AMC 10

SET 10

PRACTICE EXAMINATION

TIME ALLOWED

75:00

25 QUESTIONS • 75 MINUTES • 150 POINTS • ADVANCED LEVEL

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD TO BEGIN.

Name _____ Date _____ Start _____ Finish _____

All 25 problems are taken from past AMC 10 contests (2000–2025) and appear in their original wording. Advanced level: each problem sits two or three places earlier than it did in its original contest, and problems 21–25 come from original positions 23–25. Problems © Mathematical Association of America.

INSTRUCTIONS

1. Do not open this booklet until you are told to begin.
2. This is a 25-question multiple-choice test. Each question is followed by answers marked (A), (B), (C), (D) and (E). Only one of these is correct.
3. Mark your answer to each problem on the answer sheet at the back of this booklet. Fill in one circle completely for each answer.
4. **Scoring:** you will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer. A random guess is worth less, on average, than a blank.
5. Only scratch paper, graph paper, rulers, compasses, protractors and erasers are allowed as aids. No calculators, smartwatches, phones or computing devices.
6. Figures are not necessarily drawn to scale.
7. You will have **75 minutes** to complete the test. Note your start time on the cover and stop the moment the time is up.

Pacing guide

Problems 1–10: aim for about 20 minutes. Problems 11–20: about 30 minutes. Problems 21–25: the remaining 25 minutes. If a problem has taken three minutes with no plan in sight, move on and come back.

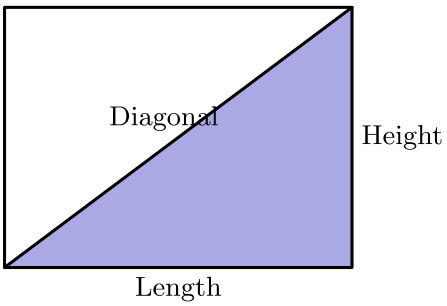
1. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

(A) 20 **(B)** 21 **(C)** 22 **(D)** 23 **(E)** 24

2. Brianna is using part of the money she earned on her weekend job to buy several equally-priced CDs. She used one fifth of her money to buy one third of the CDs. What fraction of her money will she have left after she buys all the CDs?

(A) $\frac{1}{5}$ **(B)** $\frac{1}{3}$ **(C)** $\frac{2}{5}$ **(D)** $\frac{2}{3}$ **(E)** $\frac{4}{5}$

3. Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is $4 : 3$. The horizontal length of a "27-inch" television screen is closest, in inches, to which of the following?



(A) 20 **(B)** 20.5 **(C)** 21 **(D)** 21.5 **(E)** 22

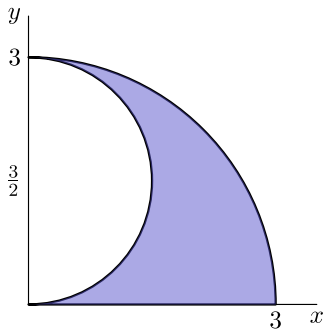
4. On a trip from the United States to Canada, Isabella took d U.S. dollars. At the border she exchanged them all, receiving 10 Canadian dollars for every 7 U.S. dollars. After spending 60 Canadian dollars, she had d Canadian dollars left. What is the sum of the digits of d?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

5. A parabola with equation $y = x^2 + bx + c$ passes through the points $(2, 3)$ and $(4, 3)$. What is c?

(A) 2 **(B)** 5 **(C)** 7 **(D)** 10 **(E)** 11

6. The shaded region below is called a shark's fin falcata, a figure studied by Leonardo da Vinci. It is bounded by the portion of the circle of radius 3 and center $(0,0)$ that lies in the first quadrant, the portion of the circle with radius $\frac{3}{2}$ and center $(0,\frac{3}{2})$ that lies in the first quadrant, and the line segment from $(0,0)$ to $(3,0)$. What is the area of the shark's fin falcata?



(A) $\frac{4\pi}{5}$ **(B)** $\frac{9\pi}{8}$ **(C)** $\frac{4\pi}{3}$ **(D)** $\frac{7\pi}{5}$ **(E)** $\frac{3\pi}{2}$

7. A radio program has a quiz consisting of 3 multiple-choice questions, each with 3 choices. A contestant wins if he or she gets 2 or more of the questions right. The contestant answers randomly to each question. What is the probability of winning?

(A) $\frac{1}{27}$ **(B)** $\frac{1}{9}$ **(C)** $\frac{2}{9}$ **(D)** $\frac{7}{27}$ **(E)** $\frac{1}{2}$

8. For which of the following integers b is the base-b number $2021_b - 221_b$ not divisible by 3?

(A) 3 **(B)** 4 **(C)** 6 **(D)** 7 **(E)** 8

9. Real numbers x and y satisfy the equation

$$x^2 + y^2 = 10x - 6y - 34.$$

What is $x + y$?

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

10. In a collection of red, blue, and green marbles, there are 25% more red marbles than blue marbles, and there are 60% more green marbles than red marbles. Suppose that there are r red marbles. What is the total number of marbles in the collection?

(A) $2.85r$ **(B)** $3r$ **(C)** $3.4r$ **(D)** $3.85r$ **(E)** $4.25r$

11. Nine athletes, no two of whom are the same height, try out for the basketball team. One at a time, they draw a wristband at random, without replacement, from a bag containing 3 blue bands, 3 red bands, and 3 green bands. They are divided into a blue group, a red group, and a green group. The tallest member of each group is named the group captain. What is the probability that the group captains are the three tallest athletes?

(A) $\frac{2}{9}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{9}{28}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{3}{8}$

12. How many ordered pairs (x, y) of real numbers satisfy the following system of equations?

$$x^2 + 3y = 9,$$
$$(|x| + |y| - 4)^2 = 1.$$

(A) 1 **(B)** 2 **(C)** 3 **(D)** 5 **(E)** 7

13. Steve wrote the digits 1, 2, 3, 4, and 5 in order repeatedly from left to right, forming a list of 10,000 digits, beginning $123451234512\ldots$ He then erased every third digit from his list (that is, the 3rd, 6th, 9th, ... digits from the left), then erased every fourth digit from the resulting list (that is, the 4th, 8th, 12th, ... digits from the left in what remained), and then erased every fifth digit from what remained at that point. What is the sum of the three digits that were then in positions 2019, 2020, and 2021?

(A) 7 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

14. Two fair coins are to be tossed once. For each head that results, one fair die is to be rolled. What is the probability that the sum of the die rolls is odd? (Note that if no die is rolled, the sum is 0.)

(A) $\frac{3}{8}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{43}{72}$ **(D)** $\frac{5}{8}$ **(E)** $\frac{2}{3}$

15. Rectangle $ABCD$ has $AB = 8$ and $BC = 6$. Point M is the midpoint of diagonal $\overline{AC}$, and E is on $\overline{AB}$ with $\overline{ME} \perp \overline{AC}$. What is the area of $\triangle AME$?

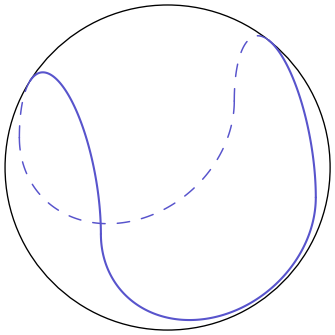
(A) $\frac{65}{8}$ **(B)** $\frac{25}{3}$ **(C)** 9 **(D)** $\frac{75}{8}$ **(E)** $\frac{85}{8}$

16. In the following table, each question mark is to be replaced by "Possible" or "Not Possible" to indicate whether a nonvertical line with the given slope can contain the given number of lattice points (points both of whose coordinates are integers). How many of the 12 entries will be "Possible"?

	zero	exactly one	exactly two	more than two
zero slope	?	?	?	?
nonzero rational slope	?	?	?	?
irrational slope	?	?	?	?

(A) 4 **(B)** 5 **(C)** 6 **(D)** 7 **(E)** 9

17. Four congruent semicircles are drawn on the surface of a sphere with radius 2, as shown, creating a closed curve that divides the surface into two congruent regions. The length of the curve is $\pi\sqrt{n}$. What is n?



(A) 32 **(B)** 12 **(C)** 48 **(D)** 36 **(E)** 27

18. A unit square is rotated $45°$ about its center. What is the area of the region swept out by the interior of the square?

(A) $1 - \frac{\sqrt{2}}{2} + \frac{\pi}{4}$ **(B)** $\frac{1}{2} + \frac{\pi}{4}$ **(C)** $2 - \sqrt{2} + \frac{\pi}{4}$

(D) $\frac{\sqrt{2}}{2} + \frac{\pi}{4}$ **(E)** $1 + \frac{\sqrt{2}}{4} + \frac{\pi}{8}$

19. Let a, b, c, and d be positive integers such that

$$\gcd(a,b) = 24,$$
$$\gcd(b,c) = 36,$$
$$\gcd(c,d) = 54,$$

and

$$70 < \gcd(d,a) < 100.$$

Which of the following must be a divisor of a?

(A) 5 **(B)** 7 **(C)** 11 **(D)** 13 **(E)** 17

20. Each vertex of convex pentagon $ABCDE$ is to be assigned a color. There are 6 colors to choose from, and the ends of each diagonal must have different colors. How many different colorings are possible?

(A) 2520 **(B)** 2880 **(C)** 3120 **(D)** 3250 **(E)** 3750

21. What is the hundreds digit of 2011^{2011}?

(A) 1 **(B)** 4 **(C)** 5 **(D)** 6 **(E)** 9

22. How many strings of length 5 formed from the digits $0, 1, 2, 3, 4$, are there such that for each $j \in \{1, 2, 3, 4\}$, at least j of the digits are less than j?

(For example, 02214 satisfies this condition because it contains at least 1 digit less than 1, at least 2 digits less than 2, at least 3 digits less than 3, and at least 4 digits less than 4. The string 23404 does not satisfy the condition because it does not contain at least 2 digits less than 2.)

(A) 500 **(B)** 625 **(C)** 1089 **(D)** 1199 **(E)** 1296

23. Let a, b, and c be positive integers with $a \geq b \geq c$ such that

$$a^2 - b^2 - c^2 + ab = 2011$$

and

$$a^2 + 3b^2 + 3c^2 - 3ab - 2ac - 2bc = -1997.$$

What is a?

(A) 249 **(B)** 250 **(C)** 251 **(D)** 252 **(E)** 253

24. How many ordered triples (x, y, z) of positive integers satisfy

$$\begin{aligned} \operatorname{lcm}(x, y) &= 72, \\ \operatorname{lcm}(x, z) &= 600, \end{aligned}$$

and

$$\operatorname{lcm}(y, z) = 900?$$

(A) 15 **(B)** 16 **(C)** 24 **(D)** 27 **(E)** 64

25. How many sequences of 0s and 1s of length 19 are there that begin with a 0, end with a 0, contain no two consecutive 0s, and contain no three consecutive 1s?

(A) 55 **(B)** 60 **(C)** 65 **(D)** 70 **(E)** 75

END OF EXAMINATION

ANSWER SHEET

AMC 10 • PRACTICE EXAMINATION • SET 10

Name _____ Date _____

1	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**14**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
2	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**15**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
3	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**16**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
4	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**17**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
5	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**18**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
6	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**19**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
7	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**20**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
8	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**21**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
9	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**22**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
10	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**23**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
11	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**24**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
12	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ	**25**	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ
13	Ⓐ	Ⓑ	Ⓒ	Ⓓ	Ⓔ										

Fill in one circle per question. Leave the row empty to skip.

Correct	Blank	Wrong	Score
$\times 6$	$\times 1.5$	$\times 0$	/ 150