
AMC 10

SET 20

HALF-LENGTH PRACTICE

SOLUTIONS

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All 10 problems come from positions 10–20 of past AMC 10 contests (2000–2019) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

Answer Key and Sources

No.	Ans.	Source	Topic
1	C	2013 AMC 10B #10	Algebra
2	C	2004 AMC 10A #11	Geometry
3	A	2012 AMC 10A #12	Number Theory
4	B	2014 AMC 10B #13	Geometry
5	B	2011 AMC 10A #14	Counting & Probability
6	A	2004 AMC 10B #15	Algebra
7	E	2013 AMC 10A #16	Geometry
8	C	2018 AMC 10A #17	Number Theory
9	C	2006 AMC 10A #18	Counting & Probability
10	C	2005 AMC 10B #20	Algebra

1 Algebra

Source: 2013 AMC 10B #10

A basketball team's players were successful on 50% of their two-point shots and 40% of their three-point shots, which resulted in 54 points. They attempted 50% more two-point shots than three-point shots. How many three-point shots did they attempt?

- (A) 10 (B) 15 (C) 20 (D) 25 (E) 30

Key idea. Let one letter stand for the number of three-point attempts and write every other quantity in terms of it.

Let t be the number of three-point attempts. Then there were $1.5t$ two-point attempts.

Three-pointers: 40% of t go in, that is $0.4t$ baskets, worth $3 \cdot 0.4t = 1.2t$ points.

Two-pointers: 50% of $1.5t$ go in, that is $0.75t$ baskets, worth $2 \cdot 0.75t = 1.5t$ points.

The total is $1.2t + 1.5t = 2.7t = 54$, so $t = 20$.

Check: 20 three-point attempts give 8 baskets (24 points); 30 two-point attempts give 15 baskets (30 points); $24 + 30 = 54$.

Watch out. The question asks for *three-point* attempts. The two-point attempts number $1.5 \cdot 20 = 30$, which is choice (E).

Answer: (C) 20
2 Geometry

Source: 2004 AMC 10A #11

A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased?

- (A) 10 (B) 25 (C) 36 (D) 50 (E) 60

Key idea. The volume of a cylinder, $V = \pi r^2 h$, depends on the *square* of the radius. Percent changes are easiest as multipliers: scale the radius by a factor, and ask what factor on h keeps V fixed.

Step 1. Increasing the diameter by 25% multiplies the diameter by $1.25 = \frac{5}{4}$. The radius is half the diameter, so the radius is also multiplied by $\frac{5}{4}$.

Step 2. Then r^2 is multiplied by $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$. For $\pi r^2 h$ to stay the same, h must be multiplied by the reciprocal:

$$\frac{16}{25} = 0.64.$$

Step 3. The new height is 64% of the old height, so the height has decreased by $100\% - 64\% = 36\%$.

Check with numbers: take $r = 4$, $h = 25$, so $V = \pi \cdot 16 \cdot 25 = 400\pi$. The new radius is 5, and the new height 16 gives $\pi \cdot 25 \cdot 16 = 400\pi$, the same volume. The height went from 25 to 16, a drop of 9, and $\frac{9}{25} = 36\%$.

Watch out. It is tempting to say “the diameter goes up 25%, so the height goes down 25%,” which is choice (B). That ignores the square: a 25% wider jar has a base $\frac{25}{16}$ times as large, not 1.25 times. Even “height goes down by the factor $\frac{1}{1.25} = 0.8$ ” forgets the square; it gives a 20% decrease, which is not offered.

Answer: (C) 36

3 Number Theory

Source: 2012 AMC 10A #12

A year is a leap year if and only if the year number is divisible by 400 (such as 2000) or is divisible by 4 but not 100 (such as 2012). The 200th anniversary of the birth of novelist Charles Dickens was celebrated on February 7, 2012, a Tuesday. On what day of the week was Dickens born?

- (A) Friday (B) Saturday (C) Sunday (D) Monday (E) Tuesday

Key idea. Only the number of days *modulo* 7 matters. Since $365 = 52 \cdot 7 + 1$, an ordinary year shifts the weekday by 1 and a leap year by 2 — so the whole job is counting the leap days between the two dates. The detailed leap-year rule in the statement is a hint that a century year will matter.

Step 1. From February 7, 1812 to February 7, 2012 there are 200 years. Each contributes 365 days, plus one more for every February 29 that falls in between.

Step 2. A February 29 lies between the two dates exactly when its year is a leap year from 1812 through 2008. (February 29, 1812 comes after February 7, 1812, so it counts; February 29, 2012 comes after February 7, 2012, so it does not.) The multiples of 4 from 1812 to 2008 number

$$\frac{2008 - 1812}{4} + 1 = 50.$$

Among them, 1900 is divisible by 100 but not 400, so it is not a leap year. That leaves 49 leap days.

Step 3. The weekday shift is $200 + 49 = 249$ days, and $249 = 35 \cdot 7 + 4$, so the shift is 4 days.

Step 4. Going *forward* 4 days from Dickens’s birth lands on Tuesday, so his birthday was 4 days *before* Tuesday: Monday, Sunday, Saturday, Friday. Dickens was born on a Friday.

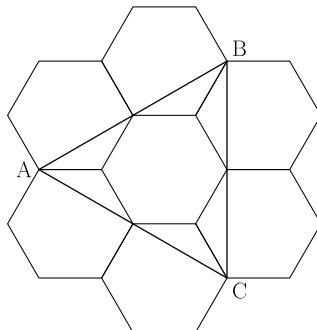
Watch out. We are moving back in time. Counting 4 days forward from Tuesday gives Saturday, choice (B).

Answer: (A) Friday

4 Geometry

Source: 2014 AMC 10B #13

Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



- (A) $2\sqrt{3}$ (B) $3\sqrt{3}$ (C) $1 + 3\sqrt{2}$ (D) $2 + 2\sqrt{3}$ (E) $3 + 2\sqrt{3}$

Key idea. The picture looks the same after a 120° turn about the center O of the middle hexagon, and that turn carries A to B to C . So $\triangle ABC$ is equilateral, and everything follows from one length: the distance from O to A .

Step 1: find OA . Look at the left vertex V of the middle hexagon, where three hexagons meet. Each hexagon has a 120° angle there, so the three edges at V are spread 120° apart. The two edges of the middle hexagon are symmetric about line OV , so the third edge — the one the two left hexagons share — points straight out along line OV . Its far end is A . Hence O, V, A are collinear and

$$OA = OV + VA = 1 + 1 = 2.$$

Step 2: coordinates. Put O at the origin with $A = (-2, 0)$. Turning A by $\pm 120^\circ$ about O gives the points at distance 2 in the directions 60° and -60° :

$$B = (1, \sqrt{3}), \quad C = (1, -\sqrt{3}).$$

These are the outer ends of the edges shared by the top and upper-right hexagons, and by the bottom and lower-right hexagons, as in the figure.

Step 3: side and area. $BC = 2\sqrt{3}$. Also $AB = \sqrt{3^2 + (\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}$, as symmetry promised. An equilateral triangle of side s has area $\frac{\sqrt{3}}{4}s^2$, so

$$[ABC] = \frac{\sqrt{3}}{4} \cdot 12 = 3\sqrt{3}.$$

Another way. Split $\triangle ABC$ from O into three congruent triangles OAB , OBC , OCA , each with two sides 2 and a 120° angle between them. Each has area $\frac{1}{2} \cdot 2 \cdot 2 \cdot \sin 120^\circ = \sqrt{3}$, so the total is $3\sqrt{3}$.

Answer: (B) $3\sqrt{3}$

5 Counting & Probability

Source: 2011 AMC 10A #14

A pair of standard 6-sided dice is rolled once. The sum of the numbers rolled determines the diameter of a circle. What is the probability that the numerical value of the area of the circle is less than the numerical value of the circle's circumference?

- (A) $\frac{1}{36}$ (B) $\frac{1}{12}$ (C) $\frac{1}{6}$ (D) $\frac{1}{4}$ (E) $\frac{5}{18}$

Key idea. Turn the geometric condition into a plain inequality on the diameter first. Area grows like the square of the size and circumference only linearly, so the condition can hold only for *small* circles — very few dice sums will work.

Step 1. Let the diameter be d , so the radius is $\frac{d}{2}$. The area is $\pi \left(\frac{d}{2}\right)^2 = \frac{\pi d^2}{4}$ and the circumference is πd . We need

$$\frac{\pi d^2}{4} < \pi d.$$

Since $d > 0$, divide both sides by πd : $\frac{d}{4} < 1$, that is, $d < 4$.

Step 2. The sum of two dice is at least 2, so the good sums are $d = 2$ and $d = 3$. Check: for $d = 3$, area $= 2.25\pi$ and circumference $= 3\pi$, so it works.

Step 3. Of the 36 equally likely ordered rolls, the sum is 2 for $(1, 1)$ and 3 for $(1, 2)$ and $(2, 1)$: three rolls in all. The probability is

$$\frac{3}{36} = \frac{1}{12}.$$

Watch out. At $d = 4$ the area and circumference are *equal* (both 4π), so $d = 4$ does not count. Including it adds the three rolls $(1, 3)$, $(2, 2)$, $(3, 1)$ and gives $\frac{6}{36} = \frac{1}{6}$, choice (C).

Answer: (B) $\frac{1}{12}$

6 Algebra

Source: 2004 AMC 10B #15

Patty has 20 coins consisting of nickels and dimes. If her nickels were dimes and her dimes were nickels, she would have 70 cents more. How much are her coins worth?

- (A) \$1.15 (B) \$1.20 (C) \$1.25 (D) \$1.30 (E) \$1.35

Key idea. The problem never tells you how many nickels or dimes there are — only what happens when they trade places. So look at what the swap does to each coin: every nickel that becomes a dime gains 5 cents, and every dime that becomes a nickel loses 5 cents. The 70-cent gain therefore measures how many *more* nickels than dimes Patty has.

Step 1. Let Patty have n nickels and d dimes, so $n + d = 20$. Swapping changes the value by

$$5n - 5d = 5(n - d) \text{ cents.}$$

This equals 70, so $5(n - d) = 70$ and $n - d = \frac{70}{5} = 14$. The gain is positive, so she has more nickels than dimes.

Step 2. Adding $n + d = 20$ and $n - d = 14$ gives $2n = 34$, so $n = 17$ and $d = 20 - 17 = 3$.

Step 3. Her coins are worth $17 \cdot 5 + 3 \cdot 10 = 85 + 30 = 115$ cents, which is \$1.15.

Check: after the swap she would have 17 dimes and 3 nickels, worth $170 + 15 = 185$ cents, which is indeed 70 cents more than 115.

Another way. Every coin is worth at least 5 cents, so 20 coins are worth 100 cents plus an extra 5 cents for each dime: the value is $100 + 5d$. After the swap it is $100 + 5n$. The choices \$1.15, \$1.20, ..., \$1.35 correspond to $d = 3, 4, 5, 6, 7$. Only $d = 3$ (so $n = 17$) makes $5n - 5d = 85 - 15 = 70$.

Watch out. Reasoning “each nickel gains 5 cents, so $\frac{70}{5} = 14$ nickels” forgets that every dime *loses* 5 cents at the same time. With 14 nickels and 6 dimes the coins are worth $70 + 60 = 130$ cents, choice (D) — but swapping them gives $140 + 30 = 170$ cents, only 40 cents more.

Answer: (A) \$1.15

7 Geometry

Source: 2013 AMC 10A #16

A triangle with vertices $(6, 5)$, $(8, -3)$, and $(9, 1)$ is reflected about the line $x = 8$ to create a second triangle. What is the area of the union of the two triangles?

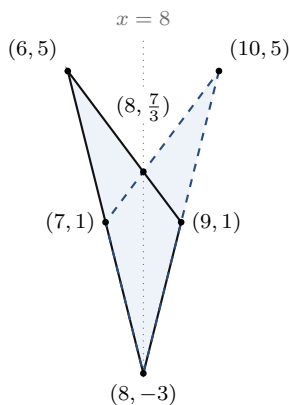
- (A) 9 (B) $\frac{28}{3}$ (C) 10 (D) $\frac{31}{3}$ (E) $\frac{32}{3}$

Key idea. Draw it. The union is symmetric about $x = 8$, so find what lies on one side of that line. The question to ask is whether the reflected triangle sticks out past the original one — and checking where $(9, 1)$ lands after reflecting settles it.

Step 1: the reflection. Reflecting about $x = 8$ sends (x, y) to $(16 - x, y)$. So the second triangle has vertices $(10, 5)$, $(8, -3)$, and $(7, 1)$.

Step 2: where the original crosses $x = 8$. The side from $(6, 5)$ to $(9, 1)$ has slope $\frac{1-5}{9-6} = -\frac{4}{3}$. At $x = 8$ it reaches $y = 5 - \frac{4}{3} \cdot 2 = \frac{7}{3}$. So the line $x = 8$ cuts the original triangle into a left piece with vertices $(6, 5)$, $(8, -3)$, $(8, \frac{7}{3})$ and a right piece with vertices $(8, -3)$, $(9, 1)$, $(8, \frac{7}{3})$.

Step 3: the reflected piece hides inside. The side from $(6, 5)$ to $(8, -3)$ passes through $(7, 1)$: it drops 8 for every 2 across, so after 1 across it is at $5 - 4 = 1$. Hence the reflection of the right piece, with vertices $(8, -3)$, $(7, 1)$, $(8, \frac{7}{3})$, lies inside the left piece. Left of $x = 8$, the union is just the left piece.



Step 4: the area. The left piece has a vertical side from $(8, -3)$ to $(8, \frac{7}{3})$, of length $\frac{7}{3} + 3 = \frac{16}{3}$, and the opposite vertex $(6, 5)$ is 2 units away from it horizontally. Its area is $\frac{1}{2} \cdot \frac{16}{3} \cdot 2 = \frac{16}{3}$. By symmetry the right side contributes the same, so the union has area $2 \cdot \frac{16}{3} = \frac{32}{3}$.

Another way. The union is the shaded “arrowhead” with vertices $(6, 5)$, $(8, -3)$, $(10, 5)$, $(8, \frac{7}{3})$. It is the big triangle $(6, 5)$, $(8, -3)$, $(10, 5)$ — base 4, height 8, area 16 — minus the notch $(6, 5)$, $(8, \frac{7}{3})$, $(10, 5)$ — base 4, height $5 - \frac{7}{3} = \frac{8}{3}$, area $\frac{16}{3}$. That leaves $16 - \frac{16}{3} = \frac{32}{3}$.

Answer: (E) $\frac{32}{3}$

8 Number Theory

Source: 2018 AMC 10A #17

Let S be a set of 6 integers taken from $\{1, 2, \dots, 12\}$ with the property that if a and b are elements of S with $a < b$, then b is not a multiple of a . What is the least possible value of an element in S ?

- (A) 2 (B) 3 (C) 4 (D) 5 (E) 7

Key idea. Group 1 to 12 into “doubling chains” by odd part: each number is an odd number times a power of 2. Inside a chain every number divides the later ones, so S takes at most one number per chain. There are exactly six odd numbers up to 12, so six chains — and a 6-element S must take *exactly* one from each. That rigid structure is what makes the problem solvable.

Step 1: the chains.

$$\{1, 2, 4, 8\}, \quad \{3, 6, 12\}, \quad \{5, 10\}, \quad \{7\}, \quad \{9\}, \quad \{11\}.$$

So 7, 9, 11 are always in S , plus one number from each of the first three chains.

Step 2: the chain $\{3, 6, 12\}$. Since $9 \in S$ and 9 is a multiple of 3, we cannot pick 3. So S contains 6 or 12.

Step 3: the chain $\{1, 2, 4, 8\}$. Both 6 and 12 are multiples of 1 and 2, so neither 1 nor 2 can be in S . The number from this chain is 4 or 8.

Step 4: the least element. Every element of S is now at least 4: the first chain gives 4 or 8, the others give numbers from $\{6, 12\}$, $\{5, 10\}$, and $\{7, 9, 11\}$. So no element is smaller than 4.

Step 5: 4 is achieved. Take $S = \{4, 5, 6, 7, 9, 11\}$. Check every pair: 4 divides none of 5, 6, 7, 9, 11; 5 divides none of 6, 7, 9, 11; 6 divides none of 7, 9, 11; and 7, 9, 11 divide nothing larger in S . So the least possible value is 4.

Watch out. A natural first try is 2 together with the odd numbers 3, 5, 7, 9, 11 — six numbers, smallest 2, choice (A). But 9 is a multiple of 3, so this set fails. In fact Step 2 shows 3 can never be used at all.

Answer: (C) 4

9 Counting & Probability

Source: 2006 AMC 10A #18

A license plate in a certain state consists of 4 digits, not necessarily distinct, and 2 letters, also not necessarily distinct. These six characters may appear in any order, except that the two letters must appear next to each other. How many distinct license plates are possible?

- (A) $10^4 \cdot 26^2$ (B) $10^3 \cdot 26^3$ (C) $5 \cdot 10^4 \cdot 26^2$ (D) $10^2 \cdot 26^4$ (E) $5 \cdot 10^3 \cdot 26^3$

Key idea. The phrase “must appear next to each other” is the signal: when objects must stay together, glue them into one block. Then count in two independent stages: *where* the letters go, and *what* fills each spot.

Step 1: the pattern. Glue the two letters into a single block LL . A plate is then 5 objects in a row: the block and four digits. The block can be first, second, third, fourth or fifth, so there are 5 patterns:

$$LLDDDD, DLLDDD, DDLLDD, DDDLLD, DDDDLL.$$

(Equivalently, the first letter can sit in any of positions 1 to 5 of the six, and the second letter is right after it.)

Step 2: fill the pattern. Whatever the pattern, there are four digit spots, each with 10 choices, and two letter spots, each with 26 choices, since repeats are allowed. That gives $10^4 \cdot 26^2$ ways to fill it.

Step 3: combine. Different patterns give different plates (the letters are in different places), so nothing is counted twice. The total is

$$5 \cdot 10^4 \cdot 26^2.$$

Watch out. Forgetting that the letter block can move gives $10^4 \cdot 26^2$, choice (A) — that counts only one of the five patterns.

Answer: (C) $5 \cdot 10^4 \cdot 26^2$

10 Algebra

Source: 2005 AMC 10B #20

What is the average (mean) of all 5-digit numbers that can be formed by using each of the digits 1, 3, 5, 7, and 8 exactly once?

- (A) 48000 (B) 49999.5 (C) 53332.8 (D) 55555 (E) 56432.8

Key idea. Do not list the 120 numbers. Average *place by place*: by symmetry, each digit appears equally often in each place, so the average digit in every place is the same. Asking “what does the average number look like in each column?” is the move whenever you must average all arrangements.

Step 1: count. There are $5! = 120$ numbers. Fix a place, say the thousands place. Each of the five digits sits there in exactly $4! = 24$ of the numbers, because the other four digits can then be arranged in 24 ways.

Step 2: average digit per place. Since each digit appears equally often in every place, the average digit in each place is the plain average of the five digits:

$$\frac{1 + 3 + 5 + 7 + 8}{5} = \frac{24}{5} = 4.8.$$

Step 3: assemble. A number is (ten-thousands digit)·10000 + ⋯ + (units digit)·1, and averages add, so the average number is

$$4.8(10000 + 1000 + 100 + 10 + 1) = 4.8 \cdot 11111 = 53332.8.$$

Check the last product: $11111 \cdot 4.8 = 44444 + 8888.8 = 53332.8$.

Another way. Add all 120 numbers. In each place every digit appears 24 times, so the digits in that place add up to $24 \cdot (1 + 3 + 5 + 7 + 8) = 24 \cdot 24 = 576$. Each place therefore contributes 576 times its place value. The total is $576 \cdot 11111 = 6399936$, and $6399936 \div 120 = 53332.8$.

Watch out. The middle digit is 5, but the *mean* digit is 4.8. Using 5 in every place gives $5 \cdot 11111 = 55555$, choice (D).

Answer: (C) 53332.8