AMC 10

SET 8

PRACTICE EXAMINATION

TIME ALLOWED

75:00

25 QUESTIONS • 75 MINUTES • 150 POINTS • ADVANCED LEVEL

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2025) and appear in their original wording. Advanced level: each problem sits two or three places earlier than it did in its original contest, and problems 21–25 come from original positions 23–25. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Alicia earns \$20 per hour, of which 1.45% is deducted to pay local taxes. How many cents per hour of Alicia's wages are used to pay local taxes?

(A) 0.0029 **(B)** 0.029 **(C)** 0.29 **(D)** 2.9 **(E)** 29

2. A month with 31 days has the same number of Mondays and Wednesdays. How many of the seven days of the week could be the first day of this month?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

3. Joey and his five brothers are ages $3, 5, 7, 9, 11$, and 13. One afternoon two of his brothers whose ages sum to 16 went to the movies, two brothers younger than 10 went to play baseball, and Joey and the 5-year-old stayed home. How old is Joey?

(A) 3 **(B)** 7 **(C)** 9 **(D)** 11 **(E)** 13

4. For how many (not necessarily positive) integer values of n is the following value an integer?

$$4000 \cdot \left(\frac{2}{5}\right)^n$$

(A) 3 **(B)** 4 **(C)** 6 **(D)** 8 **(E)** 9

5. Two years ago Pete was three times as old as his cousin Claire. Two years before that, Pete was four times as old as Claire. In how many years will the ratio of their ages be $2 : 1$?

(A) 2 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 8

6. One fair die has faces $1, 1, 2, 2, 3, 3$ and another has faces $4, 4, 5, 5, 6, 6$. The dice are rolled and the numbers on the top faces are added. What is the probability that the sum will be odd?

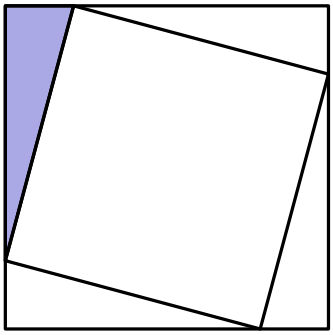
(A) $\frac{1}{3}$ **(B)** $\frac{4}{9}$ **(C)** $\frac{1}{2}$ **(D)** $\frac{5}{9}$ **(E)** $\frac{2}{3}$

7. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

(A) 2 **(B)** 4 **(C)** 7 **(D)** 8 **(E)** 9

8. A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



(A) $\frac{1}{5}$ **(B)** $\frac{1}{4}$ **(C)** $2-\sqrt{3}$ **(D)** $\sqrt{3}-\sqrt{2}$ **(E)** $\sqrt{2}-1$

9. Carl decided to fence in his rectangular garden. He bought 20 fence posts, placed one on each of the four corners, and spaced out the rest evenly along the edges of the garden, leaving exactly 4 yards between neighboring posts. The longer side of his garden, including the corners, has twice as many posts as the shorter side, including the corners. What is the area, in square yards, of Carl's garden?

(A) 256 **(B)** 336 **(C)** 384 **(D)** 448 **(E)** 512

10. Tom's age is T years, which is also the sum of the ages of his three children. His age N years ago was twice the sum of their ages then. What is $\frac{T}{N}$?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 5 **(E)** 6

11. Suppose that S is a subset of $\{1, 2, 3, \cdots, 25\}$ such that the sum of any two (not necessarily distinct) elements of S is never an element of S. What is the maximum number of elements S may contain?

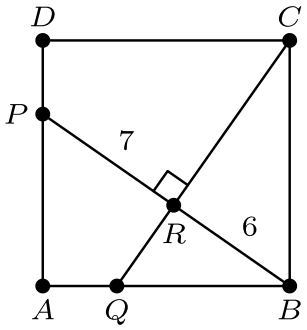
(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

12. Real numbers x and y satisfy $x + y = 4$ and $x \cdot y = -2$. What is the value of

$$x + \frac{x^3}{y^2} + \frac{y^3}{x^2} + y?$$

(A) 360 **(B)** 400 **(C)** 420 **(D)** 440 **(E)** 480

13. In square $ABCD$, points P and Q lie on $\overline{AD}$ and $\overline{AB}$, respectively. Segments $\overline{BP}$ and $\overline{CQ}$ intersect at right angles at R, with $BR = 6$ and $PR = 7$. What is the area of the square?



(A) 85 **(B)** 93 **(C)** 100 **(D)** 117 **(E)** 125

14. The graph of

$$f(x) = ||\lfloor x \rfloor| - |\lfloor 1 - x \rfloor||$$

is symmetric about which of the following? (Here $\lfloor x \rfloor$ is the greatest integer not exceeding x.)

(A) the y-axis **(B)** the line $x = 1$ **(C)** the origin

(D) the point $\left(\frac{1}{2}, 0\right)$ **(E)** the point $(1, 0)$

15. Bernardo randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

and arranges them in descending order to form a 3-digit number. Silvia randomly picks 3 distinct numbers from the set

$$\{1, 2, 3, 4, 5, 6, 7, 8\}$$

and also arranges them in descending order to form a 3-digit number. What is the probability that Bernardo's number is larger than Silvia's number?

(A) $\frac{47}{72}$ **(B)** $\frac{37}{56}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{49}{72}$ **(E)** $\frac{39}{56}$

16. Circle A has radius 100. Circle B has an integer radius $r < 100$ and remains internally tangent to circle A as it rolls once around the circumference of circle A. The two circles have the same points of tangency at the beginning and end of circle B's trip. How many possible values can r have?

(A) 4 **(B)** 8 **(C)** 9 **(D)** 50 **(E)** 90

17. Paula the painter and her two helpers each paint at constant, but different, rates. They always start at 8 : 00 AM, and all three always take the same amount of time to eat lunch.

On Monday the three of them painted 50% of a house, quitting at 4 : 00 PM. On Tuesday, when Paula wasn't there, the two helpers painted only 24% of the house and quit at 2 : 12 PM. On Wednesday Paula worked by herself and finished the house by working until 7 : 12 P.M.

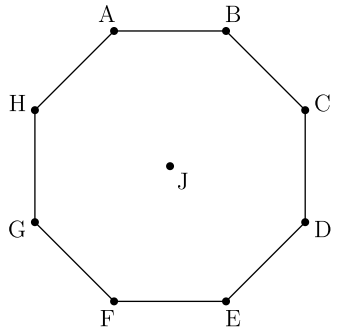
How long, in minutes, was each day's lunch break?

(A) 30 **(B)** 36 **(C)** 42 **(D)** 48 **(E)** 60

18. Rhombus $ABCD$ has side length 2 and $\angle B = 120^\circ$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{\sqrt{3}}{2}$ **(C)** $\frac{2\sqrt{3}}{3}$ **(D)** $1+\frac{\sqrt{3}}{3}$ **(E)** 2

19. The regular octagon $ABCDEFGH$ has its center at J. Each of the vertices and the center are to be associated with one of the digits 1 through 9, with each digit used once, in such a way that the sums of the numbers on the lines AJE, BJF, CJG, and DJH are all equal. In how many ways can this be done?



(A) 384 **(B)** 576 **(C)** 1152 **(D)** 1680 **(E)** 3456

20. A rectangular grid of squares has 141 rows and 91 columns. Each square has room for two numbers. Horace and Vera each fill in the grid by putting the numbers from 1 through $141 \times 91 = 12{,}831$ into the squares. Horace fills the grid horizontally: he puts 1 through 91 in order from left to right into row 1, puts 92 through 182 into row 2 in order from left to right, and continues similarly through row 141. Vera fills the grid vertically: she puts 1 through 141 in order from top to bottom into column 1, then 142 through 282 into column 2 in order from top to bottom, and continues similarly through column 91. How many squares get two copies of the same number?

(A) 7 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 19

21. A solid tetrahedron is sliced off a solid wooden unit cube by a plane passing through two nonadjacent vertices on one face and one vertex on the opposite face not adjacent to either of the first two vertices. The tetrahedron is discarded and the remaining portion of the cube is placed on a table with the cut surface face down. What is the height of this object?

(A) $\frac{\sqrt{3}}{3}$ **(B)** $\frac{2\sqrt{2}}{3}$ **(C)** 1 **(D)** $\frac{2\sqrt{3}}{3}$ **(E)** $\sqrt{2}$

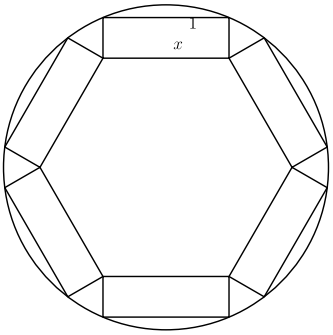
22. Call a positive integer fair if no digit is used more than once, it has no 0s, and no digit is adjacent to two greater digits. For example, 23, 196, and 12463 are fair, but 1546, 320, and 34321 are not fair. How many fair positive integers are there?

(A) 511 **(B)** 2,584 **(C)** 9,841 **(D)** 17,711 **(E)** 19,682

23. The interior of a quadrilateral is bounded by the graphs of $(x + ay)^2 = 4a^2$ and $(ax - y)^2 = a^2$, where a is a positive real number. What is the area of this region in terms of a, valid for all $a > 0$?

(A) $\frac{8a^2}{(a+1)^2}$ **(B)** $\frac{4a}{a+1}$ **(C)** $\frac{8a}{a+1}$ **(D)** $\frac{8a^2}{a^2+1}$ **(E)** $\frac{8a}{a^2+1}$

24. A round table has radius 4. Six rectangular place mats are placed on the table. Each place mat has width 1 and length x as shown. They are positioned so that each mat has two corners on the edge of the table, these two corners being end points of the same side of length x. Further, the mats are positioned so that the inner corners each touch an inner corner of an adjacent mat. What is x?



(A) $2\sqrt{5}-\sqrt{3}$ **(B)** 3 **(C)** $\frac{3\sqrt{7}-\sqrt{3}}{2}$ **(D)** $2\sqrt{3}$ **(E)** $\frac{5+2\sqrt{3}}{2}$

25. How many integers between 100 and 999, inclusive, have the property that some permutation of its digits is a multiple of 11 between 100 and 999? For example, both 121 and 211 have this property.

(A) 226 **(B)** 243 **(C)** 270 **(D)** 469 **(E)** 486

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**14**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**15**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**16**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**17**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**18**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**19**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**20**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**21**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**22**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**23**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**24**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**25**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ										

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 150