AMC 10

SET 9

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • MIDDLE BAND (PROBLEMS 10--20)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 10 to 20 of an AMC 10 — the middle band of the paper, where scores are decided. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

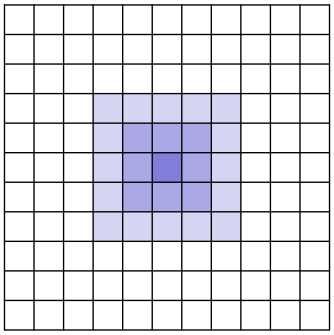
Pacing guide

Three minutes per problem is the pace of the real test in this band: problems 1–5 in about 13 minutes, problems 6–10 in the remaining 17. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Coin A is flipped three times and coin B is flipped four times. What is the probability that the number of heads obtained from flipping the two fair coins is the same?

(A) $\frac{19}{128}$ **(B)** $\frac{23}{128}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{35}{128}$ **(E)** $\frac{1}{2}$

2. Consider the dark square in an array of unit squares, part of which is shown. The first ring of squares around this center square contains 8 unit squares. The second ring contains 16 unit squares. If we continue this process, the number of unit squares in the 100th ring is



(A) 396 **(B)** 404 **(C)** 800 **(D)** 10,000 **(E)** 10,404

3. A point (x, y) is randomly picked from inside the rectangle with vertices $(0, 0)$, $(4, 0)$, $(4, 1)$, and $(0, 1)$. What is the probability that $x < y$?

(A) $\frac{1}{8}$ **(B)** $\frac{1}{4}$ **(C)** $\frac{3}{8}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{3}{4}$

4. In the United States, coins have the following thicknesses: penny, 1.55 mm; nickel, 1.95 mm; dime, 1.35 mm; quarter, 1.75 mm. If a stack of these coins is exactly 14 mm high, how many coins are in the stack?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 11

5. How many ways are there to write 2016 as the sum of twos and threes, ignoring order? (For example, $1008 \cdot 2 + 0 \cdot 3$ and $402 \cdot 2 + 404 \cdot 3$ are two such ways.)

(A) 236 **(B)** 336 **(C)** 337 **(D)** 403 **(E)** 672

6. Let @ denote the "averaged with" operation: $a@b = \frac{a+b}{2}$. Which of the following distributive laws hold for all numbers x, y, and z?

I. $x@(y+z) = (x@y) + (x@z)$

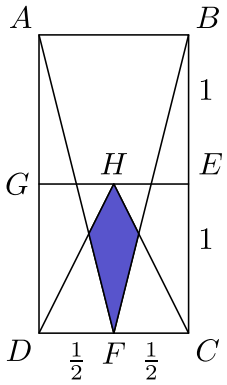
II. $x + (y@z) = (x+y)@(x+z)$

III. $x@(y@z) = (x@y)@(x@z)$

(A) I only **(B)** II only **(C)** III only

(D) I and III only **(E)** II and III only

7. In rectangle $ABCD$, $AB = 1$, $BC = 2$, and points E, F, and G are midpoints of $\overline{BC}$, $\overline{CD}$, and $\overline{AD}$, respectively. Point H is the midpoint of $\overline{GE}$. What is the area of the shaded region?



(A) $\frac{1}{12}$ **(B)** $\frac{\sqrt{3}}{18}$ **(C)** $\frac{\sqrt{2}}{12}$ **(D)** $\frac{\sqrt{3}}{12}$ **(E)** $\frac{1}{6}$

8. Every high school in the city of Euclid sent a team of 3 students to a math contest. Each participant in the contest received a different score. Andrea's score was the median among all students, and hers was the highest score on her team. Andrea's teammates Beth and Carla placed 37^{th} and 64^{th}, respectively. How many schools are in the city?

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

9. A cylindrical tank with radius 4 feet and height 9 feet is lying on its side. The tank is filled with water to a depth of 2 feet. What is the volume of the water, in cubic feet?

(A) $24\pi - 36\sqrt{2}$ **(B)** $24\pi - 24\sqrt{3}$ **(C)** $36\pi - 36\sqrt{3}$

(D) $36\pi - 24\sqrt{2}$ **(E)** $48\pi - 36\sqrt{3}$

10. The number 2013 is expressed in the form

$$2013 = \frac{a_1!a_2!\cdots a_m!}{b_1!b_2!\cdots b_n!},$$

where

$$a_1 \geq a_2 \geq \cdots \geq a_m$$

and

$$b_1 \geq b_2 \geq \cdots \geq b_n$$

are positive integers and $a_1 + b_1$ is as small as possible. What is $|a_1 - b_1|$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

END OF EXAMINATION

ANSWER SHEET

AMC 10 • HALF-LENGTH PRACTICE • SET 9

Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60