \frac{\sqrt{3}}{4}(r^2 + r + 1) + 3 \cdot \frac{\sqrt{3}}{4} r = \frac{\sqrt{3}}{4}(r^2 + 4r + 1).$$

Step 4: the condition. $[ACE] = \frac{7}{10}[ABCDEF]$; cancelling $\frac{\sqrt{3}}{4}$ and multiplying by 10:

$$10(r^2 + r + 1) = 7(r^2 + 4r + 1) \quad \Longrightarrow \quad 3r^2 - 18r + 3 = 0 \quad \Longrightarrow \quad r^2 - 6r + 1 = 0.$$

Step 5. The discriminant is $36 - 4 = 32 > 0$, so there are two real roots, $r = 3 \pm 2\sqrt{2}$. Both are positive ($2\sqrt{2} \approx 2.83 < 3$), so both give genuine hexagons, and each really does make the ratio 70%. By Vieta the sum of the roots is 6.

Watch out. Do not stop at one root. Taking only the larger value $3+2\sqrt{2} \approx 5.83$ matches no choice; the question asks for the *sum* of all possible values, and the small root $3-2\sqrt{2} \approx 0.17$ is just as valid.

Another way. For the hexagon area, extend sides $\overline{AB}$, $\overline{CD}$, $\overline{EF}$. They meet in an equilateral triangle, and each cut-off corner is an equilateral triangle of side r (its angles are all $60°$). The big triangle has side $r+1+r = 2r+1$, so $[ABCDEF] = \frac{\sqrt{3}}{4}\big((2r+1)^2 - 3r^2\big) = \frac{\sqrt{3}}{4}(r^2+4r+1)$, the same as Step 3.

Answer: (E) 6

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20 Geometry

Source: 2016 AMC 10B #20

A dilation of the plane—that is, a size transformation with a positive scale factor—sends the circle of radius 2 centered at $A(2,2)$ to the circle of radius 3 centered at $A'(5,6)$. What distance does the origin $O(0,0)$, move under this transformation?

(A) 0 **(B)** 3 **(C)** $\sqrt{13}$ **(D)** 4 **(E)** 5

Key idea. A dilation is pinned down by two things: its scale factor and its center. The radii $2 \to 3$ hand you the scale factor $k = \frac{3}{2}$, and a dilation sends the center of a circle to the center of the image circle, so $A \mapsto A'$ locates the center. Once you know the dilation, just compute where O goes.

Step 1: scale factor. Every length is multiplied by the same factor, and the radius goes from 2 to 3, so $k = \frac{3}{2}$.

Step 2: center. A dilation with center C and factor k sends a point P to $C + k(P-C)$. It must send $A = (2,2)$ to $A' = (5,6)$:

$$C + \frac{3}{2}\big((2,2) - C\big) = (5,6) \quad\Longrightarrow\quad (3,3) - \frac{1}{2}C = (5,6) \quad\Longrightarrow\quad C = (-4,-6).$$

(Check: C, A, A' are collinear, with $CA = \sqrt{6^2+8^2} = 10$ and $CA' = \sqrt{9^2+12^2} = 15 = \frac{3}{2}\cdot 10$.)

Step 3: image of the origin. $O = (0,0)$ goes to

$$C + \frac{3}{2}(O - C) = -\frac{1}{2}C = (2,3).$$

Step 4. The origin moves from $(0,0)$ to $(2,3)$, a distance $\sqrt{2^2+3^2} = \sqrt{13}$.

Watch out. A dilation is not a translation: points do not all move the same distance. Assuming O moves as far as A does gives $AA' = \sqrt{3^2+4^2} = 5$, choice (E). A point moves $(k-1)$ times its distance from the center, so points near C barely move while far ones move a lot.

Another way. Any dilation with factor k has the form $P \mapsto kP + b$ for a fixed vector b. From $A \mapsto A'$: $(5,6) = \frac{3}{2}(2,2) + b$, so $b = (5,6) - (3,3) = (2,3)$. The origin goes to $\frac{3}{2}(0,0) + b = (2,3)$, and the distance moved is $\sqrt{13}$ — no center needed.

Answer: (C) $\sqrt{13}$

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21 Counting & Probability

Source: 2005 AMC 10B #21

> Forty slips are placed into a hat, each bearing a number 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10, with each number entered on four slips. Four slips are drawn from the hat at random and without replacement. Let p be the probability that all four slips bear the same number. Let q be the probability that two of the slips bear a number a and the other two bear a number $b \neq a$. What is the value of $\frac{q}{p}$?
>
> **(A)** 162 **(B)** 180 **(C)** 324 **(D)** 360 **(E)** 720

Key idea. The question asks for a *ratio* of two probabilities. Both are "favourable draws ÷ all possible draws", and the denominator is the same $\binom{40}{4}$ for both, so it cancels: $\dfrac{q}{p}$ is simply the ratio of the two favourable counts. No large numbers are needed.

Step 1: the common denominator. Treat the 40 slips as distinct (each has its own physical identity) and count unordered sets of 4 slips. Every set of 4 is equally likely, and there are $\binom{40}{4}$ of them. So

$$p = \frac{N_p}{\binom{40}{4}}, \qquad q = \frac{N_q}{\binom{40}{4}}, \qquad \frac{q}{p} = \frac{N_q}{N_p}.$$

Step 2: count N_p. All four slips bear the same number. Choose the number (10 ways); then we must take all four of its slips, which can be done in $\binom{4}{4} = 1$ way. So $N_p = 10$.

Step 3: count N_q. Two slips show a and two show $b \neq a$. First choose the *pair* of numbers $\{a, b\}$: $\binom{10}{2} = 45$ ways. (It is an unordered pair: "two 3's and two 7's" is the same draw as "two 7's and two 3's".) Then choose which two of the four a-slips are drawn, $\binom{4}{2} = 6$ ways, and which two of the four b-slips, another 6 ways:

$$N_q = 45 \cdot 6 \cdot 6 = 1620.$$

Step 4. $\dfrac{q}{p} = \dfrac{1620}{10} = 162.$

Watch out. Choosing a first (10 ways) and then b (9 ways) counts each pair of numbers twice, once as (a, b) and once as (b, a). That gives $90 \cdot 36 = 3240$ and the ratio 324, choice (C). Since the event only says "two show one number and two show another", the order of a and b does not matter.

Another way. Draw the slips one at a time. The first slip can be anything. For p, the next three must match it: $p = \frac{3}{39} \cdot \frac{2}{38} \cdot \frac{1}{37}$. For q, the four positions split into two matching pairs, and the pair containing the first slip is $\{1,2\}$, $\{1,3\}$ or $\{1,4\}$ — 3 patterns. For the pattern $\{1,2\},\{3,4\}$: slip 2 matches slip

1 ($\frac{3}{39}$), slip 3 is a new number ($\frac{36}{38}$), slip 4 matches slip 3 ($\frac{3}{37}$). The other two patterns have the same probability, so

$$\frac{q}{p} = \frac{3 \cdot 3 \cdot 36 \cdot 3}{3 \cdot 2 \cdot 1} = \frac{972}{6} = 162.$$

Answer: (A) 162

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22 Number Theory

Source: 2006 AMC 10A #22

> Two farmers agree that pigs are worth \$300 and that goats are worth \$210. When one farmer owes the other money, he pays the debt in pigs or goats, with "change" received in the form of goats or pigs as necessary. (For example, a \$390 debt could be paid with two pigs, with one goat received in change.) What is the amount of the smallest positive debt that can be resolved in this way?
>
> **(A)** \$5 **(B)** \$10 **(C)** \$30 **(D)** \$90 **(E)** \$210

Key idea. Animals handed over count as positive, animals received as change count as negative, so a resolvable debt is exactly a number $300p+210g$ with p, g integers (of either sign). Two observations finish it: every such number is a multiple of $\gcd(300, 210)$, and running the Euclidean algorithm on 300 and 210 shows how to actually reach that gcd.

Step 1: what can be paid. If the debtor hands over p pigs and g goats, where a negative number means animals received in change, the net amount transferred is

$$D = 300p + 210g.$$

The example in the problem is $p = 2$, $g = -1$: $600 - 210 = 390$.

Step 2: a lower bound. $300 = 30 \cdot 10$ and $210 = 30 \cdot 7$, so $D = 30(10p + 7g)$ is always a multiple of 30. A positive multiple of 30 is at least 30, so no positive debt smaller than \$30 can be resolved. In particular \$5 and \$10 are impossible.

Step 3: \$30 is achievable. Imitate the Euclidean algorithm:

$$300 - 210 = 90, \qquad 210 - 2 \cdot 90 = 30.$$

Substituting the first into the second, $30 = 210 - 2(300 - 210) = 3 \cdot 210 - 2 \cdot 300$. So a \$30 debt is paid by giving 3 goats (\$630) and receiving 2 pigs (\$600) in change.

Step 4. Both halves are in place: every resolvable debt is a multiple of 30, and 30 itself works. The smallest positive debt is \$30.

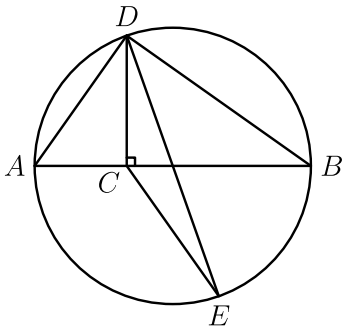
Watch out. The most obvious trade — give one pig, receive one goat in change — settles $300-210 =$ \$90, choice (D). It is a resolvable debt, but not the smallest: using more animals can bring the net amount down to \$30.

Answer: (C) \$30

23 Geometry

Source: 2005 AMC 10A #23

Let AB be a diameter of a circle and C be a point on AB with $2 \cdot AC = BC$. Let D and E be points on the circle such that $DC \perp AB$ and DE is a second diameter. What is the ratio of the area of $\triangle DCE$ to the area of $\triangle ABD$?



(A) $\frac{1}{6}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{2}{3}$

Key idea. The segment DC is the height of $\triangle ABD$, so compare everything to DC. Two features of the picture suggest how: the center O lies on AB right next to C, and O is the *midpoint* of the diameter DE — so CO is a median of $\triangle DCE$, which splits it into two equal halves, and the half $\triangle DCO$ shares the height DC with $\triangle ABD$.

Step 1: locate C. Let O be the center. From $AC + BC = AB$ and $BC = 2 \cdot AC$ we get $3 \cdot AC = AB$, so $AC = \frac{1}{3}AB$. The center is at $AO = \frac{1}{2}AB$, so

$$CO = \frac{1}{2}AB - \frac{1}{3}AB = \frac{1}{6}AB.$$

Step 2: compare $\triangle DCO$ with $\triangle ABD$. Both triangles have their base on line AB and the same height DC (because $DC \perp AB$):

$$[DCO] = \frac{1}{2} \cdot CO \cdot DC, \qquad [ABD] = \frac{1}{2} \cdot AB \cdot DC,$$

so $[DCO] = \frac{CO}{AB}[ABD] = \frac{1}{6}[ABD]$.

Step 3: from $\triangle DCO$ to $\triangle DCE$. O is the midpoint of DE, so $\overline{CO}$ is a median of $\triangle DCE$. A median cuts a triangle into two triangles with equal bases ($DO = OE$) and the same height from C, hence equal areas: $[DCE] = 2[DCO]$.

Step 4. Combining, $[DCE] = 2 \cdot \frac{1}{6}[ABD] = \frac{1}{3}[ABD]$, so the ratio is $\frac{1}{3}$. (Whether D is above or below AB does not matter; the two pictures are mirror images.)

Watch out. Stopping at Step 2 gives $[DCO] : [ABD] = \frac{1}{6}$, choice (A). The triangle asked about is $\triangle DCE$, which reaches all the way to E on the far side of the circle — twice as large as $\triangle DCO$.

Another way. Use coordinates with radius 3: $A = (-3, 0)$, $B = (3, 0)$, $O = (0, 0)$, and $AC = 2$ gives $C = (-1, 0)$. Then $D = (-1, \sqrt{8})$ lies on the circle since $1 + 8 = 9$, and $E = -D = (1, -\sqrt{8})$. $[ABD] = \frac{1}{2} \cdot 6 \cdot \sqrt{8} = 3\sqrt{8}$. In $\triangle DCE$ take the vertical side DC (length $\sqrt{8}$) as base; E is at horizontal distance $1 - (-1) = 2$ from it, so $[DCE] = \frac{1}{2} \cdot \sqrt{8} \cdot 2 = \sqrt{8}$. The ratio is $\frac{\sqrt{8}}{3\sqrt{8}} = \frac{1}{3}$.

Answer: (C) $\frac{1}{3}$

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24 Algebra

Source: 2014 AMC 10A #24

A sequence of natural numbers is constructed by listing the first 4, then skipping one, listing the next 5, skipping 2, listing 6, skipping 3, and on the nth iteration, listing $n+3$ and skipping n. The sequence begins

$$1, 2, 3, 4, 6, 7, 8, 9, 10, 13.$$

What is the 500,000th number in the sequence?

(A) 996,506 **(B)** 996,507 **(C)** 996,508 **(D)** 996,509 **(E)** 996,510

Key idea. Keep two running totals after n complete iterations: how many numbers have been *listed*, and how many natural numbers have been *used up* (listed or skipped). The listed count tells you which iteration contains term 500,000; the used-up count tells you where that iteration starts. The answer choices differ by 1, which is a loud warning that the last step needs exact counting, not estimation.

Step 1: the two totals. Iteration k lists $k + 3$ numbers and skips k, so it uses $2k + 3$ numbers. After n iterations:

$$\text{listed} = \sum_{k=1}^{n} (k+3) = \frac{n(n+1)}{2} + 3n = \frac{n(n+7)}{2}, \qquad \text{used} = \sum_{k=1}^{n} (2k+3) = n(n+1) + 3n = n^2 + 4n.$$

Check with $n = 2$: listed $= 9$ and used $= 12$, matching 1–4, skip 5, 6–10, skip 11, 12.

Step 2: which iteration? We need the largest n with $\frac{n(n+7)}{2} < 500{,}000$, i.e. $n(n+7) < 1{,}000{,}000$. Since $n(n+7) \approx n^2$, n is a little under 1000. Testing:

$$996 \cdot 1003 = 998{,}988 < 10^6, \qquad 997 \cdot 1004 = 1{,}000{,}988 > 10^6.$$

So after 996 full iterations exactly $\frac{998{,}988}{2} = 499{,}494$ numbers are listed, and term 500,000 comes during iteration 997.

Step 3: is it in the listed part? Iteration 997 lists $997 + 3 = 1000$ numbers before it skips any. We need the $500{,}000 - 499{,}494 = 506$th of them, and $506 \le 1000$, so it is a listed number of that block, not lost in a skip.

Step 4: where the block starts. After 996 iterations the numbers used are $996^2 + 4 \cdot 996 = 992{,}016 + 3{,}984 = 996{,}000$. So iteration 997 lists 996,001, 996,002, … consecutively.

Step 5. The 506th number of a run starting at 996,001 is $996{,}001 + 505 = 996{,}506$.

Watch out. The off-by-one: the 1st number of the block is $996{,}001+0$, so the 506th is $996{,}001+505$. Adding 506 instead gives 996,507, choice (B).

Answer: (A) 996,506

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25 Counting & Probability

Source: 2013 AMC 10A #25

All 20 diagonals are drawn in a regular octagon. At how many distinct points in the interior of the octagon (not on the boundary) do two or more diagonals intersect?

(A) 49 **(B)** 65 **(C)** 70 **(D)** 96 **(E)** 128

Key idea. Every crossing comes from choosing 4 vertices, which gives the "generic" count $\binom{8}{4} = 70$. But the word *distinct* and the high symmetry of a regular octagon warn that several diagonals can pass through one point — the center obviously, and maybe others. So: count 70, find *every* point where three or more diagonals meet, and remove the overcount.

Step 1: crossing pairs. Two diagonals with a common endpoint meet only at that vertex, on the boundary. Two diagonals with four different endpoints cross inside exactly when their endpoints alternate around the octagon. Of the three ways to pair up 4 chosen vertices, exactly one alternates (the two diagonals of their quadrilateral). So there are $\binom{8}{4} = 70$ crossing pairs, and 70 would be the answer if no three diagonals ever met at one point.

Step 2: coordinates and three kinds of diagonal. Let $a = 1+\sqrt{2}$ and take the regular octagon (side 2) with vertices

$$V_0 = (a,1), \quad V_1 = (1,a), \quad V_2 = (-1,a), \quad V_3 = (-a,1),$$
$$V_4 = (-a,-1), \quad V_5 = (-1,-a), \quad V_6 = (1,-a), \quad V_7 = (a,-1).$$

Up to rotation, a diagonal is *short* (like V_7V_1, 8 of them), *medium* (like V_0V_3, 8), or a *diameter* (like V_0V_4, 4). Call a point *crowded* if three or more diagonals pass through it. A crowded point lies on some diagonal ℓ that two other diagonals cross at that same point, so we check each kind of ℓ.

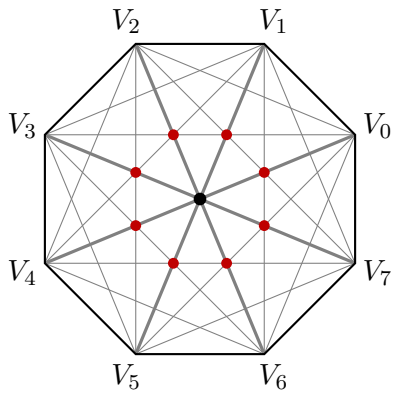
Step 3: short diagonals. V_7V_1 cuts off only V_0, so the diagonals crossing it are the 5 from V_0 to $V_2, \ldots, V_6$. Any two of these meet only at V_0, so they cross V_7V_1 at 5 different points. No crowded point lies on a short diagonal.

Step 4: medium diagonals. V_0V_3 is the line $y = 1$. It is crossed by the 8 diagonals from V_1 or V_2 to $V_4, \ldots, V_7$. Here is where each one meets $y = 1$ (the x-coordinate):

	to V_4	to V_5	to V_6	to V_7
from V_1	$1-\sqrt{2}$	$\sqrt{2}-1$	1	$3-\sqrt{2}$
from V_2	$\sqrt{2}-3$	-1	$1-\sqrt{2}$	$\sqrt{2}-1$

For instance V_1V_6 is $x = 1$; V_1V_4 is $y = x+\sqrt{2}$, giving $x = 1-\sqrt{2}$; the diameter V_1V_5 is $y = ax$, giving $x = \frac{1}{a} = \sqrt{2}-1$; the second row is the mirror image of the first in the y-axis. Exactly two values repeat, $x = \pm(\sqrt{2}-1)$, and those two points lie on the diameters V_2V_6 and V_1V_5. So every crowded point lies on a diameter.

Step 5: diameters. V_0V_4 is $y = (\sqrt{2}-1)x$. The diagonals crossing it join one of V_1, V_2, V_3 to one of V_5, V_6, V_7. Three are diameters, meeting it at the center O. Of the other six, V_1V_6 ($x = 1$) and V_2V_7 ($x + y = \sqrt{2}$) both meet it at $(1, \sqrt{2}-1)$; V_2V_5 and V_3V_6 both meet it at $(-1, 1-\sqrt{2})$. Finally V_1V_7 and V_3V_5 each meet it alone, at $x = \pm\frac{2+\sqrt{2}}{2}$. So each diameter carries the center (on 4 diagonals) and exactly two other crowded points, each on exactly 3 diagonals. Two diameters share only O, so these are $4 \cdot 2 = 8$ different points (red below).



Step 6: correct the count. A point on m diagonals was counted $\binom{m}{2}$ times among the 70 pairs. The center was counted $\binom{4}{2} = 6$ times and each red point $\binom{3}{2} = 3$ times, but each should count once:

$$70 - (6-1) - 8\,(3-1) = 70 - 5 - 16 = 49.$$

Watch out. Assuming no three diagonals are concurrent gives $\binom{8}{4} = 70$, choice (C). Spotting the center but missing the eight triple points gives $70 - 5 = 65$, choice (B).

Answer: (A) 49