AMC 10

SET 22

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • UPPER BAND (PROBLEMS 13--23)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 13–23 of past AMC 10 contests (2000–2025) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

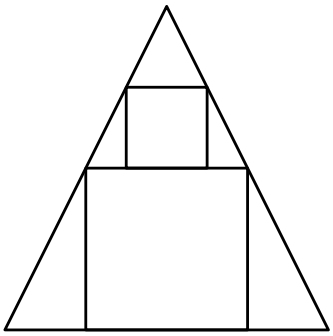
INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 13 to 23 of an AMC 10 — the upper band of the paper, where strong scores are separated from good ones. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

Pacing guide

This band is harder than the time allows comfortably: aim for problems 1–5 in about 12 minutes and give problems 6–10 the remaining 18. Skipping one hard problem to finish the rest cleanly is a winning strategy. If a problem has taken three minutes with no plan in sight, move on and come back.

1. A square with side length 3 is inscribed in an isosceles triangle with one side of the square along the base of the triangle. A square with side length 2 has two vertices on the other square and the other two on sides of the triangle, as shown. What is the area of the triangle?



(A) $19\frac{1}{4}$ **(B)** $20\frac{1}{4}$ **(C)** $21\frac{3}{4}$ **(D)** $22\frac{1}{2}$ **(E)** $23\frac{3}{4}$

2. A list of 9 real numbers consists of $1, 2.2, 3.2, 5.2, 6.2, 7$, as well as x, y, z with $x \leq y \leq z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3

(D) 4 **(E)** infinitely many

3. In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that $EF \perp DB$, and A and C lie on DE and DF, respectively. What is EF?

(A) 9 **(B)** 10 **(C)** $\frac{125}{12}$ **(D)** $\frac{103}{9}$ **(E)** 12

5. What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor$? (Recall that $\lfloor x\rfloor$ denotes the greatest integer less than or equal to x.)

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

6. A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?

(A) $\frac{1}{120}$ **(B)** $\frac{1}{32}$ **(C)** $\frac{1}{20}$ **(D)** $\frac{3}{20}$ **(E)** $\frac{1}{6}$

7. Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

8. For how many positive integers $n \leq 1000$ is

$$\left\lfloor \frac{998}{n} \right\rfloor + \left\lfloor \frac{999}{n} \right\rfloor + \left\lfloor \frac{1000}{n} \right\rfloor$$

not divisible by 3? (Recall that $\lfloor x\rfloor$ is the greatest integer less than or equal to x.)

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

9. A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

10. Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1)$, $B(-1,1)$, $C(-1,-1)$, and $D(1,-1)$. Consider the following four transformations:

- L, a rotation of 90° counterclockwise around the origin;
- R, a rotation of 90° clockwise around the origin;
- H, a reflection across the x-axis; and
- V, a reflection across the y-axis.

Each of these transformations maps the square onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1,1)$ to $(-1,-1)$ and would send the vertex B at $(-1,1)$ to itself. How many sequences of 20 transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, R, R, V, H is one sequence of 4 transformations that will send the vertices back to their original positions.)

(A) 2^{37} **(B)** $3 \cdot 2^{36}$ **(C)** 2^{38} **(D)** $3 \cdot 2^{37}$ **(E)** 2^{39}

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60