AMC 10

SET 28

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • UPPER BAND (PROBLEMS 13--23)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 13–23 of past AMC 10 contests (2000–2025) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 13 to 23 of an AMC 10 — the upper band of the paper, where strong scores are separated from good ones. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

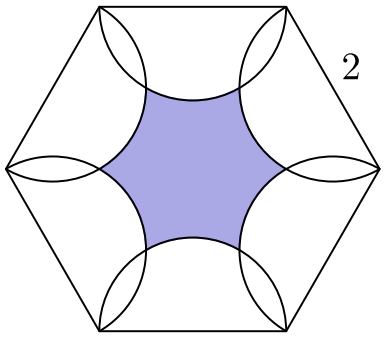
Pacing guide

This band is harder than the time allows comfortably: aim for problems 1–5 in about 12 minutes and give problems 6–10 the remaining 18. Skipping one hard problem to finish the rest cleanly is a winning strategy. If a problem has taken three minutes with no plan in sight, move on and come back.

1. Positive integers x and y satisfy the equation $\sqrt{x} + \sqrt{y} = \sqrt{1183}$. What is the minimum possible value of $x + y$?

(A) 585 **(B)** 595 **(C)** 623 **(D)** 700 **(E)** 791

2. As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region—inside the hexagon but outside all of the semicircles?

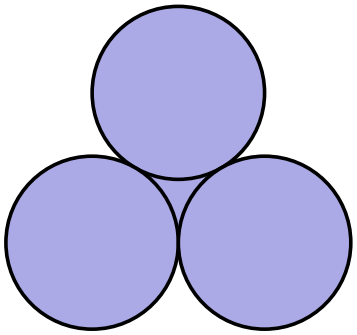


(A) $6\sqrt{3} - 3\pi$ **(B)** $\frac{9\sqrt{3}}{2} - 2\pi$ **(C)** $\frac{3\sqrt{3}}{2} - \frac{\pi}{3}$ **(D)** $3\sqrt{3} - \pi$ **(E)** $\frac{9\sqrt{3}}{2} - \pi$

3. Five balls are arranged around a circle. Chris chooses two adjacent balls at random and interchanges them. Then Silva does the same, with her choice of adjacent balls to interchange being independent of Chris's. What is the expected number of balls that occupy their original positions after these two successive transpositions?

(A) 1.6 **(B)** 1.8 **(C)** 2.0 **(D)** 2.2 **(E)** 2.4

4. Three circles with radius 2 are mutually tangent. What is the total area of the circles and the region bounded by them, as shown in the figure?



(A) $10\pi + 4\sqrt{3}$ **(B)** $13\pi - \sqrt{3}$ **(C)** $12\pi + \sqrt{3}$ **(D)** $10\pi + 9$ **(E)** 13π

5. Define

$$P(x) = (x - 1^2)(x - 2^2) \cdots (x - 100^2).$$

How many integers n are there such that $P(n) \leq 0$?

(A) 4900 **(B)** 4950 **(C)** 5000 **(D)** 5050 **(E)** 5100

6. The line segment formed by $A(1, 2)$ and $B(3, 3)$ is rotated to the line segment formed by $A'(3, 1)$ and $B'(4, 3)$ about the point $P(r, s)$. What is $|r - s|$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3}{4}$ **(D)** $\frac{2}{3}$ **(E)** 1

7. In how many ways can the sequence $1, 2, 3, 4, 5$ be rearranged so that no three consecutive terms are increasing and no three consecutive terms are decreasing?

(A) 10 **(B)** 18 **(C)** 24 **(D)** 32 **(E)** 44

8. Mary chose an even 4-digit number n. She wrote down all the divisors of n in increasing order from left to right: $1, 2, \ldots, \frac{n}{2}, n$. At some moment Mary wrote 323 as a divisor of n. What is the smallest possible value of the next divisor written to the right of 323?

(A) 324 **(B)** 330 **(C)** 340 **(D)** 361 **(E)** 646

9. Integers a, b, and c satisfy

$$\begin{aligned} ab + c &= 100, \\ bc + a &= 87, \\ ca + b &= 60. \end{aligned}$$

What is $ab + bc + ca$?

(A) 212 **(B)** 247 **(C)** 258 **(D)** 276 **(E)** 284

10. Let

$$N = 123456789101112 \ldots 4344$$

be the 79-digit number that is formed by writing the integers from 1 to 44 in order, one after the other. What is the remainder when N is divided by 45?

(A) 1 **(B)** 4 **(C)** 9 **(D)** 18 **(E)** 44

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60