
AMC 10

SET 13

HALF-LENGTH PRACTICE

TIME ALLOWED

30 : 00

10 QUESTIONS • 30 MINUTES • 60 POINTS • MIDDLE BAND (PROBLEMS 10--20)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____

Date _____

Start _____

Finish _____

All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 10 to 20 of an AMC 10 — the middle band of the paper, where scores are decided. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Three minutes per problem is the pace of the real test in this band: problems 1–5 in about 13 minutes, problems 6–10 in the remaining 17. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Suppose that real number x satisfies

$$\sqrt{49 - x^2} - \sqrt{25 - x^2} = 3.$$

What is the value of

$$\sqrt{49 - x^2} + \sqrt{25 - x^2}?$$

- (A) 8 (B) $\sqrt{33} + 8$ (C) 9 (D) $2\sqrt{10} + 4$ (E) 12

2. Which of the following expressions is never a prime number when p is a prime number?

- (A) $p^2 + 16$ (B) $p^2 + 24$ (C) $p^2 + 26$ (D) $p^2 + 46$ (E) $p^2 + 96$

3. Two tour guides are leading six tourists. The guides decide to split up. Each tourist must choose one of the guides, but with the stipulation that each guide must take at least one tourist. How many different groupings of guides and tourists are possible?

- (A) 56 (B) 58 (C) 60 (D) 62 (E) 64

4. Doug can paint a room in 5 hours. Dave can paint the same room in 7 hours. Doug and Dave paint the room together and take a one-hour break for lunch. Let t be the total time, in hours, required for them to complete the job working together, including lunch. Which of the following equations is satisfied by t ?

- (A) $\left(\frac{1}{5} + \frac{1}{7}\right)(t + 1) = 1$ (B) $\left(\frac{1}{5} + \frac{1}{7}\right)t + 1 = 1$ (C) $\left(\frac{1}{5} + \frac{1}{7}\right)t = 1$
 (D) $\left(\frac{1}{5} + \frac{1}{7}\right)(t - 1) = 1$ (E) $(5 + 7)t = 1$

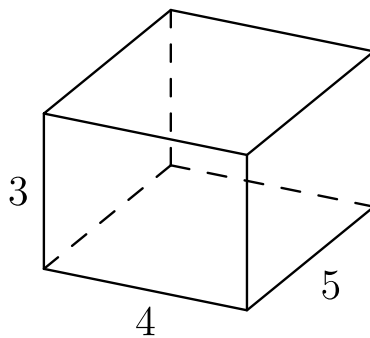
5. Triangle ABC has $AB = 2 \cdot AC$. Let D and E be on $\overline{AB}$ and $\overline{BC}$, respectively, such that $\angle BAE = \angle ACD$. Let F be the intersection of segments AE and CD , and suppose that $\triangle CFE$ is equilateral. What is $\angle ACB$?

- (A) 60° (B) 75° (C) 90° (D) 105° (E) 120°

6. There are 100 players in a singles tennis tournament. The tournament is single elimination, meaning that a player who loses a match is eliminated. In the first round, the strongest 28 players are given a bye, and the remaining 72 players are paired off to play. After each round, the remaining players play in the next round. The match continues until only one player remains unbeaten. The total number of matches played is

(A) a prime number (B) divisible by 2 (C) divisible by 5
 (D) divisible by 7 (E) divisible by 11

7. The centers of the faces of the right rectangular prism shown below are joined to create an octahedron. What is the volume of this octahedron?



(A) $\frac{75}{12}$ (B) 10 (C) 12 (D) $10\sqrt{2}$ (E) 15

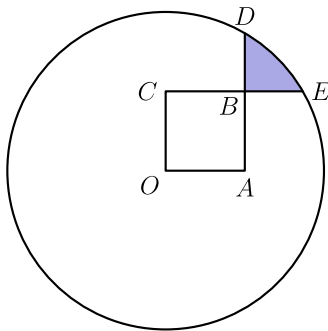
8. Suppose that one of every 500 people in a certain population has a particular disease, which displays no symptoms. A blood test is available for screening for this disease. For a person who has this disease, the test always turns out positive.

For a person who does not have the disease, however, there is a 2% false positive rate. In other words, for such people, 98% of the time the test will turn out negative, but 2% of the time the test will turn out positive and will incorrectly indicate that the person has the disease.

Let p be the probability that a person who is chosen at random from this population and gets a positive test result actually has the disease. Which of the following is closest to p ?

(A) $\frac{1}{98}$ (B) $\frac{1}{9}$ (C) $\frac{1}{11}$ (D) $\frac{49}{99}$ (E) $\frac{98}{99}$

9. A circle of radius 2 is centered at O . Square $OABC$ has side length 1. Sides $\overline{AB}$ and $\overline{CB}$ are extended past B to meet the circle at D and E , respectively. What is the area of the shaded region in the figure, which is bounded by $\overline{BD}$, $\overline{BE}$, and the minor arc connecting D and E ?



- (A) $\frac{\pi}{3} + 1 - \sqrt{3}$ (B) $\frac{\pi}{2}(2 - \sqrt{3})$ (C) $\pi(2 - \sqrt{3})$
 (D) $\frac{\pi}{6} + \frac{\sqrt{3} - 1}{2}$ (E) $\frac{\pi}{3} - 1 + \sqrt{3}$
10. The faces of a cubical die are marked with the numbers 1, 2, 2, 3, 3, and 4. The faces of a second cubical die are marked with the numbers 1, 3, 4, 5, 6, and 8. Both dice are thrown. What is the probability that the sum of the two top numbers will be 5, 7, or 9?
- (A) $\frac{5}{18}$ (B) $\frac{7}{18}$ (C) $\frac{11}{18}$ (D) $\frac{3}{4}$ (E) $\frac{8}{9}$

END OF EXAMINATION

ANSWER SHEET

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Name _____

Date _____

1 ☐ A ☐ B ☐ C ☐ D ☐ E

2 ☐ A ☐ B ☐ C ☐ D ☐ E

3 ☐ A ☐ B ☐ C ☐ D ☐ E

4 ☐ A ☐ B ☐ C ☐ D ☐ E

5 ☐ A ☐ B ☐ C ☐ D ☐ E

6 ☐ A ☐ B ☐ C ☐ D ☐ E

7 ☐ A ☐ B ☐ C ☐ D ☐ E

8 ☐ A ☐ B ☐ C ☐ D ☐ E

9 ☐ A ☐ B ☐ C ☐ D ☐ E

10 ☐ A ☐ B ☐ C ☐ D ☐ E

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
×6	×1.5	×0	/ 60