AMC 10

SET 6

PRACTICE EXAMINATION

TIME ALLOWED

75:00

25 QUESTIONS • 75 MINUTES • 150 POINTS • ADVANCED LEVEL

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2025) and appear in their original wording. Advanced level: each problem sits two or three places earlier than it did in its original contest, and problems 21–25 come from original positions 23–25. Problems © Mathematical Association of America.

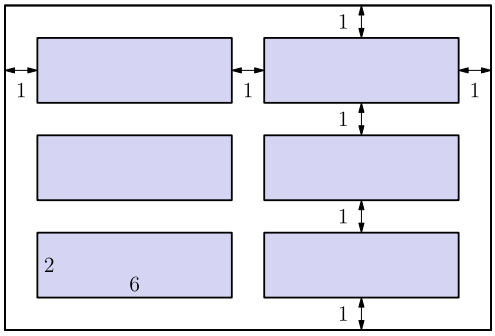
INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Tamara has three rows of two 6-feet by 2-feet flower beds in her garden. The beds are separated and also surrounded by 1-foot-wide walkways, as shown on the diagram. What is the total area of the walkways, in square feet?



(A) 72 **(B)** 78 **(C)** 90 **(D)** 120 **(E)** 150

2. Camilla had twice as many blueberry jelly beans as cherry jelly beans. After eating 10 pieces of each kind, she now has three times as many blueberry jelly beans as cherry jelly beans. How many blueberry jelly beans did she originally have?

(A) 10 **(B)** 20 **(C)** 30 **(D)** 40 **(E)** 50

3. The six-digit number $\underline{2}\,\underline{0}\,\underline{2}\,\underline{1}\,\underline{0}\,\underline{A}$ is prime for only one digit A. What is A?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

4. The 2007 AMC 10 will be scored by awarding 6 points for each correct response, 0 points for each incorrect response, and 1.5 points for each problem left unanswered. After looking over the 25 problems, Sarah has decided to attempt the first 22 and leave only the last 3 unanswered. How many of the first 22 problems must she solve correctly in order to score at least 100 points?

(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

5. How many positive even multiples of 3 less than 2020 are perfect squares?

(A) 7 **(B)** 8 **(C)** 9 **(D)** 10 **(E)** 12

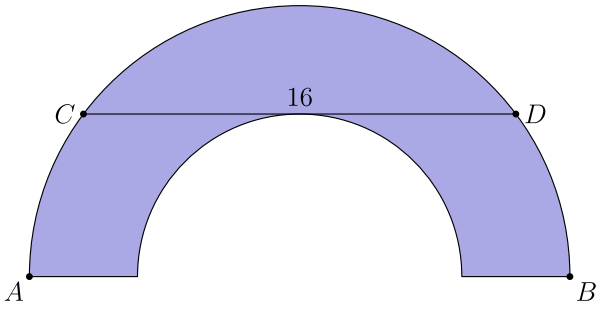
6. Positive integers a, b, and 2009, with $a < b < 2009$, form a geometric sequence with an integer ratio. What is a?

(A) 7 **(B)** 41 **(C)** 49 **(D)** 287 **(E)** 2009

7. Seven cubes, whose volumes are $1, 8, 27, 64, 125, 216$, and 343 cubic units, are stacked vertically to form a tower in which the volumes of the cubes decrease from bottom to top. Except for the bottom cube, the bottom face of each cube lies completely on top of the cube below it. What is the total surface area of the tower (including the bottom) in square units?

(A) 644 **(B)** 658 **(C)** 664 **(D)** 720 **(E)** 749

8. A semicircle has diameter AB and chord CD of length 16 parallel to AB. A smaller semicircle with diameter on AB and tangent to CD is cut from the larger semicircle, as shown below.



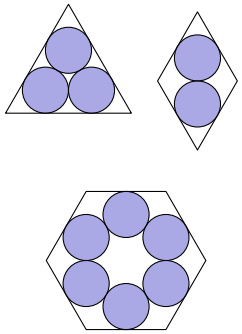
What is the area of the resulting figure, shown shaded?

(A) 16π **(B)** 24π **(C)** 32π **(D)** 48π **(E)** 64π

9. Twelve fair dice are rolled. What is the probability that the product of the numbers on the top faces is prime?

(A) $\left(\frac{1}{12}\right)^{12}$ **(B)** $\left(\frac{1}{6}\right)^{12}$ **(C)** $2\left(\frac{1}{6}\right)^{11}$ **(D)** $\frac{5}{2}\left(\frac{1}{6}\right)^{11}$ **(E)** $\left(\frac{1}{6}\right)^{10}$

10. The figure below shows an equilateral triangle, a rhombus with a $60°$ angle, and a regular hexagon, each of them containing some mutually tangent congruent disks. Let T, R, and H, respectively, denote the ratio in each case of the total area of the disks to the area of the enclosing polygon.



Which of the following is true?

(A) $T = H = R$ **(B)** $H < R = T$ **(C)** $H = R < T$

(D) $H < R < T$ **(E)** $H < T < R$

11. An *iterative average* of the numbers $1, 2, 3, 4$, and 5 is computed the following way. Arrange the five numbers in some order. Find the mean of the first two numbers, then find the mean of that with the third number, then the mean of that with the fourth number, and finally the mean of that with the fifth number. What is the difference between the largest and smallest possible values that can be obtained using this procedure?

(A) $\frac{31}{16}$ **(B)** 2 **(C)** $\frac{17}{8}$ **(D)** 3 **(E)** $\frac{65}{16}$

12. A wire is cut into two pieces, one of length a and the other of length b. The piece of length a is bent to form an equilateral triangle, and the piece of length b is bent to form a regular hexagon. The triangle and the hexagon have equal area. What is $\frac{a}{b}$?

(A) 1 **(B)** $\frac{\sqrt{6}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3\sqrt{2}}{2}$

13. The sum of an infinite geometric series is a positive number S, and the second term in the series is 1. What is the smallest possible value of S?

(A) $\frac{1+\sqrt{5}}{2}$ **(B)** 2 **(C)** $\sqrt{5}$ **(D)** 3 **(E)** 4

14. Points A and C lie on a circle centered at O, each of $\overline{BA}$ and $\overline{BC}$ are tangent to the circle, and $\triangle ABC$ is equilateral. The circle intersects $\overline{BO}$ at D. What is $\frac{BD}{BO}$?

(A) $\frac{\sqrt{2}}{3}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{\sqrt{3}}{3}$ **(D)** $\frac{\sqrt{2}}{2}$ **(E)** $\frac{\sqrt{3}}{2}$

15. Let points $A = (0,0), B = (1,2), C = (3,3)$, and $D = (4,0)$. Quadrilateral $ABCD$ is cut into equal area pieces by a line passing through A. This line intersects $\overline{CD}$ at point $\left(\frac{p}{q}, \frac{r}{s}\right)$, where these fractions are in lowest terms. What is $p+q+r+s$?

(A) 54 **(B)** 58 **(C)** 62 **(D)** 70 **(E)** 75

16. Suppose a, b, and c are positive integers such that

$$\frac{a}{14} + \frac{b}{15} = \frac{c}{210}.$$

Which of the following statements are necessarily true?

I. If $\gcd(a,14) = 1$ or $\gcd(b,15) = 1$ or both, then $\gcd(c,210) = 1$.

II. If $\gcd(c,210) = 1$, then $\gcd(a,14) = 1$ or $\gcd(b,15) = 1$ or both.

III. $\gcd(c,210) = 1$ if and only if $\gcd(a,14) = \gcd(b,15) = 1$.

(A) I, II, and III **(B)** I only **(C)** I and II only

(D) III only **(E)** II and III only

17. Three different pairs of shoes are placed in a row so that no left shoe is next to a right shoe from a different pair. In how many ways can these six shoes be lined up?

(A) 60 **(B)** 72 **(C)** 90 **(D)** 108 **(E)** 120

18. Cozy the Cat and Dash the Dog are going up a staircase with a certain number of steps. However, instead of walking up the steps one at a time, both Cozy and Dash jump.

Cozy goes two steps up with each jump (though if necessary, he will just jump the last step).

Dash goes five steps up with each jump (though if necessary, he will just jump the last steps if there are fewer than 5 steps left).

Suppose Dash takes 19 fewer jumps than Cozy to reach the top of the staircase. Let s denote the sum of all possible numbers of steps this staircase can have. What is the sum of the digits of s?

(A) 9 **(B)** 11 **(C)** 12 **(D)** 13 **(E)** 15

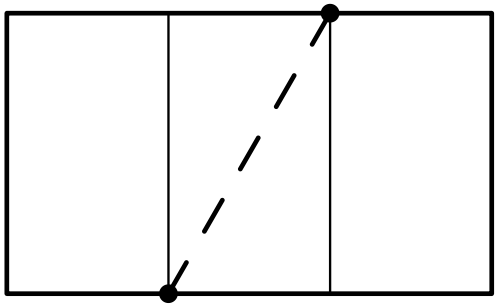
19. Let $(a_1, a_2, \ldots, a_{10})$ be a list of the first 10 positive integers such that for each $2 \leq i \leq 10$ either $a_i + 1$ or $a_i - 1$ or both appear somewhere before a_i in the list. How many such lists are there?

(A) 120 **(B)** 512 **(C)** 1024 **(D)** 181,440 **(E)** 362,880

20. Real numbers x and y are chosen independently and uniformly at random from the interval $[0, 1]$. Which of the following numbers is closest to the probability that x, y, and 1 are the side lengths of an obtuse triangle?

(A) 0.21 **(B)** 0.25 **(C)** 0.29 **(D)** 0.50 **(E)** 0.79

21. A rectangular piece of paper whose length is $\sqrt{3}$ times the width has area A. The paper is divided into three equal sections along the opposite lengths, and then a dotted line is drawn from the first divider to the second divider on the opposite side as shown. The paper is then folded flat along this dotted line to create a new shape with area B. What is the ratio $B : A$?



(A) 1 : 2 **(B)** 3 : 5 **(C)** 2 : 3 **(D)** 3 : 4 **(E)** 4 : 5

22. The numbers $1, 2, 3, 4, 5$ are to be arranged in a circle. An arrangement is *bad* if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5.

23. Consider functions f that satisfy

$$|f(x) - f(y)| \leq \frac{1}{2}|x - y|$$

for all real numbers x and y. Of all such functions that also satisfy the equation $f(300) = f(900)$, what is the greatest possible value of

$$f(f(800)) - f(f(400))?$$

(A) 25 **(B)** 50 **(C)** 100 **(D)** 150 **(E)** 200

24. Jim starts with a positive integer n and creates a sequence of numbers. Each successive number is obtained by subtracting the largest possible integer square less than or equal to the current number until zero is reached. For example, if Jim starts with $n = 55$, then his sequence contains 5 numbers:

$$\begin{array}{rcrcr} & & & & 55 \\ 55 & - & 7^2 & = & 6 \\ 6 & - & 2^2 & = & 2 \\ 2 & - & 1^2 & = & 1 \\ 1 & - & 1^2 & = & 0 \end{array}$$

Let N be the smallest number for which Jim's sequence has 8 numbers. What is the units digit of N?

(A) 1 **(B)** 3 **(C)** 5 **(D)** 7 **(E)** 9

25. Let S be a square of side length 1. Two points are chosen independently at random on the sides of S. The probability that the straight-line distance between the points is at least $\frac{1}{2}$ is $\frac{a - b\pi}{c}$, where a, b, and c are positive integers with $\gcd(a, b, c) = 1$. What is $a + b + c$?

(A) 59 **(B)** 60 **(C)** 61 **(D)** 62 **(E)** 63

END OF EXAMINATION

ANSWER SHEET

AMC 10 • PRACTICE EXAMINATION • SET 6

Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**14**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**15**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**16**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**17**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**18**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**19**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**20**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**21**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**22**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**23**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**24**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**25**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ										

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 150