
AMC 10

SET 19

HALF-LENGTH PRACTICE

SOLUTIONS

[Answer key](#) • [Sources](#) • [Step-by-step solutions](#)

All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

Answer Key and Sources

No.	Ans.	Source	Topic
1	D	2016 AMC 10B #10	Algebra
2	A	2003 AMC 10B #11	Geometry
3	E	2002 AMC 10B #12	Algebra
4	B	2000 AMC 10 #13	Counting & Probability
5	D	2014 AMC 10A #14	Geometry
6	D	2006 AMC 10A #15	Algebra
7	E	2018 AMC 10B #16	Number Theory
8	C	2019 AMC 10B #18	Algebra
9	C	2012 AMC 10B #19	Geometry
10	A	2015 AMC 10B #20	Counting & Probability

1 Algebra

Source: 2016 AMC 10B #10

A thin piece of wood of uniform density in the shape of an equilateral triangle with side length 3 inches weighs 12 ounces. A second piece of the same type of wood, with the same thickness, also in the shape of an equilateral triangle, has side length 5 inches. Which of the following is closest to the weight, in ounces, of the second piece?

- (A) 14.0 (B) 16.0 (C) 20.0 (D) 33.3 (E) 55.6

Key idea. Same wood, same thickness: the weight is proportional to the *area* of the triangle, and areas of similar figures scale with the *square* of the side ratio.

The two triangles are similar, with side ratio $\frac{5}{3}$. So the area of the second piece is $\left(\frac{5}{3}\right)^2 = \frac{25}{9}$ times the area of the first.

Because the density and thickness are the same, the weight grows by the same factor:

$$12 \cdot \frac{25}{9} = \frac{300}{9} = \frac{100}{3} \approx 33.3 \text{ ounces.}$$

Watch out. Scaling the weight by the side ratio $\frac{5}{3}$ gives $12 \cdot \frac{5}{3} = 20$, choice (C); scaling by the cube $\left(\frac{5}{3}\right)^3$, as if the thickness also grew, gives about 55.6, choice (E). Only the area changes here.

Answer: (D) 33.3

2 Geometry

Source: 2003 AMC 10B #11

A line with slope 3 intersects a line with slope 5 at the point (10, 15). What is the distance between the x -intercepts of these two lines?

- (A) 2 (B) 5 (C) 7 (D) 12 (E) 20

Key idea. Slope is “rise over run”. From (10, 15) each line has to drop 15 units to reach the x -axis, and its slope tells you how far left it must run to do that.

Line with slope 3. Every 1 unit left, the line drops 3 units. To drop 15 it runs $\frac{15}{3} = 5$ units left, so its x -intercept is $10 - 5 = 5$.

Line with slope 5. Every 1 unit left, it drops 5. To drop 15 it runs $\frac{15}{5} = 3$ units left, so its x -intercept is $10 - 3 = 7$.

Distance. The intercepts are (5, 0) and (7, 0), which are $7 - 5 = 2$ apart.

With equations: the lines are $y - 15 = 3(x - 10)$ and $y - 15 = 5(x - 10)$. Putting $y = 0$ gives $x - 10 = -5$ and $x - 10 = -3$, the same two intercepts.

Watch out. Finding the y -intercepts instead: $15 - 3 \cdot 10 = -15$ and $15 - 5 \cdot 10 = -35$ are 20 apart, which is choice (E). The question asks where the lines cross the x -axis.

Answer: (A) 2

3 Algebra

Source: 2002 AMC 10B #12

For which of the following values of k does the equation

$$\frac{x-1}{x-2} = \frac{x-k}{x-6}$$

have no solution for x ?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Key idea. Clear the fractions. The x^2 terms cancel, leaving a *linear* equation $ax = b$, and a linear equation has no solution exactly when $a = 0$ but $b \neq 0$. Keep in mind that $x = 2$ and $x = 6$ are forbidden.

Step 1. Cross-multiplying (allowed for $x \neq 2, 6$):

$$(x-1)(x-6) = (x-2)(x-k).$$

Expanding both sides gives $x^2 - 7x + 6 = x^2 - (k+2)x + 2k$.

Step 2. Subtract x^2 from both sides, leaving $-7x + 6 = -(k+2)x + 2k$. Now add $(k+2)x$ to both sides and subtract 6:

$$(k+2)x - 7x = 2k - 6, \quad \text{so} \quad (k-5)x = 2k - 6.$$

Step 3. If $k = 5$, the left side is $0 \cdot x$ and the right side is $2 \cdot 5 - 6 = 4$, so the equation reads $0 \cdot x = 4$. That is never true, so there is no solution.

For the other choices, $x = \frac{2k-6}{k-5}$: $k = 1$ gives $x = 1$, $k = 2$ gives $x = \frac{2}{3}$, $k = 3$ gives $x = 0$, and $k = 4$ gives $x = -2$. None of these is 2 or 6, so each one really solves the original equation.

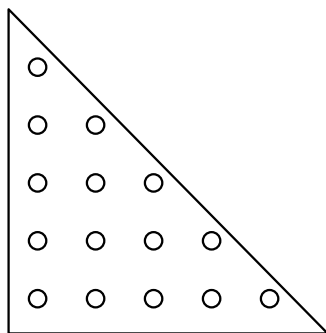
Remark. There is one more value with no solution: $k = 6$ gives $x = 6$, which is forbidden. It is not among the choices.

Answer: (E) 5

4 Counting & Probability

Source: 2000 AMC 10 #13

There are 5 yellow pegs, 4 red pegs, 3 green pegs, 2 blue pegs, and 1 orange peg to be placed on a triangular peg board. In how many ways can the pegs be placed so that no (horizontal) row or (vertical) column contains two pegs of the same color?



- (A) 0 (B) 1 (C) $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!$
 (D) $\frac{15!}{5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!}$ (E) $15!$

Key idea. Start with the color that has the *most* pegs. Five yellow pegs, five rows and five columns, at most one yellow per row or column: yellow must use every row and every column exactly once, and the short rows leave it almost no freedom.

Number the rows 1 to 5 from the top (row r has r holes) and the columns 1 to 5 from the left (column 1 has 5 holes, column 5 has only 1).

Yellow. There is exactly one yellow in each row and each column. Row 1 has a single hole, in column 1, so it is yellow. Row 2 has holes in columns 1 and 2; column 1 already has its yellow, so the yellow goes in column 2. In the same way row 3 is forced into column 3, and so on: the yellows fill the long diagonal.

Red. Row 1 is now full, so the four reds need one each in rows 2–5. Column 5 (one hole) is full too, so they also take one each in columns 1–4. Row 2 has only column 1 left, so red goes there; then row 3 is forced into column 2, and so on. The reds form the next diagonal.

The rest. The same argument places the greens, then the blues, then the orange, each with no choice at all:

Y				
R	Y			
G	R	Y		
B	G	R	Y	
O	B	G	R	Y

Pegs of one color are identical, so this is exactly one placement (Y = yellow, R = red, and so on).

Watch out. Counting all ways to place the colors with no restriction gives $15!/(5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!)$, choice (D). Treating the five yellow pegs (and the others) as different multiplies the one pattern by $5! \cdot 4! \cdot 3! \cdot 2! \cdot 1!$, choice (C).

Answer: (B) 1

5 Geometry

Source: 2014 AMC 10A #14

The y -intercepts, P and Q , of two perpendicular lines intersecting at the point $A(6, 8)$ have a sum of zero. What is the area of $\triangle APQ$?

- (A) 45 (B) 48 (C) 54 (D) 60 (E) 72

Key idea. “Sum of zero” means the intercepts are $(0, p)$ and $(0, -p)$, symmetric about the origin. One unknown and one condition (perpendicular slopes multiply to -1) pins p down.

Step 1. Let $P = (0, p)$ and $Q = (0, -p)$. The slope of line AP is $\frac{8-p}{6}$ and the slope of line AQ is $\frac{8+p}{6}$.

Step 2. Perpendicular lines have slopes with product -1 :

$$\frac{(8-p)(8+p)}{36} = -1 \implies 64 - p^2 = -36 \implies p^2 = 100.$$

So the intercepts are 10 and -10 , and $PQ = 10 - (-10) = 20$. Check: the slopes are then $\frac{8-10}{6} = -\frac{1}{3}$ and $\frac{8+10}{6} = 3$, whose product is indeed -1 .

Step 3. Take $\overline{PQ}$, which lies on the y -axis, as the base. The height is the horizontal distance from A to the y -axis, which is 6. So

$$[APQ] = \frac{1}{2} \cdot 20 \cdot 6 = 60.$$

Watch out. The height is the x -coordinate 6, not the y -coordinate 8. Using 8 gives 80.

Another way. Since $\angle PAQ = 90^\circ$, the midpoint of the hypotenuse $\overline{PQ}$ — the origin — is the center of a circle through P , Q and A . So $OP = OQ = OA = \sqrt{6^2 + 8^2} = 10$, giving $PQ = 20$ again.

Answer: (D) 60

6 Algebra

Source: 2006 AMC 10A #15

Odell and Kershaw run for 30 minutes on a circular track. Odell runs clockwise at 250 m/min and uses the inner lane with a radius of 50 meters. Kershaw runs counterclockwise at 300 m/min and uses the outer lane with a radius of 60 meters, starting on the same radial line as Odell. How many times after the start do they pass each other?

- (A) 29 (B) 42 (C) 45 (D) 47 (E) 50

Key idea. The runners are in different lanes, so their distances in meters cannot be compared directly. What matters is the *angle* each one turns through: they pass whenever they are on the same radial line again.

Step 1. Find each runner's time per lap.

Odell's lap is $2\pi \cdot 50 = 100\pi$ meters, which takes $\frac{100\pi}{250} = \frac{2\pi}{5}$ minutes.

Kershaw's lap is $2\pi \cdot 60 = 120\pi$ meters, which takes $\frac{120\pi}{300} = \frac{2\pi}{5}$ minutes.

So both turn at the same rate: one full turn (360°) every $\frac{2\pi}{5}$ minutes.

Step 2. They turn in opposite directions, so the angle between them grows twice as fast as either one turns: by a full turn every $\frac{\pi}{5}$ minutes. Each time it completes another full turn, they are on the same radial line again and pass. So they pass at

$$t = \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \dots$$

Step 3. The k -th pass happens at $t = \frac{k\pi}{5}$, and we need $\frac{k\pi}{5} \leq 30$, that is,

$$k \leq \frac{150}{\pi} \approx 47.7.$$

So $k = 1, 2, \dots, 47$: they pass 47 times.

Watch out. Adding the speeds, $250 + 300 = 550$ m/min, and dividing into one lane's length ignores that the lanes differ: with the inner lane you would get about 52 passes, with the outer about 43.

Answer: (D) 47

7 Number Theory

Source: 2018 AMC 10B #16

Let $a_1, a_2, \dots, a_{2018}$ be a strictly increasing sequence of positive integers such that

$$a_1 + a_2 + \dots + a_{2018} = 2018^{2018}.$$

What is the remainder when $a_1^3 + a_2^3 + \dots + a_{2018}^3$ is divided by 6?

- (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

Key idea. We know nothing about the individual a_i , only their sum. So the cubes must be tied to the numbers themselves: the fact to look for is that n^3 and n always leave the same remainder when divided by 6.

Step 1. For any integer n ,

$$n^3 - n = n(n^2 - 1) = (n - 1)n(n + 1),$$

a product of three consecutive integers. Among three consecutive integers one is a multiple of 3 and at least one is even, so the product is a multiple of 6. Hence n^3 and n have the same remainder mod 6.

Step 2. Apply this to every term and add:

$$a_1^3 + \dots + a_{2018}^3 \equiv a_1 + \dots + a_{2018} = 2018^{2018} \pmod{6}.$$

Step 3. Since $2018 = 6 \cdot 336 + 2$, we have $2018 \equiv 2 \pmod{6}$, so $2018^{2018} \equiv 2^{2018} \pmod{6}$. The powers of 2 leave remainders

$$2, 4, 2, 4, \dots$$

because $4 \cdot 2 = 8 \equiv 2$. Odd exponents give 2 and even exponents give 4. The exponent 2018 is even, so the remainder is 4.

Watch out. Stopping at “2018 leaves remainder 2” and answering 2, choice (C), forgets the exponent: 2^{2018} leaves remainder 4, not 2.

Answer: (E) 4

8 Algebra

Source: 2019 AMC 10B #18

Henry decides one morning to do a workout, and he walks $\frac{3}{4}$ of the way from his home to his gym. The gym is 2 kilometers away from Henry's home. At that point, he changes his mind and walks $\frac{3}{4}$ of the way from where he is back toward home. When he reaches that point, he changes his mind again and walks $\frac{3}{4}$ of the distance from there back toward the gym. If Henry keeps changing his mind when he has walked $\frac{3}{4}$ of the distance toward either the gym or home from the point where he last changed his mind, he will get very close to walking back and forth between a point A kilometers from home and a point B kilometers from home. What is $|A - B|$?

- (A) $\frac{2}{3}$ (B) 1 (C) $1\frac{1}{5}$ (D) $1\frac{1}{4}$ (E) $1\frac{1}{2}$

Key idea. Do not follow the walk step by step. “Walking back and forth between A and B ” means that walking from one of those points by the rule lands exactly on the other. That gives two equations in two unknowns.

Let A be the turning point near home and B the turning point near the gym, both measured in kilometers from home.

Step 1: from A toward the gym. The gym is $2 - A$ away, and Henry walks $\frac{3}{4}$ of that, so he stops $\frac{1}{4}(2 - A)$ short of the gym:

$$B = 2 - \frac{2 - A}{4}.$$

Step 2: from B toward home. Home is B away, and he walks $\frac{3}{4}$ of it, so he stops $\frac{1}{4}B$ from home:

$$A = \frac{B}{4}, \quad \text{that is,} \quad B = 4A.$$

Step 3: solve. Substituting $B = 4A$:

$$4A = 2 - \frac{2 - A}{4} \implies 16A = 8 - 2 + A \implies 15A = 6 \implies A = \frac{2}{5}.$$

Then $B = \frac{8}{5}$, and $|A - B| = \frac{8}{5} - \frac{2}{5} = \frac{6}{5} = 1\frac{1}{5}$.

Check: from $\frac{2}{5}$ the gym is $\frac{8}{5}$ away, and $\frac{3}{4} \cdot \frac{8}{5} = \frac{6}{5}$ brings him to $\frac{8}{5}$; from $\frac{8}{5}$, $\frac{3}{4} \cdot \frac{8}{5} = \frac{6}{5}$ brings him back to $\frac{2}{5}$.

The actual walk starts at $\frac{3}{2}$, then $\frac{3}{8}$, then $\frac{51}{32}$, ..., and these approach $\frac{8}{5}$ and $\frac{2}{5}$.

Watch out. Reading “ $\frac{3}{4}$ of the way” as $\frac{3}{4}$ of the full 2 km every time makes each leg $\frac{3}{2}$ km, which is choice (E). Each leg is $\frac{3}{4}$ of the distance from the current point to the destination.

Answer: (C) $1\frac{1}{5}$

9 Geometry

Source: 2012 AMC 10B #19

In rectangle $ABCD$, $AB = 6$, $AD = 30$, and G is the midpoint of $\overline{AD}$. Segment AB is extended 2 units beyond B to point E , and F is the intersection of $\overline{ED}$ and $\overline{BC}$. What is the area of quadrilateral $BFDG$?

- (A) $\frac{133}{2}$ (B) 67 (C) $\frac{135}{2}$ (D) 68 (E) $\frac{137}{2}$

Key idea. Sketch it: $\overline{BF}$ lies on side $\overline{BC}$ and $\overline{GD}$ lies on side $\overline{AD}$, which are parallel. So $BFDG$ is a trapezoid whose height is $AB = 6$, and all that is missing is the length BF . A line cutting across a right angle like this suggests similar triangles.

Step 1: the easy pieces. G is the midpoint of $\overline{AD}$, so $GD = \frac{30}{2} = 15$. The parallel sides $\overline{BF}$ and $\overline{GD}$ are $AB = 6$ apart.

Step 2: find BF . Triangles EBF and EAD share the angle at E and both have a right angle (at B and at A), so they are similar. Here $EB = 2$ and $EA = 6 + 2 = 8$, so

$$\frac{BF}{AD} = \frac{EB}{EA} \implies \frac{BF}{30} = \frac{2}{8} \implies BF = \frac{15}{2}.$$

Step 3: the trapezoid.

$$[BFDG] = \frac{6 \left(15 + \frac{15}{2}\right)}{2} = \frac{6 \cdot \frac{45}{2}}{2} = \frac{135}{2}.$$

Another way. Use coordinates $A = (0, 0)$, $B = (6, 0)$, $D = (0, 30)$, $E = (8, 0)$. Line ED is $y = 30 - \frac{30}{8}x$, which at $x = 6$ gives $y = \frac{15}{2}$, so $F = \left(6, \frac{15}{2}\right)$ and $G = (0, 15)$. The shoelace formula on B, F, D, G gives $\frac{135}{2}$.

Watch out. In the similar triangles, the side matching BF is AD , and EA is 8, not $AB = 6$. Using $\frac{BF}{30} = \frac{2}{6}$ gives $BF = 10$ and an area of 75.

Answer: (C) $\frac{135}{2}$

10 Counting & Probability

Source: 2015 AMC 10B #20

Erin the ant starts at a given corner of a cube and crawls along exactly 7 edges in such a way that she visits every corner exactly once and then finds that she is unable to return along an edge to her starting point. How many paths are there meeting these conditions?

- (A) 6 (B) 9 (C) 12 (D) 18 (E) 24

Key idea. First ask *where* the path can end. Color the corners in two colors so that every edge joins different colors (like a checkerboard). An odd number of edges, 7, means Erin finishes on the other color — and the only corner of that color not joined to the start is the opposite corner.

Step 1: the endpoint. Label the start 000 and the other corners by three-digit strings of 0s and 1s, with an edge between two corners when they differ in exactly one digit. Each edge changes the digit sum by 1, so after 7 edges the digit sum is odd: Erin ends at 100, 010, 001 or 111. The first three are joined to 000, so she must end at 111.

Step 2: the first two edges. The first edge has 3 choices and the second 2. Say she goes $000 \rightarrow 100 \rightarrow 110$. These three corners lie on the bottom face, whose fourth corner is 010.

Step 3: no more choices. The neighbors of 010 are 000, 110 and 011. If Erin skips 010 now, she can only reach it later from 011, and then she would be stuck there — but the path must end at 111. So she goes to 010 next, then to 011 (its only unvisited neighbor). From 011, going to 111 would reach the required endpoint while 001 and 101 are still unvisited, so she goes to 001, then 101, then 111.

So each of the $3 \cdot 2 = 6$ starts finishes in exactly one way: 6 paths.

Watch out. Counting every path that visits all eight corners, ignoring “unable to return”, gives 18, choice (D); 12 of those 18 end next to the start.

Answer: (A) 6