AMC 10

SET 4

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • MIDDLE BAND (PROBLEMS 10--20)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 10 to 20 of an AMC 10 — the middle band of the paper, where scores are decided. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Three minutes per problem is the pace of the real test in this band: problems 1–5 in about 13 minutes, problems 6–10 in the remaining 17. If a problem has taken three minutes with no plan in sight, move on and come back.

1. The lines with equations $ax - 2y = c$ and $2x + by = -c$ are perpendicular and intersect at $(1, -5)$. What is c?

(A) -13 **(B)** -8 **(C)** 2 **(D)** 8 **(E)** 13

2. There are 52 people in a room. What is the largest value of n such that the statement "At least n people in this room have birthdays falling in the same month" is always true?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 12

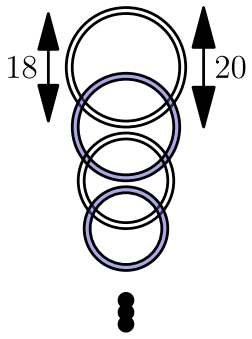
3. Al, Betty, and Clare split \$1000 among them to be invested in different ways. Each begins with a different amount. At the end of one year they have a total of \$1500. Betty and Clare have both doubled their money, whereas Al has managed to lose \$100. What was Al's original portion?

(A) \$250 **(B)** \$350 **(C)** \$400 **(D)** \$450 **(E)** \$500

4. Two circles of radius 2 are centered at $(2, 0)$ and at $(0, 2)$. What is the area of the intersection of the interiors of the two circles?

(A) $\pi - 2$ **(B)** $\frac{\pi}{2}$ **(C)** $\frac{\pi\sqrt{3}}{3}$ **(D)** $2(\pi - 2)$ **(E)** π

5. A number of linked rings, each 1 cm thick, are hanging on a peg. The top ring has an outside diameter of 20 cm. The outside diameter of each of the other rings is 1 cm less than that of the ring above it. The bottom ring has an outside diameter of 3 cm. What is the distance, in cm, from the top of the top ring to the bottom of the bottom ring?



(A) 171 **(B)** 173 **(C)** 182 **(D)** 188 **(E)** 210

6. All the numbers $1, 2, 3, 4, 5, 6, 7, 8, 9$ are written in a 3×3 array of squares, one number in each square, in such a way that if two numbers are consecutive then they occupy squares that share an edge. The numbers in the four corners add up to 18. What is the number in the center?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

7. A triangle with vertices $A(0, 2)$, $B(-3, 2)$, and $C(-3, 0)$ is reflected about the x-axis, then the image $\triangle A'B'C'$ is rotated counterclockwise about the origin by 90° to produce $\triangle A''B''C''$. Which of the following transformations will return $\triangle A''B''C''$ to $\triangle ABC$?

(A) counterclockwise rotation about the origin by 90°

(B) clockwise rotation about the origin by 90°

(C) reflection about the x-axis

(D) reflection about the line $y = x$

(E) reflection about the y-axis

8. The number 2013 has the property that its units digit is the sum of its other digits, that is $2 + 0 + 1 = 3$. How many integers less than 2013 but greater than 1000 have this property?

(A) 33 **(B)** 34 **(C)** 45 **(D)** 46 **(E)** 58

9. On a certain math exam, 10% of the students got 70 points, 25% got 80 points, 20% got 85 points, 15% got 90 points, and the rest got 95 points. What is the difference between the mean and the median score on this exam?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 4 **(E)** 5

10. A scanning code consists of a 7×7 grid of squares, with some of its squares colored black and the rest colored white. There must be at least one square of each color in this grid of 49 squares.

A scanning code is called *symmetric* if its look does not change when the entire square is rotated by a multiple of 90° counterclockwise around its center, nor when it is reflected across a line joining opposite corners or a line joining midpoints of opposite sides.

What is the total number of possible symmetric scanning codes?

(A) 510 **(B)** 1022 **(C)** 8190 **(D)** 8192 **(E)** 65,534

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60