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# AMC 10

# SET 2

HALF-LENGTH PRACTICE

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TIME ALLOWED

# 30 : 00

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10 QUESTIONS • 30 MINUTES • 60 POINTS • MIDDLE BAND (PROBLEMS 10--20)

**DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.**

Name \_\_\_\_\_ Date \_\_\_\_\_ Start \_\_\_\_\_ Finish \_\_\_\_\_

All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

## INSTRUCTIONS

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1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 10 to 20 of an AMC 10 — the middle band of the paper, where scores are decided. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

### Pacing guide

Three minutes per problem is the pace of the real test in this band: problems 1–5 in about 13 minutes, problems 6–10 in the remaining 17. If a problem has taken three minutes with no plan in sight, move on and come back.

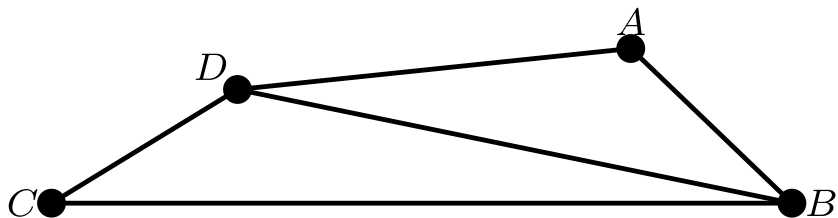
1. Shelby drives her scooter at a speed of 30 miles per hour if it is not raining, and 20 miles per hour if it is raining. Today she drove in the sun in the morning and in the rain in the evening, for a total of 16 miles in 40 minutes. How many minutes did she drive in the rain?

(A) 18                      (B) 21                      (C) 24                      (D) 27                      (E) 30

2. A dessert chef prepares the dessert for every day of a week starting with Sunday. The dessert each day is either cake, pie, ice cream, or pudding. The same dessert may not be served two days in a row. There must be cake on Friday because of a birthday. How many different dessert menus for the week are possible?

(A) 729                      (B) 972                      (C) 1024                      (D) 2187                      (E) 2304

3. In quadrilateral  $ABCD$ ,  $AB = 5$ ,  $BC = 17$ ,  $CD = 5$ ,  $DA = 9$ , and  $BD$  is an integer. What is  $BD$ ?

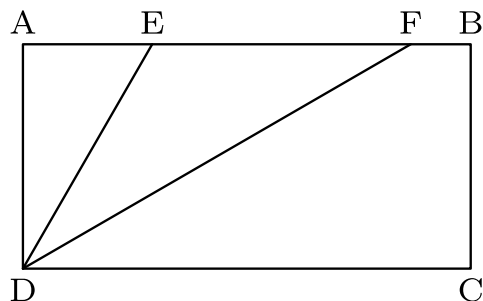


(A) 11                      (B) 12                      (C) 13                      (D) 14                      (E) 15

4. Let  $a$  and  $b$  be the roots of the equation  $x^2 - mx + 2 = 0$ . Suppose that  $a + \frac{1}{b}$  and  $b + \frac{1}{a}$  are the roots of the equation  $x^2 - px + q = 0$ . What is  $q$ ?

(A)  $\frac{5}{2}$                       (B)  $\frac{7}{2}$                       (C) 4                      (D)  $\frac{9}{2}$                       (E) 8

5. In rectangle  $ABCD$ ,  $DC = 2 \cdot CB$  and points  $E$  and  $F$  lie on  $\overline{AB}$  so that  $\overline{ED}$  and  $\overline{FD}$  trisect  $\angle ADC$  as shown. What is the ratio of the area of  $\triangle DEF$  to the area of rectangle  $ABCD$ ?



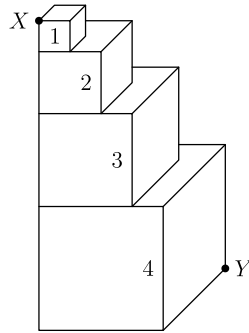
- (A)  $\frac{\sqrt{3}}{6}$       (B)  $\frac{\sqrt{6}}{8}$       (C)  $\frac{3\sqrt{3}}{16}$       (D)  $\frac{1}{3}$       (E)  $\frac{\sqrt{2}}{4}$
6. A teacher gave a test to a class in which 10% of the students are juniors and 90% are seniors. The average score on the test was 84. The juniors all received the same score, and the average score of the seniors was 83. What score did each of the juniors receive on the test?
- (A) 85      (B) 88      (C) 93      (D) 94      (E) 98
7. Let  $N$  be a positive multiple of 5. One red ball and  $N$  green balls are arranged in a line in random order. Let  $P(N)$  be the probability that at least  $\frac{3}{5}$  of the green balls are on the same side of the red ball. Observe that  $P(5) = 1$  and that  $P(N)$  approaches  $\frac{4}{5}$  as  $N$  grows large. What is the sum of the digits of the least value of  $N$  such that  $P(N) < \frac{321}{400}$ ?
- (A) 12      (B) 14      (C) 16      (D) 18      (E) 20
8. For some positive integer  $k$ , the repeating base- $k$  representation of the (base-ten) fraction  $\frac{7}{51}$  is

$$0.\overline{23}_k = 0.232323 \dots_k.$$

What is  $k$ ?

- (A) 13      (B) 14      (C) 15      (D) 16      (E) 17

9. Four cubes with edge lengths 1, 2, 3, and 4 are stacked as shown. What is the length of the portion of  $\overline{XY}$  contained in the cube with edge length 3?



- (A)  $\frac{3\sqrt{33}}{5}$       (B)  $2\sqrt{3}$       (C)  $\frac{2\sqrt{33}}{3}$       (D) 4      (E)  $3\sqrt{2}$
10. Let  $A$ ,  $M$ , and  $C$  be nonnegative integers such that  $A + M + C = 10$ . What is the maximum value of
- $$A \cdot M \cdot C + A \cdot M + M \cdot C + C \cdot A?$$
- (A) 49      (B) 59      (C) 69      (D) 79      (E) 89

END OF EXAMINATION

# ANSWER SHEET

AMC 10 • HALF-LENGTH PRACTICE • SET 2

Name \_\_\_\_\_

Date \_\_\_\_\_

1    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

2    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

3    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

4    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

5    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

6    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

7    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

8    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

9    ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

10   ☐ A   ☐ B   ☐ C   ☐ D   ☐ E

Fill in one circle per question. Leave the row empty to skip.

| Correct | Blank | Wrong | Score |
|---------|-------|-------|-------|
| ×6      | ×1.5  | ×0    | / 60  |