AMC 10

SET 23

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • UPPER BAND (PROBLEMS 13--23)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 13–23 of past AMC 10 contests (2000–2025) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 13 to 23 of an AMC 10 — the upper band of the paper, where strong scores are separated from good ones. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

Pacing guide

This band is harder than the time allows comfortably: aim for problems 1–5 in about 12 minutes and give problems 6–10 the remaining 18. Skipping one hard problem to finish the rest cleanly is a winning strategy. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Let n be a positive integer and d be a digit such that the value of the numeral $\underline{32d}$ in base n equals 263, and the value of the numeral $\underline{324}$ in base n equals the value of the numeral $\underline{11d1}$ in base six. What is $n + d$?

(A) 10 **(B)** 11 **(C)** 13 **(D)** 15 **(E)** 16

2. Quadrilateral $ABCD$ with side lengths $AB = 7, BC = 24, CD = 20, DA = 15$ is inscribed in a circle. The area interior to the circle but exterior to the quadrilateral can be written in the form $\frac{a\pi - b}{c}$, where a, b, and c are positive integers such that a and c have no common prime factor. What is $a + b + c$?

(A) 260 **(B)** 855 **(C)** 1235 **(D)** 1565 **(E)** 1997

3. On a 50-question multiple choice math contest, students receive 4 points for a correct answer, 0 points for an answer left blank, and -1 point for an incorrect answer. Jesse's total score on the contest was 99. What is the maximum number of questions that Jesse could have answered correctly?

(A) 25 **(B)** 27 **(C)** 29 **(D)** 31 **(E)** 33

4. There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other 9 people: the 2 people standing next to her or him, as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?

(A) 11 **(B)** 12 **(C)** 13 **(D)** 14 **(E)** 15

5. In rectangle $PQRS$, $PQ = 8$ and $QR = 6$. Points A and B lie on PQ, points C and D lie on QR, points E and F lie on RS, and points G and H lie on SP so that $AP = BQ < 4$ and the convex octagon $ABCDEFGH$ is equilateral. The length of a side of this octagon can be expressed in the form $k + m\sqrt{n}$, where k, m, and n are integers and n is not divisible by the square of any prime. What is $k + m + n$?

(A) 1 **(B)** 7 **(C)** 21 **(D)** 92 **(E)** 106

6. The first three terms of a geometric sequence are the integers a, 720, and b, where $a < 720 < b$. What is the sum of the digits of the least possible value of b?

(A) 9 **(B)** 12 **(C)** 16 **(D)** 18 **(E)** 21

7. Regular polygons with 5, 6, 7, and 8 sides are inscribed in the same circle. No two of the polygons share a vertex, and no three of their sides intersect at a common point. At how many points inside the circle do two of their sides intersect?

(A) 52 **(B)** 56 **(C)** 60 **(D)** 64 **(E)** 68

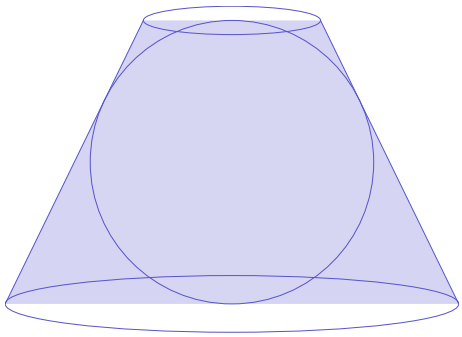
8. Positive integers a and b are such that the graphs of $y = ax + 5$ and $y = 3x + b$ intersect the x-axis at the same point. What is the sum of all possible x-coordinates of these points of intersection?

(A) -20 **(B)** -18 **(C)** -15 **(D)** -12 **(E)** -8

9. Two cubical dice each have removable numbers 1 through 6. The twelve numbers on the two dice are removed, put into a bag, then drawn one at a time and randomly reattached to the faces of the cubes, one number to each face. The dice are then rolled and the numbers on the two top faces are added. What is the probability that the sum is 7?

(A) $\frac{1}{9}$ **(B)** $\frac{1}{8}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{2}{11}$ **(E)** $\frac{1}{5}$

10. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



(A) $\frac{3}{2}$ **(B)** $\frac{1+\sqrt{5}}{2}$ **(C)** $\sqrt{3}$ **(D)** 2 **(E)** $\frac{3+\sqrt{5}}{2}$

END OF EXAMINATION

ANSWER SHEET

AMC 10 • HALF-LENGTH PRACTICE • SET 23

Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60