
AMC 10

SET 17

HALF-LENGTH PRACTICE

SOLUTIONS

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All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

Answer Key and Sources

No.	Ans.	Source	Topic
1	E	2013 AMC 10A #10	Algebra
2	C	2000 AMC 10 #11	Number Theory
3	E	2019 AMC 10A #12	Algebra
4	B	2008 AMC 10B #14	Geometry
5	E	2002 AMC 10B #15	Number Theory
6	D	2006 AMC 10A #16	Geometry
7	D	2002 AMC 10A #17	Algebra
8	A	2005 AMC 10A #18	Counting & Probability
9	C	2015 AMC 10B #19	Geometry
10	B	2016 AMC 10A #20	Counting & Probability

1 Algebra

Source: 2013 AMC 10A #10

A flower bouquet contains pink roses, red roses, pink carnations, and red carnations. One third of the pink flowers are roses, three fourths of the red flowers are carnations, and six tenths of the flowers are pink. What percent of the flowers are carnations?

- (A) 15 (B) 30 (C) 40 (D) 60 (E) 70

Key idea. The answer is a percent, so pretend the bouquet has exactly 100 flowers and count each of the four kinds.

Six tenths of the flowers are pink, so there are 60 pink flowers and $100 - 60 = 40$ red flowers.

One third of the pink flowers are roses: $\frac{1}{3} \cdot 60 = 20$ pink roses, which leaves $60 - 20 = 40$ pink carnations.

Three fourths of the red flowers are carnations: $\frac{3}{4} \cdot 40 = 30$ red carnations.

All together there are $40 + 30 = 70$ carnations out of 100 flowers, which is 70%.

Watch out. Choices (C) 40 and (B) 30 are each only *one* kind of carnation. The question asks for all carnations, pink and red together.

Answer: (E) 70**2** Number Theory

Source: 2000 AMC 10 #11

Two different prime numbers between 4 and 18 are chosen. When their sum is subtracted from their product, which of the following numbers could be obtained?

- (A) 21 (B) 60 (C) 119 (D) 180 (E) 231

Key idea. “Product minus sum” is $xy - x - y$, which is one step away from a factorization: adding 1 gives $(x - 1)(y - 1)$. A product of two small known numbers is far easier to test than the original expression.

Step 1. The primes between 4 and 18 are 5, 7, 11, 13, 17. Call the two chosen primes x and y . Then

$$xy - (x + y) = xy - x - y + 1 - 1 = (x - 1)(y - 1) - 1.$$

So a choice can be obtained exactly when that choice plus 1 is a product of two different numbers from the list 4, 6, 10, 12, 16 (the possible values of $x - 1$).

Step 2. Test the choices.

- $21 + 1 = 22$, whose only factor pairs are $1 \cdot 22$ and $2 \cdot 11$: neither uses two numbers from the list.
- $60 + 1 = 61$ and $180 + 1 = 181$ are odd, but $(x - 1)(y - 1)$ is a product of even numbers. (Equivalently: odd $\times$ odd minus even is odd, so 60 and 180 are impossible.)

- $119 + 1 = 120 = 10 \cdot 12$, so $x = 11$, $y = 13$ works: $11 \cdot 13 - (11 + 13) = 143 - 24 = 119$.
- $231 + 1 = 232$, but the largest possible product is $12 \cdot 16 = 192$, so 231 is too big.

Another way. Odd primes give odd product and even sum, so the result is odd: this rules out 60 and 180. The smallest result is $5 \cdot 7 - 12 = 23 > 21$ and the largest is $13 \cdot 17 - 30 = 191 < 231$, leaving only 119.

Answer: (C) 119

3 Algebra

Source: 2019 AMC 10A #12

Melanie computes the mean μ , the median M , and the modes of the 365 values that are the dates in the months of 2019. Thus her data consist of 12 copies of 1, 12 copies of 2, $\dots$, 12 copies of 28, 11 copies of 29, 11 copies of 30, and 7 copies of 31. Let d be the median of the modes. Which of the following statements is true?

- (A) $\mu < d < M$ (B) $M < d < \mu$ (C) $d = M = \mu$ (D) $d < M < \mu$ (E) $d < \mu < M$

Key idea. Find d and M by counting positions, then compare μ with M by asking which values are “missing”: the short months remove only large dates, which drags the mean down.

Step 1: the modes. The most frequent values are the ones that appear 12 times: 1, 2, $\dots$, 28. Their median is the average of the middle two, $d = \frac{14+15}{2} = 14.5$.

Step 2: the median. With 365 values in order, the median is the 183rd. The dates 1 through 15 fill $15 \cdot 12 = 180$ positions, so positions 181 to 192 are all 16. Hence $M = 16$.

Step 3: the mean. If every date 1 to 31 appeared 12 times, the 372 values would have mean exactly 16, with sum $372 \cdot 16 = 5952$. Our data is missing one 29, one 30 and five 31s, a total of $29 + 30 + 5 \cdot 31 = 214$. So

$$\mu = \frac{5952 - 214}{365} = \frac{5738}{365} \approx 15.72.$$

This is below 16 because every removed value was above 16; it is above 14.5 since $14.5 \cdot 365 = 5292.5 < 5738$.

Therefore $d < \mu < M$.

Watch out. The median is the 183rd value, and $183 \div 12 = 15.25$. Rounding this down to “ $M = 15$ ” gives $d < M < \mu$, choice (D). The .25 means we are *past* the fifteenth block, so $M = 16$.

Answer: (E) $d < \mu < M$

4 Geometry

Source: 2008 AMC 10B #14

Triangle OAB has $O = (0, 0)$, $B = (5, 0)$, and A in the first quadrant. In addition, $\angle ABO = 90^\circ$ and $\angle AOB = 30^\circ$. Suppose that $\overline{OA}$ is rotated 90° counterclockwise about O . What are the coordinates of the image of A ?

- (A) $\left(-\frac{10}{3}\sqrt{3}, 5\right)$ (B) $\left(-\frac{5}{3}\sqrt{3}, 5\right)$ (C) $(\sqrt{3}, 5)$
 (D) $\left(\frac{5}{3}\sqrt{3}, 5\right)$ (E) $\left(\frac{10}{3}\sqrt{3}, 5\right)$

Key idea. First pin down A exactly — the right angle at B makes $\overline{AB}$ vertical, so A is directly above B . Then a quarter turn counterclockwise is a simple rule on coordinates: $(x, y) \mapsto (-y, x)$.

Step 1: find A . Since $\angle ABO = 90^\circ$ and $\overline{OB}$ lies on the x -axis, $\overline{AB}$ is vertical, so $A = (5, h)$ for some $h > 0$. In right triangle OAB , the leg opposite the 30° angle is $AB = h$ and the adjacent leg is $OB = 5$, so

$$h = 5 \tan 30^\circ = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3}.$$

Thus $A = \left(5, \frac{5\sqrt{3}}{3}\right)$.

Step 2: rotate. Turning a point 90° counterclockwise about the origin sends (x, y) to $(-y, x)$. (Check with $(1, 0)$: it goes to $(0, 1)$, a quarter turn counterclockwise.) So the image of A is

$$\left(-\frac{5\sqrt{3}}{3}, 5\right).$$

As a sanity check, the image must lie in the second quadrant (negative x , positive y), since A was in the first quadrant and we turned a quarter turn counterclockwise. Only (A) and (B) are in the second quadrant.

Watch out. Forgetting the minus sign — writing (y, x) instead of $(-y, x)$ — reflects A in the line $y = x$ rather than rotating it, and gives (D).

Watch out. Using the hypotenuse $OA = \frac{5}{\cos 30^\circ} = \frac{10}{3}\sqrt{3}$ in place of the leg AB gives (A). The image's distance from the y -axis is AB , not OA .

Answer: (B) $\left(-\frac{5\sqrt{3}}{3}, 5\right)$

5 Number Theory

Source: 2002 AMC 10B #15

The positive integers A , B , $A - B$, and $A + B$ are all prime numbers. The sum of these four primes is

- (A) even (B) divisible by 3 (C) divisible by 5 (D) divisible by 7 (E) prime

Key idea. With four primes linked by sums and differences, look at *parity*: 2 is the only even prime, so parity forces one of the numbers to be 2. After that, look at remainders mod 3.

Step 1: one of them is 2. The numbers $A - B$ and $A + B$ differ by $2B$, so they are both even or both odd. They cannot both be even (only one even prime exists, and $A - B < A + B$), so both are odd. That means one of A, B is even and the other odd, so one of them equals 2.

Step 2: it must be B . If $A = 2$, then $A - B = 2 - B \leq 0$ for any prime B , which is not a prime. So $B = 2$, and the three numbers $A - 2$, A , $A + 2$ are all prime.

Step 3: the triple is 3, 5, 7. Among three numbers spaced 2 apart, one is divisible by 3 (since $n + 4 = (n + 1) + 3$, the numbers n , $n + 2$, $n + 4$ leave the same remainders on division by 3 as n , $n + 2$, $n + 1$ — three consecutive integers, one of which is a multiple of 3). That one is prime, so it equals 3. Since $A - 2$ is a prime, $A - 2 \geq 2$, so A and $A + 2$ are both at least 4; the member equal to 3 must be $A - 2$. This gives 3, 5, 7, so $A = 5$.

Step 4. The four primes are $A = 5$, $B = 2$, $A - B = 3$, $A + B = 7$, with sum

$$5 + 2 + 3 + 7 = 17,$$

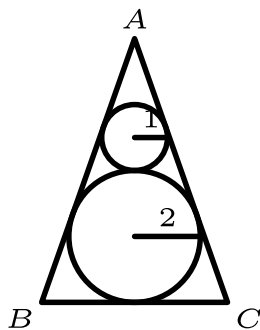
which is prime — and is not even, nor divisible by 3, 5, or 7.

Answer: (E) prime

6 Geometry

Source: 2006 AMC 10A #16

A circle of radius 1 is tangent to a circle of radius 2. The sides of $\triangle ABC$ are tangent to the circles as shown, and the sides AB and AC are congruent. What is the area of $\triangle ABC$?



- (A) $\frac{35}{2}$ (B) $15\sqrt{2}$ (C) $\frac{64}{3}$ (D) $16\sqrt{2}$ (E) 24

Key idea. Whenever a line is tangent to a circle, draw the radius to the point of tangency: it is perpendicular to the line, so you get right triangles. Here both circles touch side $\overline{AC}$, and the two right triangles share the angle at A , so they are similar.

Step 1: the axis. Because $AB = AC$, the figure is symmetric about the altitude from A ; both centers lie on it. Call the centers P (radius 1) and Q (radius 2), and let F be the midpoint of $\overline{BC}$. The circles are tangent, so $PQ = 1 + 2 = 3$.

Step 2: similar triangles. Drop radii from P and Q to their tangent points D and E on $\overline{AC}$. Right triangles ADP and AEQ share the angle at A , so

$$\frac{AP}{1} = \frac{AQ}{2} = \frac{AP + 3}{2}.$$

Then $2AP = AP + 3$, so $AP = 3$ and $AQ = 6$.

Step 3: the height. The large circle touches $\overline{BC}$ at F , so $AF = AQ + QF = 6 + 2 = 8$.

Step 4: the base. In triangle ADP , $AD = \sqrt{3^2 - 1^2} = \sqrt{8} = 2\sqrt{2}$. Triangle AFC is right-angled at F and shares the angle at A , so it is similar to ADP :

$$\frac{FC}{AF} = \frac{DP}{AD} = \frac{1}{2\sqrt{2}}, \quad FC = \frac{8}{2\sqrt{2}} = 2\sqrt{2}.$$

So $BC = 4\sqrt{2}$.

Step 5. The area is $\frac{1}{2} \cdot BC \cdot AF = \frac{1}{2} \cdot 4\sqrt{2} \cdot 8 = 16\sqrt{2}$.

Watch out. The height is $AF = 8$, not $AQ = 6$: the large circle's radius 2 still lies between its center and the base. Stopping at 6 gives area $9\sqrt{2}$, which is not a choice.

Answer: (D) $16\sqrt{2}$

7 Algebra

Source: 2002 AMC 10A #17

Sarah pours four ounces of coffee into an eight-ounce cup and four ounces of cream into a second cup of the same size. She then transfers half the coffee from the first cup to the second and, after stirring thoroughly, transfers half the liquid in the second cup back to the first. What fraction of the liquid in the first cup is now cream?

- (A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{2}{5}$ (E) $\frac{1}{2}$

Key idea. Mixing problems become bookkeeping: track the ounces of coffee and of cream in each cup after every pour. “Stirring thoroughly” is the key phrase — it means any portion poured out has the same coffee-to-cream ratio as the whole cup.

Start. Cup 1: 4 oz coffee. Cup 2: 4 oz cream.

First pour. Half the coffee, 2 oz, moves to cup 2. Cup 1: 2 oz coffee. Cup 2: 2 oz coffee + 4 oz cream = 6 oz.

Second pour. Half of cup 2 is 3 oz. Since cup 2 is well stirred, half of each ingredient moves: 1 oz coffee and 2 oz cream. Cup 1 now holds $2 + 1 = 3$ oz coffee and $0 + 2 = 2$ oz cream, a total of 5 oz.

Answer. The fraction of cup 1 that is cream is

$$\frac{2}{3 + 2} = \frac{2}{5}.$$

Watch out. The “eight-ounce cup” is just the capacity, not the amount of liquid. Dividing the 2 oz of cream by 8 gives $\frac{1}{4}$, choice (A); the cup actually holds only 5 oz.

Answer: (D) $\frac{2}{5}$

8 Counting & Probability

Source: 2005 AMC 10A #18

Team A and team B play a series. The first team to win three games wins the series. Each team is equally likely to win each game, there are no ties, and the outcomes of the individual games are independent. If team B wins the second game and team A wins the series, what is the probability that team B wins the first game?

- (A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{3}$ (D) $\frac{1}{2}$ (E) $\frac{2}{3}$

Key idea. This is a *conditional* probability: list every way the series can go with B winning game 2 and A winning the series, weight each by its probability, and see what share has B winning game 1. A slicker trick is to pretend all five games are always played, which makes every outcome equally likely.

Step 1: list the series. Game 2 is a B win. If A wins game 1, A then needs 2 more wins before B gets 2 more:

$$ABAA, \quad ABABA, \quad ABBAA.$$

If B wins game 1, B leads 2–0 and A must win the next three: BBAAA.

Step 2: weight them. A series of k games has probability $(\frac{1}{2})^k$. So the four series have probabilities $\frac{1}{16}, \frac{1}{32}, \frac{1}{32}, \frac{1}{32}$, totalling $\frac{2+1+1+1}{32} = \frac{5}{32}$.

Step 3. Only BBAAA has B winning game 1, so the probability is

$$\frac{1/32}{5/32} = \frac{1}{5}.$$

Another way. Imagine all five games are played even after the series is decided; the winner does not change, and all 32 sequences are equally likely. Sequences with B in game 2 and at least three A wins are ABAAA, ABAAB, ABABA, ABBAA, BBAAA — five of them, one starting with B. So the answer is $\frac{1}{5}$.

Watch out. Counting the four real series as equally likely gives $\frac{1}{4}$, choice (B). The short series ABAA is twice as likely as each five-game one.

Watch out. “Games are independent, so game 1 is still $\frac{1}{2}$ ” gives (D). The games are independent of each other, but the series result is not independent of game 1: a B win in game 1 makes an A series win less likely, so knowing that A won the series changes the odds for game 1.

Answer: (A) $\frac{1}{5}$

9 Geometry

Source: 2015 AMC 10B #19

In $\triangle ABC$, $\angle C = 90^\circ$ and $AB = 12$. Squares $ABXY$ and $ACWZ$ are constructed outside of the triangle. The points X, Y, Z , and W lie on a circle. What is the perimeter of the triangle?

- (A) $12 + 9\sqrt{3}$ (B) $18 + 6\sqrt{3}$ (C) $12 + 12\sqrt{2}$ (D) 30 (E) 32

Key idea. The center of a circle through two points lies on their perpendicular bisector. Since $\overline{XY}$ is the side of a square opposite $\overline{AB}$, it has the *same* perpendicular bisector as $\overline{AB}$ — so the unknown circle is centered somewhere you already know.

Step 1: find the center. The center is on the perpendicular bisector of $\overline{XY}$, which is the perpendicular bisector of $\overline{AB}$. Likewise it is on the perpendicular bisector of $\overline{ZW}$, which is that of $\overline{AC}$. These two lines meet at the circumcenter of $\triangle ABC$, and for a right triangle that is the midpoint O of the hypotenuse $\overline{AB}$.

Step 2: coordinates. Put $C = (0, 0)$, $A = (0, b)$, $B = (a, 0)$ with $a, b > 0$ and $a^2 + b^2 = 144$. Then $O = (\frac{a}{2}, \frac{b}{2})$. The square on $\overline{AC}$ lies on the left: $W = (-b, 0)$ and $Z = (-b, b)$.

Step 3: equal radii. X is a corner of the square on $\overline{AB}$: from O it lies 6 along the direction of $\overline{AB}$ and 12 perpendicular to it, so $OX^2 = 6^2 + 12^2 = 180$. Also

$$OZ^2 = \left(\frac{a}{2} + b\right)^2 + \left(\frac{b}{2}\right)^2 = \frac{a^2+b^2}{4} + ab + b^2 = 36 + ab + b^2.$$

Setting $OZ^2 = 180$ gives $ab + b^2 = 144 = a^2 + b^2$, so $ab = a^2$; dividing by $a > 0$ gives $a = b$.

Step 4. The triangle is isosceles right with hypotenuse 12, so $a = b = \frac{12}{\sqrt{2}} = 6\sqrt{2}$. The perimeter is

$$12 + 6\sqrt{2} + 6\sqrt{2} = 12 + 12\sqrt{2}.$$

Answer: (C) $12 + 12\sqrt{2}$

10 Counting & Probability

Source: 2016 AMC 10A #20

For some particular value of N , when $(a + b + c + d + 1)^N$ is expanded and like terms are combined, the resulting expression contains exactly 1001 terms that include all four variables a, b, c , and d , each to some positive power. What is N ?

- (A) 9 (B) 14 (C) 16 (D) 17 (E) 19

Key idea. After combining like terms, each term is fixed by its exponents, so “count the terms” means “count the exponent lists”. Exponents adding up to a fixed total is exactly what stars and bars counts — and the “+1” in the base contributes a fifth, invisible exponent.

Step 1: what a term looks like. Every term of the expansion is a constant times $a^v b^w c^x d^y 1^z$ with

$$v + w + x + y + z = N,$$

where $z \geq 0$ is the number of times 1 was chosen. Different lists (v, w, x, y) give different terms, and every list with $v + w + x + y \leq N$ does occur.

Step 2: count. We need $v, w, x, y \geq 1$ and $z \geq 0$. Write $v = v' + 1$ and so on; then

$$v' + w' + x' + y' + z = N - 4$$

with all five unknowns ≥ 0 . By stars and bars ($N - 4$ stars, 4 bars), the number of solutions is

$$\binom{(N-4)+4}{4} = \binom{N}{4}.$$

Step 3: solve. We need $\binom{N}{4} = 1001$. Try the choices: $\binom{14}{4} = \frac{14 \cdot 13 \cdot 12 \cdot 11}{24} = \frac{24024}{24} = 1001$. So $N = 14$. (The others give $\binom{9}{4} = 126$, $\binom{16}{4} = 1820$, $\binom{17}{4} = 2380$, $\binom{19}{4} = 3876$.)

Watch out. Forgetting the 1^z factor means counting only $v + w + x + y = N$, giving $\binom{N-1}{3}$. No N makes that 1001, since $\binom{19}{3} = 969$ and $\binom{20}{3} = 1140$, so a solver who drops z gets stuck.

Answer: (B) 14