AMC 10

SET 22

HALF-LENGTH PRACTICE

SOLUTIONS

Answer key • Sources • Step-by-step solutions

All 10 problems come from positions 13–23 of past AMC 10 contests (2000–2025) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

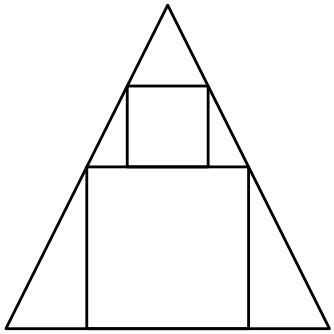
Answer Key and Sources

No.	Ans.	Source	Topic
1	**B**	2021 Fall AMC 10B #13	Geometry
2	**C**	2024 AMC 10B #15	Algebra
3	**D**	2010 AMC 10A #15	Counting & Probability
4	**C**	2009 AMC 10A #17	Geometry
5	**D**	2025 AMC 10B #18	Algebra
6	**C**	2021 AMC 10B #18	Counting & Probability
7	**C**	2015 AMC 10A #21	Geometry
8	**A**	2020 AMC 10A #22	Algebra
9	**B**	2008 AMC 10B #23	Number Theory
10	**C**	2020 AMC 10B #23	Counting & Probability

1 Geometry

Source: 2021 Fall AMC 10B #13

A square with side length 3 is inscribed in an isosceles triangle with one side of the square along the base of the triangle. A square with side length 2 has two vertices on the other square and the other two on sides of the triangle, as shown. What is the area of the triangle?



(A) $19\frac{1}{4}$ **(B)** $20\frac{1}{4}$ **(C)** $21\frac{3}{4}$ **(D)** $22\frac{1}{2}$ **(E)** $23\frac{3}{4}$

Key idea. Every horizontal segment across the triangle has width proportional to its distance from the top vertex (similar triangles). The two squares give us two such widths at two known heights — enough to find the height of the triangle.

Let the triangle have height H and base B, with the base on the ground.

Step 1. The large square stands on the base, so its top side is at height 3 and has width 3. The small square sits on top of it, so its top side is at height $3+2=5$ and has width 2. Both top sides have their ends on the slanted sides of the triangle.

Step 2. A horizontal segment at height h cuts off a small triangle at the top that is similar to the whole triangle, with height $H-h$. So the width is proportional to $H-h$:

$$\frac{3}{H-3}=\frac{2}{H-5}.$$

Cross-multiplying, $3(H-5)=2(H-3)$, so $3H-15=2H-6$ and $H=9$.

Step 3. Compare the full base with the top of the large square: $\dfrac{B}{H}=\dfrac{3}{H-3}$, so $\dfrac{B}{9}=\dfrac{3}{6}$ and $B=\dfrac{9}{2}$.

Check: at height 5 the width is $\frac{9}{2}\cdot\frac{9-5}{9}=2$, as it should be.

Step 4. The area is

$$\frac{1}{2}\cdot B\cdot H=\frac{1}{2}\cdot\frac{9}{2}\cdot 9=\frac{81}{4}=20\tfrac{1}{4}.$$

Answer: (B) $20\frac{1}{4}$

2 Algebra

Source: 2024 AMC 10B #15

A list of 9 real numbers consists of $1, 2.2, 3.2, 5.2, 6.2, 7$, as well as x, y, z with $x \le y \le z$. The range of the list is 7, and the mean and median are both positive integers. How many ordered triples (x, y, z) are possible?

(A) 1 **(B)** 2 **(C)** 3

(D) 4 **(E)** infinitely many

Key idea. The six fixed numbers already run from 1 to 7, a span of 6. A range of 7 means the smallest number drops below 1, or the largest rises above 7, or both — three cases. In each case the integer mean fixes the sum of the unknowns, and the integer median then pins them down.

The fixed numbers sum to $1 + 2.2 + 3.2 + 5.2 + 6.2 + 7 = 24.8$. If the mean is the integer k, then $x + y + z = 9k - 24.8$. The median is the 5th smallest of the nine numbers.

Case 1: $z \le 7$. The largest number is 7, so the smallest is 0: $x = 0$. Then $y + z = 9k - 24.8$ lies between 0 and 14, so $k = 3$ or 4.

If $k = 3$, then $y + z = 2.2$, so $0, 1, y, z$ are all at most 2.2 and the median is 2.2 — not an integer.

If $k = 4$, then $y + z = 11.2$; with $z \le 7$ this forces $y \ge 4.2$. Four numbers $0, 1, 2.2, 3.2$ lie below y, so the median is the smaller of y and 5.2. It is an integer only if $y = 5$, giving $z = 6.2$: the triple $(0, 5, 6.2)$.

Case 2: $x \ge 1$. The smallest number is 1, so $z = 8$. Then $x + y = 9k - 32.8$ lies between 2 and 16, so $k = 4$ or 5.

If $k = 4$, then $x + y = 3.2$, so $x, y \le 2.2$ and the median is 3.2 — not an integer.

If $k = 5$, then $x + y = 12.2$ with $y \le 8$, so $4.2 \le x \le 6.1 \le y$. Below x lie only $1, 2.2, 3.2$, so the median is the second smallest of $x, y, 5.2, 6.2, 7, 8$. If $x \le 5.2$ that is 5.2; if $x > 5.2$ it is x itself. So $x = 6$, $y = 6.2$: the triple $(6, 6.2, 8)$.

Case 3: $x < 1$ and $z > 7$. Now $z = x + 7$, and $x > 0$. The total $31.8 + 2x + y$ is greater than 31.8 and less than $31.8 + 2 + 8 = 41.8$, so it must be 36: $k = 4$ and $y = 4.2 - 2x$, which lies strictly between 2.2 and 4.2. The four smallest are $x, 1, 2.2$ and the smaller of y and 3.2, so the median is the larger of y and 3.2. It is an integer only if $y = 4$, giving $x = 0.1$, $z = 7.1$: the triple $(0.1, 4, 7.1)$.

Exactly 3 triples work.

Watch out. Case 3, where *both* ends move, is easy to forget. Checking only the first two cases gives 2, choice (B).

Answer: (C) 3

3 Counting & Probability

Source: 2010 AMC 10A #15

In a magical swamp there are two species of talking amphibians: toads, whose statements are always true, and frogs, whose statements are always false. Four amphibians, Brian, Chris, LeRoy, and Mike live together in this swamp, and they make the following statements.

Brian: "Mike and I are different species."

Chris: "LeRoy is a frog."

LeRoy: "Chris is a frog."

Mike: "Of the four of us, at least two are toads."

How many of these amphibians are frogs?

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Key idea. Start with the pair whose statements are about each other: Chris and LeRoy. Pinning down that pair first tells us how many toads there are among them, and the rest follows.

Step 1. If Chris and LeRoy were both frogs, then "LeRoy is a frog" would be true — but a frog cannot say something true. If both were toads, "LeRoy is a frog" would be false — but a toad cannot lie. So *exactly one* of Chris and LeRoy is a toad, and the other is a frog.

Step 2. Suppose Brian is a toad. Then his statement is true, so Mike is a frog. But then Brian and the toad from Step 1 are two toads, which makes Mike's statement "at least two are toads" true — impossible for a frog. So Brian is a frog.

Step 3. Brian's statement is false, so Mike is the *same* species as Brian: Mike is a frog. Check Mike: the only toad is the one from Step 1, so "at least two are toads" is false, as a frog's statement must be. ✓

Step 4. The frogs are Brian, Mike, and one of Chris and LeRoy: 3 frogs. (Either choice of toad in Step 1 works, and both give the same count.)

Answer: (D) 3

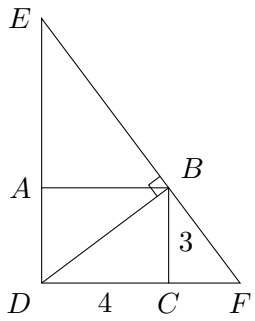
4 Geometry

Source: 2009 AMC 10A #17

Rectangle $ABCD$ has $AB = 4$ and $BC = 3$. Segment EF is constructed through B so that $EF \perp DB$, and A and C lie on DE and DF, respectively. What is EF?

(A) 9 **(B)** 10 **(C)** $\frac{125}{12}$ **(D)** $\frac{103}{9}$ **(E)** 12

Key idea. A line perpendicular to the diagonal at B creates right angles at B, and the rectangle's sides drop perpendiculars from B onto DE and DF. Right triangles with altitudes inside them mean similar triangles.



Step 1. The diagonal is $DB = \sqrt{4^2 + 3^2} = 5$. Since A lies on DE and C lies on DF, the point E is on line DA beyond A, and F is on line DC beyond C.

Step 2. Triangle EBD has a right angle at B, and $BA \perp ED$, so BA is its altitude. Triangles EAB and BAD are then similar (each shares an acute angle with $\triangle EBD$), with $E \leftrightarrow B$, $A \leftrightarrow A$, $B \leftrightarrow D$. Hence

$$\frac{EB}{BD} = \frac{AB}{AD} \quad \Longrightarrow \quad \frac{EB}{5} = \frac{4}{3} \quad \Longrightarrow \quad EB = \frac{20}{3}.$$

Step 3. In the same way, triangle FBD has a right angle at B with altitude BC, so $\triangle FCB \sim \triangle BCD$ and

$$\frac{BF}{BD} = \frac{BC}{CD} \quad \Longrightarrow \quad \frac{BF}{5} = \frac{3}{4} \quad \Longrightarrow \quad BF = \frac{15}{4}.$$

Step 4. B lies between E and F, so

$$EF = EB + BF = \frac{20}{3} + \frac{15}{4} = \frac{80}{12} + \frac{45}{12} = \frac{125}{12}.$$

Another way. Use coordinates $D = (0,0)$, $C = (4,0)$, $B = (4,3)$, $A = (0,3)$. Line DB has slope $\frac{3}{4}$, so line EF has slope $-\frac{4}{3}$: $y - 3 = -\frac{4}{3}(x - 4)$. Setting $x = 0$ gives $E = \left(0, \frac{25}{3}\right)$; setting $y = 0$ gives $F = \left(\frac{25}{4}, 0\right)$. Then

$$EF = \sqrt{\left(\frac{25}{4}\right)^2 + \left(\frac{25}{3}\right)^2} = 25\sqrt{\frac{1}{16} + \frac{1}{9}} = 25 \cdot \frac{5}{12} = \frac{125}{12}.$$

Answer: (C) $\dfrac{125}{12}$

5 Algebra

Source: 2025 AMC 10B #18

What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor+\lfloor\sqrt{2}\rfloor+\lfloor\sqrt{3}\rfloor+\cdots+\lfloor\sqrt{2024}\rfloor+\lfloor\sqrt{2025}\rfloor$? (Recall that $\lfloor x\rfloor$ denotes the greatest integer less than or equal to x.)

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 8

Key idea. The value $\lfloor\sqrt{n}\rfloor$ stays constant for long stretches and only jumps at perfect squares. Count how many n give each value, then add up block by block. Noticing that $2025 = 45^2$ tells you where the blocks stop.

Step 1: the blocks. $\lfloor\sqrt{n}\rfloor = m$ exactly when $m^2 \le n \le (m+1)^2-1$. That block contains $(m+1)^2-m^2 = 2m+1$ integers. For example, $m=1$ covers $n=1,2,3$ (three terms).

Step 2: where it ends. Since $44^2 = 1936$ and $45^2 = 2025$, the blocks $m = 1, 2, \ldots, 44$ fill $n = 1$ to 2024 exactly, and the last term $n = 2025$ contributes 45 on its own.

Step 3: add. The total is

$$\sum_{m=1}^{44} m(2m+1) + 45 = 2\sum_{m=1}^{44} m^2 + \sum_{m=1}^{44} m + 45.$$

Using $\sum m^2 = \frac{44\cdot 45\cdot 89}{6} = 29370$ and $\sum m = \frac{44\cdot 45}{2} = 990$:

$$2\cdot 29370 + 990 + 45 = 58740 + 990 + 45 = 59775.$$

The ones digit is 5.

Watch out. Forgetting the lone final term $\lfloor\sqrt{2025}\rfloor = 45$ gives 59730, whose ones digit 0 is not a choice.

Answer: (D) 5

6 Counting & Probability

Source: 2021 AMC 10B #18

A fair 6-sided die is repeatedly rolled until an odd number appears. What is the probability that every even number appears at least once before the first occurrence of an odd number?

(A) $\frac{1}{120}$ **(B)** $\frac{1}{32}$ **(C)** $\frac{1}{20}$ **(D)** $\frac{3}{20}$ **(E)** $\frac{1}{6}$

Key idea. Repeated values do not matter here — rolling a 4 twice is the same as rolling it once. So ignore repeats and watch only the *order in which new numbers first appear*. The question becomes: are the first three new numbers all even?

Step 1: only new numbers matter. Throw away every roll that repeats a number already seen. What is left is the list of numbers in the order they first appear. The event happens exactly when 2, 4, 6 all show up before any odd number, that is, when the first three *new* numbers are 2, 4, 6 in some order.

Step 2: each new number is equally likely to be any unseen face. When a new number appears, the die is fair, so it is equally likely to be any of the faces not yet seen.

- First new number: 3 of the 6 faces are even, probability $\frac{3}{6}$.
- Second new number: 2 of the 5 unseen faces are even, probability $\frac{2}{5}$.
- Third new number: 1 of the 4 unseen faces is even, probability $\frac{1}{4}$.

Step 3. Multiply:

$$\frac{3}{6}\cdot\frac{2}{5}\cdot\frac{1}{4}=\frac{3\cdot 2\cdot 1}{6\cdot 5\cdot 4}=\frac{6}{120}=\frac{1}{20}.$$

Another way. All $6! = 720$ orders of first appearance are equally likely. The good ones put the three evens first (3! ways) and the three odds after (3! ways), so the probability is $\frac{3!\cdot 3!}{6!}=\frac{36}{720}=\frac{1}{20}$.

Watch out. The evens may come in *any* order. Insisting on a specific order, such as 2 then 4 then 6, gives $\frac{1}{6}\cdot\frac{1}{5}\cdot\frac{1}{4}=\frac{1}{120}$, which is choice (A).

Answer: (C) $\frac{1}{20}$

7 Geometry

Source: 2015 AMC 10A #21

Tetrahedron $ABCD$ has $AB = 5$, $AC = 3$, $BC = 4$, $BD = 4$, $AD = 3$, and $CD = \frac{12}{5}\sqrt{2}$. What is the volume of the tetrahedron?

(A) $3\sqrt{2}$ **(B)** $2\sqrt{5}$ **(C)** $\frac{24}{5}$ **(D)** $3\sqrt{3}$ **(E)** $\frac{24}{5}\sqrt{2}$

Key idea. The numbers $3, 4, 5$ appear twice: triangles ABC and ABD are congruent 3-4-5 right triangles sharing the hypotenuse $\overline{AB}$. That suggests dropping the altitudes from C and D to $\overline{AB}$ — and the odd-looking length $CD = \frac{12}{5}\sqrt{2}$ is a hint that those two altitudes meet at a right angle.

Step 1. Since $3^2 + 4^2 = 5^2$, triangle ABC has a right angle at C. Its altitude from C meets $\overline{AB}$ at a point P with

$$CP = \frac{AC \cdot BC}{AB} = \frac{3 \cdot 4}{5} = \frac{12}{5}, \qquad AP = \frac{AC^2}{AB} = \frac{9}{5}.$$

(The first formula is twice the area computed two ways; the second comes from similar triangles $APC \sim ACB$.)

Step 2. Triangle ABD has the same side lengths ($AD = 3$, $BD = 4$), so its altitude from D also has length $\frac{12}{5}$ and lands at distance $\frac{9}{5}$ from A — the *same* point P. So $CP \perp AB$ and $DP \perp AB$ with $CP = DP = \frac{12}{5}$.

Step 3. Look at triangle CPD:

$$CP^2 + DP^2 = \frac{144}{25} + \frac{144}{25} = \frac{288}{25} = \left(\frac{12}{5}\sqrt{2}\right)^2 = CD^2.$$

By the converse of the Pythagorean theorem, $\angle CPD = 90°$.

Step 4. Now $\overline{CP}$ is perpendicular to both $\overline{AB}$ and $\overline{PD}$, two non-parallel lines in plane ABD. Hence $\overline{CP}$ is perpendicular to plane ABD, and $CP = \frac{12}{5}$ is the height of the tetrahedron over base ABD.

Step 5. The base is a right triangle with legs 3 and 4, so $[ABD] = \frac{1}{2} \cdot 3 \cdot 4 = 6$. Therefore

$$V = \frac{1}{3} \cdot [ABD] \cdot CP = \frac{1}{3} \cdot 6 \cdot \frac{12}{5} = \frac{24}{5}.$$

Watch out. The height must be perpendicular to the base plane. $\overline{CD}$ is not: using $CD = \frac{12}{5}\sqrt{2}$ as the height gives $\frac{1}{3} \cdot 6 \cdot \frac{12}{5}\sqrt{2} = \frac{24}{5}\sqrt{2}$, choice (E).

Another way. Use coordinates: $A = (0, 0, 0)$, $B = (5, 0, 0)$, $C = \left(\frac{9}{5}, \frac{12}{5}, 0\right)$ (check: $AC = 3$, $BC = 4$), and $D = (x, y, z)$. Subtracting $BD^2 = 16$ from $AD^2 = 9$ gives $x^2 - (x - 5)^2 = -7$, so $10x - 25 = -7$ and $x = \frac{9}{5}$. Then $y^2 + z^2 = 9 - \frac{81}{25} = \frac{144}{25}$, and

$$CD^2 = \left(y - \tfrac{12}{5}\right)^2 + z^2 = \tfrac{288}{25} - \tfrac{24}{5}y,$$

which equals $\frac{288}{25}$ only when $y = 0$. So $z = \pm\frac{12}{5}$: point D is $\frac{12}{5}$ above the plane of ABC, whose area is $\frac{1}{2} \cdot 3 \cdot 4 = 6$, and again $V = \frac{1}{3} \cdot 6 \cdot \frac{12}{5} = \frac{24}{5}$.

Answer: (C) $\frac{24}{5}$

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8 Algebra

Source: 2020 AMC 10A #22

For how many positive integers $n \leq 1000$ is

$$\left\lfloor \frac{998}{n} \right\rfloor + \left\lfloor \frac{999}{n} \right\rfloor + \left\lfloor \frac{1000}{n} \right\rfloor$$

not divisible by 3? (Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to x.)

(A) 22 **(B)** 23 **(C)** 24 **(D)** 25 **(E)** 26

Key idea. The three numerators are consecutive, so the three floors are almost always equal — and three equal numbers sum to a multiple of 3. The sum can only break when 998, 999, 1000 straddle a multiple of n, so focus on the remainder of 1000 when divided by n.

Step 1. Divide: $1000 = qn + r$ with $0 \leq r < n$, so $\left\lfloor \frac{1000}{n} \right\rfloor = q$.

Step 2. Remainder $r \geq 2$. Then $999 = qn + (r-1)$ and $998 = qn + (r-2)$ with both remainders between 0 and $n-1$. All three floors equal q, and the sum $3q$ *is* divisible by 3.

Step 3. Remainder $r = 1$ (so $n \geq 2$). Then $999 = qn$ gives floor q, while $998 = (q-1)n + (n-1)$ gives floor $q-1$. The sum is $3q - 1$: not divisible by 3.

Step 4. Remainder $r = 0$. Then $999 = (q-1)n + (n-1)$ gives $q-1$. If $n \geq 2$, $998 = (q-1)n + (n-2)$ also gives $q-1$, so the sum is $3q - 2$: not divisible by 3. But if $n = 1$ the sum is $998 + 999 + 1000 = 2997 = 3 \cdot 999$, which *is* divisible by 3.

Step 5. Count. We need the $n \geq 2$ that divide 1000 or divide 999 (these are the cases $r = 0$ and $r = 1$; no $n \geq 2$ divides both, since 1000 and 999 are consecutive).

- $1000 = 2^3 \cdot 5^3$ has $4 \cdot 4 = 16$ divisors; dropping $n = 1$ leaves 15.
- $999 = 3^3 \cdot 37$ has $4 \cdot 2 = 8$ divisors; dropping $n = 1$ leaves 7.

Total: $15 + 7 = 22$.

Watch out. Forgetting that $n = 1$ fails: counting all $16 + 8 = 24$ divisors gives (C), and counting $n = 1$ once gives 23, choice (B).

Answer: (A) 22

9 Number Theory

Source: 2008 AMC 10B #23

A rectangular floor measures a feet by b feet, where a and b are positive integers with $b > a$. An artist paints a rectangle on the floor with the sides of the rectangle parallel to the sides of the floor. The unpainted part of the floor forms a border of width 1 foot around the painted rectangle and occupies half the area of the entire floor. How many possibilities are there for the ordered pair (a, b)?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

Key idea. The border has width 1, so the painted rectangle is $(a-2) \times (b-2)$. "Half the floor" gives an equation in a and b with an ab term; the move for such equations is to factor them as (something)×(something) = constant, the way one completes a square.

Step 1. A border of width 1 foot trims 1 foot from each side, so the painted rectangle measures $(a-2)$ by $(b-2)$ feet (and we need $a > 2$). The unpainted border is half the floor, so the painted part is the other half:

$$(a-2)(b-2) = \frac{ab}{2} \quad \Longrightarrow \quad 2(ab - 2a - 2b + 4) = ab.$$

Step 2. Expanding and collecting terms: $2ab - 4a - 4b + 8 = ab$, so $ab - 4a - 4b + 8 = 0$. Adding 8 to both sides makes the left side factor:

$$ab - 4a - 4b + 16 = 8 \quad \Longrightarrow \quad (a-4)(b-4) = 8.$$

Step 3. If $a - 4$ were negative, it would be -1 (since $a \geq 3$), and then $b - 4 = -8$, so $b = -4$, which is impossible. And $a = 4$ gives $0 = 8$. So $a - 4$ and $b - 4$ are positive integers with product 8 and $a - 4 < b - 4$ (because $b > a$):

$$1 \cdot 8 \;\Rightarrow\; (a, b) = (5, 12), \qquad 2 \cdot 4 \;\Rightarrow\; (a, b) = (6, 8).$$

Check: $3 \cdot 10 = 30 = \frac{60}{2}$ and $4 \cdot 6 = 24 = \frac{48}{2}$.

So there are 2 possibilities.

Watch out. The condition $b > a$ matters. Also counting $(8, 6)$ and $(12, 5)$ gives 4, choice (D).

Answer: (B) 2

10 Counting & Probability

Source: 2020 AMC 10B #23

Square $ABCD$ in the coordinate plane has vertices at the points $A(1,1)$, $B(-1,1)$, $C(-1,-1)$, and $D(1,-1)$. Consider the following four transformations:

- L, a rotation of 90° counterclockwise around the origin;
- R, a rotation of 90° clockwise around the origin;
- H, a reflection across the x-axis; and
- V, a reflection across the y-axis.

Each of these transformations maps the square onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1,1)$ to $(-1,-1)$ and would send the vertex B at $(-1,1)$ to itself. How many sequences of 20 transformations chosen from $\{L,R,H,V\}$ will send all of the labeled vertices back to their original positions? (For example, R,R,V,H is one sequence of 4 transformations that will send the vertices back to their original positions.)

(A) 2^{37} **(B)** $3\cdot 2^{36}$ **(C)** 2^{38} **(D)** $3\cdot 2^{37}$ **(E)** 2^{39}

Key idea. Counting 4^{20} sequences directly is hopeless, so look for a reason the last move is *forced*. All four moves send every vertex to a neighbouring corner, which suggests colouring the corners like a checkerboard and tracking parity.

Step 1: a parity colouring. Colour the corners $(1,1)$ and $(-1,-1)$ black, and $(-1,1)$ and $(1,-1)$ white. Each of L,R,H,V moves every vertex to an adjacent corner, which has the other colour. For example, L sends $(1,1)$ to $(-1,1)$ and H sends $(1,1)$ to $(1,-1)$.

So after an odd number of moves, every labelled vertex sits on a corner of the opposite colour to where it started, which in a square means an adjacent corner.

Step 2: where we are after 19 moves. Take any first 19 moves. Composing symmetries of the square gives a symmetry of the square, call it g, and by Step 1, g sends every vertex to an adjacent corner. The square has eight symmetries: the identity, L, R, the half-turn, H, V, and the reflections in the two diagonals. The identity fixes every vertex, the half-turn sends each vertex to the opposite corner, and each diagonal reflection fixes two vertices. So g is one of L, R, H, V.

Step 3: the last move is forced. The 20th move t must undo g, that is t must be the inverse of g. The inverses are $L \leftrightarrow R$, $H \leftrightarrow H$, $V \leftrightarrow V$, all in our list. So for each choice of the first 19 moves there is exactly one good 20th move.

Step 4: count. The first 19 moves can be chosen freely in 4^{19} ways, so the number of sequences is

$$4^{19} = (2^2)^{19} = 2^{38}.$$

Check with the example: R,R is a half-turn, then V makes $g = H$ after 3 moves, and indeed the forced 4th move is H.

Watch out. Assuming the 4^{20} sequences are spread evenly over all 8 symmetries gives $\frac{4^{20}}{8} = 2^{37}$, choice (A). They are not: after an even number of moves only the 4 "even" positions (identity, half-turn, two diagonal reflections) are possible, so the right fraction is $\frac{1}{4}$.

Answer: (C) 2^{38}