AMC 10

SET 29

HALF-LENGTH PRACTICE

TIME ALLOWED

30:00

10 QUESTIONS • 30 MINUTES • 60 POINTS • UPPER BAND (PROBLEMS 13--23)

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 10 problems come from positions 13–23 of past AMC 10 contests (2000–2025) and appear in their original wording, in order of their original position. Problems © Mathematical Association of America.

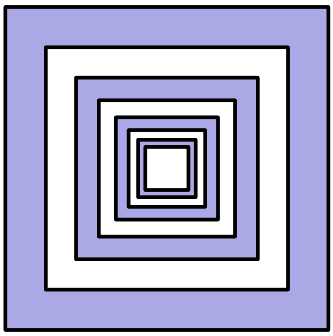
INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 10-question multiple-choice test. Every problem was originally problem 13 to 23 of an AMC 10 — the upper band of the paper, where strong scores are separated from good ones. They appear in order of their original position, so the difficulty rises from the first problem to the last.
3. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct. Mark your answers on the answer sheet at the back of this booklet.
4. **Scoring:** 6 points for each correct answer, 1.5 points for each problem left unanswered, 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **30 minutes**. Note your start time on the cover and stop the moment the time is up.

Pacing guide

This band is harder than the time allows comfortably: aim for problems 1–5 in about 12 minutes and give problems 6–10 the remaining 18. Skipping one hard problem to finish the rest cleanly is a winning strategy. If a problem has taken three minutes with no plan in sight, move on and come back.

1. In the figure below, the outside square contains infinitely many squares, each of them with the same center and sides parallel to the outside square. The ratio of the side length of a square to the side length of the next inner square is k, where $0 < k < 1$. The spaces between squares are alternately shaded, as shown in the figure (which is not necessarily drawn to scale).



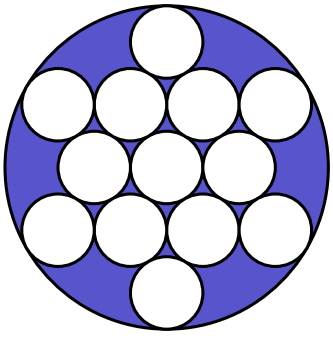
The area of the shaded portion of the figure is 64% of the area of the original square. What is k?

(A) $\frac{3}{5}$ **(B)** $\frac{16}{25}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{4}{5}$

2. Una rolls 6 standard 6-sided dice simultaneously and calculates the product of the 6 numbers obtained. What is the probability that the product is divisible by 4?

(A) $\frac{3}{4}$ **(B)** $\frac{57}{64}$ **(C)** $\frac{59}{64}$ **(D)** $\frac{187}{192}$ **(E)** $\frac{63}{64}$

3. The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1?



(A) $4\pi\sqrt{3}$ **(B)** 7π **(C)** $\pi\left(3\sqrt{3}+2\right)$

(D) $10\pi\left(\sqrt{3}-1\right)$ **(E)** $\pi\left(\sqrt{3}+6\right)$

4. In the following list of numbers, the integer n appears n times in the list for $1 \leq n \leq 200$.

$$1, 2, 2, 3, 3, 3, 4, 4, 4, 4, \ldots, 200, 200, \ldots, 200$$

What is the median of the numbers in this list?

(A) 100.5 **(B)** 134 **(C)** 142 **(D)** 150.5 **(E)** 167

5. Let T_k be the transformation of the coordinate plane that first rotates the plane k degrees counterclockwise around the origin and then reflects the plane across the y-axis. What is the least positive integer n such that performing the sequence of transformations $T_1, T_2, T_3, \cdots, T_n$ returns the point $(1, 0)$ back to itself?

(A) 359 **(B)** 360 **(C)** 719 **(D)** 720 **(E)** 721

6. In a certain card game, a player is dealt a hand of 10 cards from a deck of 52 distinct cards. The number of distinct (unordered) hands that can be dealt to the player can be written as $158A00A4AA0$. What is the digit A?

(A) 2 **(B)** 3 **(C)** 4 **(D)** 6 **(E)** 7

7. Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of $ABCD$?

(A) $10\sqrt{6}$ **(B)** 25 **(C)** $8\sqrt{10}$ **(D)** $18\sqrt{2}$ **(E)** 26

8. A seven-digit positive integer is chosen at random. What is the probability that the number is divisible by 11, given that the sum of its digits is 61?

(A) $\frac{3}{14}$ **(B)** $\frac{3}{11}$ **(C)** $\frac{2}{7}$ **(D)** $\frac{4}{11}$ **(E)** $\frac{3}{7}$

9. How many distinct values of x satisfy

$$\lfloor x \rfloor^2 - 3x + 2 = 0,$$

where $\lfloor x \rfloor$ denotes the largest integer less than or equal to x?

(A) an infinite number **(B)** 4 **(C)** 2

(D) 3 **(E)** 0

10. The entries in a 3×3 array include all the digits from 1 through 9, arranged so that the entries in every row and column are in increasing order. How many such arrays are there?

(A) 18 **(B)** 24 **(C)** 36 **(D)** 42 **(E)** 60

END OF EXAMINATION

ANSWER SHEET

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Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**6**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**7**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**8**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**9**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**10**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 60